

An Eigen-Approach to the Design of Near-Optimum Time Domain Equalizer for DMT Transceivers

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Abstract— Time domain equalizer (TEQ) is used in the discrete multitone (DMT) transceivers in order to reduce the duration of the overall response of the transmission system, so that a shorter length cyclic prefix could be used. The optimum TEQ is the one that results in maximum bit allocation to each block of DMT. Al-Dhahir and Cioffi [3] have proposed an optimization scheme which can achieve this goal under certain conditions. This scheme is rather involved and thus its implementation in an on-line system is not feasible. In this paper, we present a novel approach which results in TEQ designs with comparable performance to those of [3], but at a very affordable computational cost.

Keywords— Equalizers, Data Communications, Subscriber loops, Discrete multitone

I. INTRODUCTION

The discrete multitone (DMT) has attracted considerable attention as a practical and viable technology for high-speed data transmission over spectrally shaped noisy channels [1]. Modems employing this technology are already available in the market. The DMT based modems have, in particular, been found very useful in transmitting high-speed data over digital subscriber lines (DSL). In DMT, channel distortion is taken care of by cyclically extending the output of the inverse FFT (IFFT) modulator so that the input sequence looks periodic to the channel. This is referred to as *cyclic prefix* method [1] (see Fig. 1). The length of the cyclic prefix should be at least equal to the duration of the channel impulse response minus one. However, we note that the addition of the cyclic prefix reduces the throughput of the channel as it carries redundant data. To minimize this reduction of the throughput, a channel equalizer whose goal is to reduce the overall duration of the system (channel plus equalizer) impulse response to a predefined length is used. In the DMT literature, this type of equalizers are called time-domain equalizer (TEQ).

The ultimate goal in the design of the TEQ is to achieve maximum bit-rate over the channel. However, development of a practical design method which can achieve this goal turns out to be very difficult. Most of the studies in TEQ design have set the goal of mean-square error (MSE) minimization which means the shorten impulse response (known as target impulse response - TIR) and TEQ are jointly optimized so that the difference between the outputs of the TEQ and TIR is minimized in the MSE sense. This leads to an analytically tractable problem with a unique close-form solution. In the present literature, the only attempt that has been made in the design of TEQ with the

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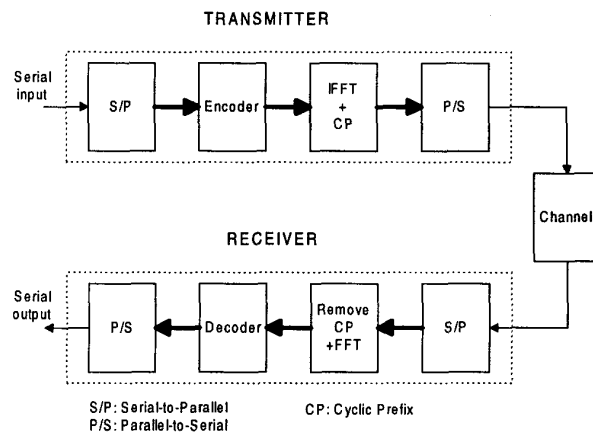


Fig. 1. Block diagram of a DMT transceiver

goal of bit-rate maximization is the work of Al-Dhahir and Cioffi [3], [4], where the authors have proposed a rather complicated optimization procedure. Convergence of this procedure to the corresponding optimum global solution is not guaranteed. However, numerical examples of actual digital subscriber line (DSL) channels have shown that the results obtained are superior to those of the MMSE TEQ.

In this paper, we consider a novel approach to the design of TEQ for DMT transceivers. Through a numerical example, we first explore the problem of TEQ design and find the general guidelines that one should follow in order to arrive at a near-optimum design for TEQ. We show that the solution suggested by Al-Dhahir and Cioffi [3], [4] may be viewed as an attempt to satisfy these guidelines in a systematic way. We then propose an eigen-approach to the design of TEQ and through numerical examples demonstrate that this results in TEQ designs which are comparable to those of [3] and [4], however, they are obtained with a much lower computational cost.

II. MMSE TEQ

As was noted earlier, in the MMSE TEQ, the criterion used for the design is the MMSE at the equalizer output. The solution to this problem is well understood [2]. Let \mathbf{c} denote the length L_s column vector of the samples of the TIR. To prevent the trivial solution of $\mathbf{c} = \mathbf{0}$, the unit-norm constraint is commonly applied to the TIR. That is, the variation of \mathbf{c} is limited by applying the constraint

$\mathbf{c}^T \mathbf{c} = 1$, where the superscript T denotes transposition.¹ The optimum TIR, \mathbf{c}_{MMSE} , which results in the MMSE is then obtained according to the following optimization procedure:

$$\mathbf{c}_{MMSE} = \arg \min \{ \mathbf{c}^T \mathbf{R}_\Delta \mathbf{c} \} \quad (1)$$

subject to the constraint

$$\mathbf{c}_{MMSE}^T \mathbf{c}_{MMSE} = 1. \quad (2)$$

Here,

$$\mathbf{R}_\Delta = \mathbf{I}_{L_s} - \mathbf{H}_\Delta^T \mathbf{R}^{-1} \mathbf{H}_\Delta \quad (3)$$

where \mathbf{R} is $N \times N$ correlation matrix of the TEQ input, $\mathbf{H}_\Delta = \mathbf{H} \times [\mathbf{0}_{L_s \times \Delta} \quad \mathbf{I}_{L_s} \quad \mathbf{0}_{L_s \times L_r}]^T$ is an $N \times L_s$ matrix,

$$\mathbf{H} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{L-1} & 0 & \cdots & 0 \\ 0 & h_0 & h_1 & \cdots & h_{L-1} & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_{L-1} \end{bmatrix} \quad (4)$$

is an $N \times (N + L - 1)$ matrix, $\mathbf{0}_{m \times n}$ is the $m \times n$ null matrix, \mathbf{I}_m is the identity matrix of size m , Δ is the equalizer delay, and $L_r = N + L - \Delta - L_s - 1$. The solution to the above optimization is well understood. The optimum solution, \mathbf{c}_{MMSE} , is the eigenvector that corresponds to the minimum eigenvalue of the matrix \mathbf{R}_Δ .

Once the optimum TIR is obtained, the TEQ coefficients can be calculated by solving the corresponding Wiener-Hopf equation or using an adaptive algorithm. Also, for our later reference, we note that for a given TIR, \mathbf{c} , the MSE at the TIR output is given by

$$\xi = \mathbf{c}^T \mathbf{R}_\Delta \mathbf{c}. \quad (5)$$

III. AN EXAMPLE OF MMSE TEQ

In this section, we demonstrate the short coming of the MMSE TEQ, through a numerical example. We consider loop 4 of the 8 carrier-serving-area (CSA) loops whose configuration are presented in Fig. 5 and will be used for an extensive study/comparison of various TEQ design methods in Section VII. Figs. 2 and 3 present the magnitude responses of this loop and the corresponding MMSE TEQ, respectively. From these plots, we note that the MMSE TEQ has a very narrow band. It emphasizes only on a portion of the spectrum of the received signal which has the highest power. This observation may be interpreted as follows:

- Subject to the TIR duration and unit-length constraint, the MMSE TEQ does its best to pick-up as much as possible of the less noisy part of the received signal, while suppressing the channel noise over the rest of the band as much as possible.

Although this maximizes the signal-to-noise ratio (SNR) of the received signal at the TEQ output and thus achieves

¹In this paper, with the exception of the matrix \mathbf{G}_i which is introduced in Section 5, we assume that all vectors and matrices are real-valued. When the involved vectors/matrices are complex-valued all transpositions are replaced by Hermitian.

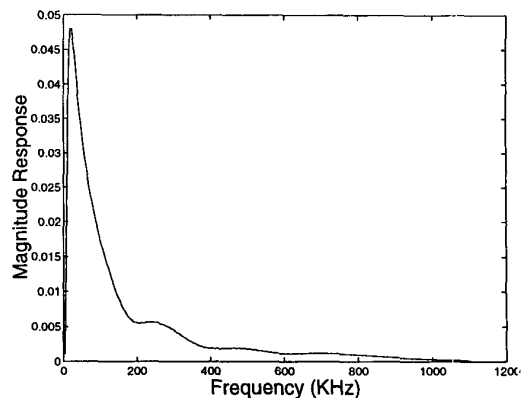


Fig. 2. Amplitude response of loop4

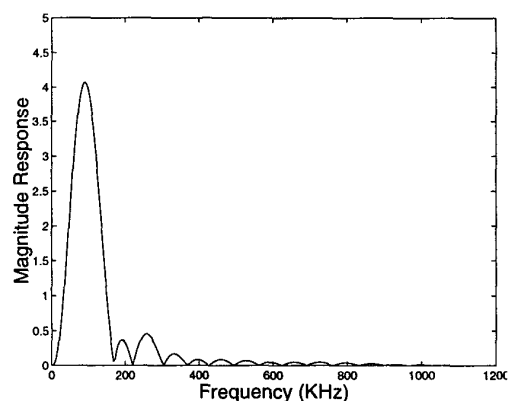


Fig. 3. Magnitude response of MMSE TEQ for loop4

the goal of the more conventional communication schemes, it does not necessarily results in the highest transmission bit-rate in the DMT transceivers. To achieve the maximum bit-rate in a DMT transceiver, one should strike a balance in the profile of the SNRs of all subcarriers after demodulation, so that a reasonably good number of bits can be allocated to most of the subcarriers, thus, overall, a large number of bits can be allocated to each block of DMT. As one may see by direct inspection of Fig. 3, the fact that the MMSE TEQ suppresses the received signal over most of its spectral band may result in a significant loss of information by lowering SNR in most of the subcarriers and thus an overall reduction in the total bits that could be allocated to each block of DMT. In particular, the subcarriers which may coincide with the nulls of the equalizer response will suffer a significant loss in their SNRs and thus the number of bits allocated to them.

To see this phenomenon clearly, Fig. 4 presents the bit allocation profile of the DMT transceiver for the above example. As was just predicted, we note that all subcarriers that coincide with the nulls of TEQ or are near the nulls have lower number of bits allocated to them. However, the subcarriers which are located near the middle of the side-lobes of the MMSE TEQ response still can support relatively fair numbers of bits, even though the peaks

of these side-lobes are about an order of magnitude lower than the peak of the main-lobe of the response. This observation which in first glance may seem contradicting our earlier arguments needs some explanation.

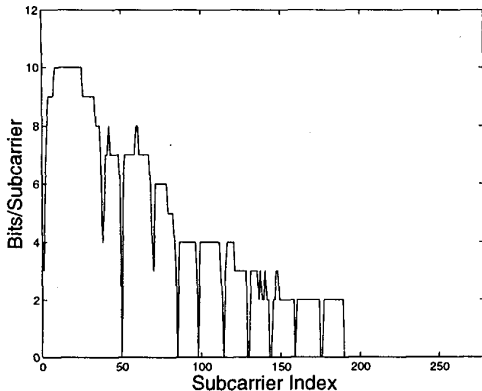


Fig. 4. Bit allocation after MMSE TEQ in loop4

Over each subcarrier band, the TEQ attenuate both the noise free part of the received (DMT) signal and the channel noise equally. As a result, if in the demodulation process one could use a bank of ideal narrow-band filters to separate different subcarriers, the receiver performance would be independent of the equalizer response. However, in a DMT system we use a DFT filter bank to separate the subcarriers. On the other hand, we recall that the DFT filters have relatively large side-lobes. As a result, there will be some leakage of noise energy from other bands to the main bands of the DFT filters. Those subcarrier whose amplitude have been significantly attenuated (like those near the nulls of the TEQ), can thus be greatly affected by this noise leakage process. However, the effect of noise leakage to those bands whose amplitude attenuation is moderate, like those near the middle of the first or second side-lobes of TEQ, is insignificant.

IV. DESIGN GUIDELINES FOR NEAR-OPTIMUM TEQ DESIGN

From our discussion in the last section, we may conclude that to enhance the performance of the DMT transceivers, we shall choose a TEQ whose gain does not experience any null over the useful portion of the received signal band. We also note that the TEQ response is directly related to the TIR, \mathbf{c} . In particular, the nulls in the MMSE TEQ which were observed in Fig. 3 and found to be undesirable, arise because of their presence in the amplitude response of the selected TIR, \mathbf{c}_{MMSE} . On the other hand, the MSE at the TEQ output varies with \mathbf{c} (see (5)). With this understanding of the problem, we may now propose the following general guidelines for the design of TEQ in the DMT transceivers:

- The amplitude response associated with the TIR, \mathbf{c} , shall not have any null.
- At the same time, \mathbf{c} should be chosen so that the MSE given by (5) remains relatively low.

The guidelines set above are relatively vague and seem to be hard to quantify. However, as we proceed in the rest of this paper, we find that the DMT transceivers are very robust to variations of the TIR, thus, many designs could be provided based on the above guidelines with all performing about the same.

V. AL-DHAHIR AND CIOFFI'S SOLUTION

In this section, after a short review of the method of Al-Dhahir and Cioffi [3], we give an interpretation of that and show how it is related to the guidelines set in the last section.

To optimize the TIR, \mathbf{c} , Al-Dhahir and Cioffi [3] have proposed the following nonlinear optimization procedure:

$$\mathbf{c}_{opt} = \arg \max \left\{ \sum_i \ln(\mathbf{c}^T \mathbf{G}_i \mathbf{c}) \right\} \quad (6)$$

subject to the constraints

$$\mathbf{c}_{opt}^T \mathbf{c}_{opt} = 1 \quad (7)$$

and

$$\mathbf{c}_{opt}^T \mathbf{R}_\Delta \mathbf{c}_{opt} \leq \xi_{max} \quad (8)$$

where ξ_{max} is the maximum acceptable MSE at the TEQ output, and \mathbf{G}_i is an $L_s \times L_s$ matrix defined as

$$\mathbf{G}_i = \begin{bmatrix} 1 & e^{j2\pi i/N} & \dots & e^{j2\pi i(L_s-1)/N} \\ e^{-j2\pi i/N} & 1 & e^{j2\pi i/N} & \dots \\ \vdots & \ddots & \ddots & \vdots \\ e^{-j2\pi i(L_s-1)/N} & \dots & e^{-j2\pi i/N} & 1 \end{bmatrix} \quad (9)$$

This problem can be solved using an appropriate numerical software package. In [3] and [4] and also for the results presented in this paper the MATLAB optimization toolbox is used.

The above procedure suggests joint maximization of the terms $\ln(\mathbf{c}^T \mathbf{G}_i \mathbf{c})$, for $i = 1, 2, 3, \dots$, subject to the constraints (7) and (8). On the other hand, we note that the term $\mathbf{c}^T \mathbf{G}_i \mathbf{c}$ can be interpreted as the square of the gain of an FIR filter with tap-weight vector \mathbf{c} when its input is the complex sinusoid $x_i(n) = e^{j2\pi i n/N}$, since \mathbf{G}_i is the autocorrelation function of $x_i(n)$. We thus may say, the Al-Dhahir and Cioffi's procedure makes an attempt to choose the samples of the TIR (i.e., the elements of \mathbf{c}) so that its gain values over a dense grid of frequencies, defined by $\omega_i = 2\pi i/N$, $i = 1, 2, 3, \dots$, are maximized subject to the constraints (7) and (8). This procedure will converge to a TIR which has no null within the band of interest, since any null in the TIR response will force a few of the terms under the summation in (6) to some large negative values (because of the dense grid of frequencies) and this clearly cannot be an optimal solution to the maximization procedure.

The above interpretation of the Al-Dhahir and Cioffi's solution matches nicely with the guidelines (design rules) that were set in the previous section. That is, subject to the unit-length and MSE constraints set by (7) and (8),

the optimum TIR, \mathbf{c}_{opt} , is found so that for all values of i which are of interest the terms $\mathbf{c}_{opt}^T \mathbf{G}_i \mathbf{c}_{opt}$ be non-zero.

VI. TEQ DESIGN BASED ON EIGEN-APPROACH

In this section, we propose a new TEQ design scheme which uses the eigenvalues and eigenvectors of the matrix \mathbf{R}_Δ in order to select TIRs which satisfy the design guidelines of Section IV. The computational complexity of this scheme is much lower than the method of Al-Dhahir and Cioffi [3], [4], but the results of the two methods are comparable.

Let the column vectors $\mathbf{q}_0, \mathbf{q}_1, \dots, \mathbf{q}_{L_s-1}$ and the scalars $\lambda_0, \lambda_1, \dots, \lambda_{L_s-1}$, be the unit-norm eigenvectors and the corresponding eigenvalues of \mathbf{R}_Δ , respectively. Then according to the unitary similarity transformation [6], [7],

$$\mathbf{R}_\Delta = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^T \quad (10)$$

where $\mathbf{Q} = [\mathbf{q}_0 \ \mathbf{q}_1 \ \dots \ \mathbf{q}_{L_s-1}]$, and $\mathbf{\Lambda}$ is a diagonal matrix consisting of $\lambda_0, \lambda_1, \dots, \lambda_{L_s-1}$. We also note that the eigenvectors $\mathbf{q}_0, \mathbf{q}_1, \dots, \mathbf{q}_{L_s-1}$ are a set of mutually orthogonal vectors and, thus, may be considered as a set of bases vectors that can be used to express the vector \mathbf{c} as

$$\mathbf{c} = \sum_{i=0}^{L_s-1} \alpha_i \mathbf{q}_i = \mathbf{Q}\boldsymbol{\alpha} \quad (11)$$

where $\boldsymbol{\alpha} = [\alpha_0 \ \alpha_1 \ \dots \ \alpha_{L_s-1}]^T$ and $\alpha_i = \mathbf{q}_i^T \mathbf{c}$. A good (near-optimum) value of \mathbf{c} may thus equivalently be obtained by choosing a set of α_i s which satisfy the guidelines of Section 4.

The term eigenfilter is often used to refer to FIR filters whose tap-weight vectors are the eigenvectors of a correlation matrix [6], [7]. Numerical examples show that the set of eigenfilters of \mathbf{R}_Δ happen to be a set of (almost) equally spaced narrow band filters. Any linear combination of these filters with some non-zero coefficients will thus result in a filter whose gain (with a good chance) remains non-zero over all frequencies. This implies that all α_i s should be non-zero. However, α_i s cannot be chosen arbitrarily since they may result in a large undesirable MSE.

Next, we suggest a simple *rule of thumb* for the selection of α_i s. To this end, we substitute (10) and (11) in (5), and rearrange the result, to obtain

$$\xi = \sum_{i=0}^{L_s-1} \alpha_i^2 \lambda_i. \quad (12)$$

We also note that the unit-norm constraint on \mathbf{c} implies that $\sum_{i=0}^{L_s-1} \alpha_i^2 = 1$. Hence, we may say the MSE of TEQ is given by a weighted average of the eigenvalues of \mathbf{R}_Δ . The weight factors are α_i^2 s. To keep the MSE of TEQ relatively low we may simply choose α_i s proportional to some negative power of λ_i s. That is,

$$\alpha_i = k \lambda_i^{-m}, \quad \text{for } i = 0, 1, \dots, L_s - 1 \quad (13)$$

where k is a common factor which is chosen so that $\sum_i \alpha_i^2 = 1$. A good choice of the parameter m may be found experimentally - see next section.

VII. SIMULATION RESULTS

To evaluate the performance of the proposed eigen-approach scheme of the TEQ design and also to compare that with the MMSE TEQ and the results of Al-Dhahir and Cioffi [3], [4], we present the results of a number of TEQs which we have designed for 8 typical CSA loops. These loops which are presented in Fig. 5 are those that have been considered in [3] and [4]. The DMT setup that we consider is an ADSL transceiver. Accordingly, the IFFT and FFT lengths are 512 and a cyclic prefix length of 32 is assumed [10]. The signal power at the transmitter output is set equal to 14.1dBm. An additive white Gaussian noise (AWGN) with -110 dBm/Hz power and a near-end crosstalk (NEXT) whose power spectral density (PSD) is given by

$$S_{NEXT}(f) = k_{NEXT} f^{3/2} \times \frac{[\sin(\pi f/f_o) \sin(\pi f/2f_o)]^2}{[1 + (f/f_{1,3dB})^6](f^2 + f_{2,3dB}^2)} \quad (14)$$

where $f_o = 1.455$ MHz, $f_{1,3dB} = 3$ MHz, $f_{2,3dB} = 40$ kHz, and $k_{NEXT} = 2.1581 \times 10^{-9}$, are assumed. The NEXT PSD has been obtained from the ANSI T1.413-1995 standard [10]. The selected value of k_{NEXT} corresponds to 20 disturbers.

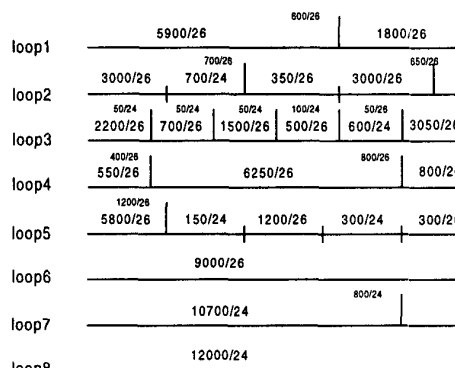


Fig. 5. Configurations of 8 CSA loops

The bit allocation to different subcarriers are calculated according to the equation

$$b_i = \left\lfloor \log_2 \left(1 + \frac{\text{SNR}_i}{10} \right) \right\rfloor, \quad (15)$$

where i varies over all data carrying subcarriers, SNR_i is the SNR at the i th subcarrier and $\lfloor x \rfloor$ denotes the largest integer less than or equal to x . Equation (15) corresponds to bit error probability of 10^{-7} for uncoded data. The total number of bits in each block of DMT is then $\sum_i b_i$, excluding the cases where $b_i = 1$. In the computation of the values of b_i s we have considered and used the bandwidth optimizing algorithm of [5]. That is, the subchannels which are unable to carry any information bit are taken note of and thus their corresponding signal powers are distributed among the data carrying channels [5], [4].

We performed an extensive set of simulations to find the best value of m to be used in (13). We found that $m = 1$ which also gives the simplest equation for computation of the TIR is a good value.

Table 1 presents a summary of the results that we obtained for MMSE TEQ, the optimum design of Al-Dhahir and Cioffi [4], and the proposed eigen-approach. From these results, we see that the eigen-approach method performs very similar to the optimum design of [4]. Both methods succeed to perform better than the MMSE TEQ.

TABLE I
NUMBER OF BITS PER BLOCK OF DMT FOR VARIOUS CSA LOOPS AND DIFFERENT DESIGNS.

loop	MMSE TEQ	Reference [4]	Eigen-Approach
1	1104	1179	1181
2	973	1114	1112
3	895	1049	1047
4	890	961	953
5	957	1153	1161
6	854	1011	1012
7	523	560	540
8	421	492	496

In our initial study of the TEQ design, we observed that although the TIR obtained by our method looked very different from the one obtained through the optimization method of [4] (or [3]), in both the time and, to some extent, frequency domain, there was almost no difference between the number of bits allocated to each block of DMT in the two designs. This interesting observation show that DMT transceivers are robust to some variation of the TIR. This observation is also found to be in line with our arguments in Section IV where the proposed guidelines were noted to be relatively vague. To explore this further we generated 1000 different target responses according to the equation

$$\mathbf{c} = \left(\sum_{i=0}^{L_s-1} \frac{\beta_i}{\lambda_i} \mathbf{q}_i \right) / \sqrt{\sum_{i=0}^{L_s-1} \frac{1}{\lambda_i^2}} \quad (16)$$

where β_i s are a set of independent random numbers taking values of +1 and -1. Each TIR was then used to obtain the corresponding TEQ and accordingly to calculate the number of bits allocated to each block of the DMT. Table 2 presents a summary of the results of this experiment which was repeated for all the 8 loops of Fig. 5. In this table we have given the mean of the number of bits allocated to each loop and their corresponding standard deviations, plus the maximum and minimum bit allocations that was observed in each experiment. The results clearly show the insensitivity of the DMT to these variations of the TIR. It is also instructive to compare these results with those of Table 1 and observe that:

1. In all cases, on the average, the results obtained by the eigen-approach are superior to those of the MMSE design.
2. The results confirm the non-optimality of the designs obtained by the method of Al-Dhahir and Cioffi [3], [4], as in all cases superior designs are found.

3. Most of the designs obtained through the TIR randomization perform about the same. The standard deviation of the number of bits allocated to each block in all cases remain within 1 to 2% of the respective averages.

TABLE II
NUMBER OF BITS PER BLOCK OF DMT RESULTING FROM RANDOMLY SELECTED TIRs, ACCORDING TO (16) FOR VARIOUS CSA LOOPS.

loop	mean	std. deviation	maximum	minimum
1	1166	12.1	1194	1091
2	1101	14.0	1138	1046
3	1035	10.4	1056	986
4	954	9.6	980	914
5	1139	11.4	1165	1083
6	996	10.7	1018	921
7	550	11.1	572	506
8	488	11.8	505	425

VIII. CONCLUSIONS

In this paper, we studied the problem of designing near-optimum time domain equalizers (TEQ) in the application of DMT transceivers. We noted that the optimal design of TEQ is a very difficult problem and thus one has to resort to suboptimal solutions. To this end, based on an understanding of a short-coming of the MMSE TEQ, we set some design guidelines for the selection of good (near-optimum) target impulse response (TIR) in TEQ design. We noted that the optimization method proposed in [3] is one possible method of achieving a design which satisfies the proposed guidelines. We also showed that there are other ways that could be used for selection of TIR which also satisfy the proposed guidelines and thus achieve good performance. In particular, we presented an eigen-approach and showed that it achieves similar performance to the designs obtained by the method of [3], however, at a much lower computational cost.

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