Channel Classification and Time-Domain Equalizer Design for ADSL Transceivers

B. Farhang-Boroujeny and Ming Ding
Department of Electrical Engineering, National University of Singapore, Singapore 119260.
Tel: (++65)874 6682, Fax: (++65)779 1103, email: elefarhg@leonis.nus.edu.sg

Abstract— **Time domain equalizer (TEQ) is used in the discrete multitone (DMT) transceivers in order to reduce the duration of the overall response of the transmission system. The optimum TEQ is the one that results in maximum bit allocation to each block of DMT. However, the optimum design of TEQ turns out to be a very difficult task. In an earlier work, we noted that there are some general guidelines that one should follow in the design of TEQ for achieving a near-optimum performance. There, we proposed an eigen-approach which could result in TEQs with comparable performance to those of a previously reported method, but at a much lower computational cost. In this paper, we propose a second design method which is even simpler than our first method, but still results in designs comparable to the best available methods.**

Keywords— **Equalizers, Data Communications, Subscriber loops, Discrete multitone**

I. Introduction

The discrete multitone (DMT) has attracted considerable attention as a practical and viable technology for high-speed data transmission over spectrally shaped noisy channels [1]. Modems employing this technology are already available in the market. The DMT based modems have, in particular, been found very useful in transmitting high-speed data over digital subscriber lines (DSL). DMT is a special multicarrier data transmission technique which uses the properties of the discrete Fourier transform (DFT) in an elegant way so as to achieve a computationally efficient realization. Fig. 1 depicts a block diagram of a DMT modem. In the transmitter, the data sequence is partitioned into a number of parallel streams. Each stream of data is modulated via a particular subcarrier. The modulated subcarriers are summed to obtain the transmit signal. The use of DFT in DMT allows an efficient realization of the subcarrier modulators in a parallel processing structure which benefits from the computational efficiency of the fast Fourier transform (FFT). A similar DFT-based structure is used for efficient realization of the subcarrier demodulators in the receiver part of the DMT modem.

In DMT, channel distortion is taken care of by cyclically extending the output of the inverse FFT (IFFT) modulator so that the input sequence looks periodic to the channel. This is referred to as *cyclic prefix* method [1] (see Fig. 1). The length of the cyclic prefix should be at least equal to the duration of the channel impulse response minus one. However, we note that the addition of the cyclic prefix reduces the throughput of the channel as it carries redundant data. To minimize this reduction of the throughput, a channel equalizer whose goal is to reduce the overall duration of the system (channel plus equalizer) impulse response to a predefined length is used. In the DMT literature, this type of equalizers are called time-domain equalizer (TEQ).

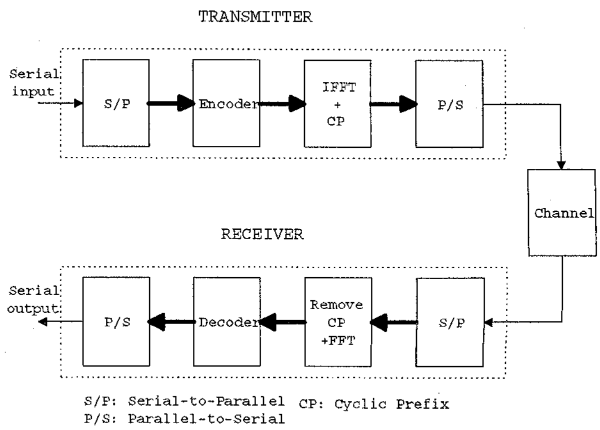

Fig. 1. Block diagram of a DMT transceiver

The problem of TEQ design in the DMT transceivers may be formulated as follows. Given a channel with the impulse response samples $h_0, h_1, \cdots, h_{L-1}$ and corrupted with some additive noise, we wish to find the coefficients $w_0, w_1, \cdots, w_{N-1}$ of a transversal equalizer that results in a combined channel-equalizer response which is shortened to a duration of L_s samples, where L_s can be at most equal to the length of cyclic prefix plus one. In this design, the known parameters are the channel response, the channel noise (usually its autocorrelation coefficients), and the expected duration, L_s, of the equalized response. In

This work is protected by a patent application.
This work has been partially supported by the Center for Wireless Communications, Singapore.

the design process, we usually begin with the selection of a proper shorten impulse response known as target impulse response (TIR). The TEQ coefficients are then selected so that the combined response of the channel and equalizer be as close as possible to the TIR. The criterion used for the selection of the TIR and TEQ may vary.

The ultimate goal in the design of the TEQ is to achieve maximum bit-rate over the channel. However, development of a practical design method which can achieve this goal turns out to be very difficult. Most of the studies in TEQ design have set the goal of mean-square error (MSE) minimization which means the TIR and TEQ are jointly optimized so that the difference between the outputs of the TEQ and TIR is minimized in the MSE sense. This leads to an analytically tractable problem with a unique close-form solution.

Recently there has been some efforts which aim at the goal of bit-rate maximization [3], [4] and [10]. In [3], [4], the authors have proposed a rather complexptimization procedure. Convergence of this procedure to the corresponding optimum global solution is not guaranteed. However, numerical examples of actual digital subscriber line (DSL) channels have shown that the results obtained are superior to those of the minimum MSE (MMSE) TEQ. In [10], we have proposed an approach which is much simpler than the method of [3] and [4], but achieves about the same performance. The method proposed in [10] chooses the TIR as a linear combination of the eigenvectors of a positive definite matrix which is related to the correlation coefficients of the channel response.

In this paper, we note that in the design of TEQ a good choice of the TIR weakly depends on the channel response. Noting this, we propose a second approach for the design of TEQs. This approach involves classification of the class of channels into a few subclasses and use of a common TIR for all the channels in each subclass. This method which involves the use of some look-up tables is even simpler than our first method, thus is more useful for practical implementation of the DMT transceivers in real time.

II. Previous Works

In the MMSE TEQ, the optimum TIR, $\mathbf{c}_{MMSE}$, which results in the MMSE is obtained according to the following optimization procedure [2]:

$$\mathbf{c}_{MMSE} = \arg\min\left\{\mathbf{c}^T\mathbf{R}_\Delta\mathbf{c}\right\} \tag{1}$$

subject to the constraint $\mathbf{c}_{MMSE}^T\mathbf{c}_{MMSE} = 1$. Here,

$$\mathbf{R}_\Delta = \mathbf{I}_{L_s} - \mathbf{H}_\Delta^T\mathbf{R}^{-1}\mathbf{H}_\Delta \tag{2}$$

where $\mathbf{R}$ is the $N\times N$ correlation matrix of the TEQ input, $\mathbf{H}_\Delta = \mathbf{H}\times[\mathbf{0}_{L_s\times\Delta}\ \ \mathbf{I}_{L_s}\ \ \mathbf{0}_{L_s\times L_r}]^T$ is an $N\times L_s$ matrix,

$$\mathbf{H} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{L-1} & 0 & \cdots & 0 \\ 0 & h_0 & h_1 & \cdots & h_{L-1} & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_{L-1} \end{bmatrix} \tag{3}$$

is an $N\times(N+L-1)$ matrix, $\mathbf{0}_{m\times n}$ is the $m\times n$ null matrix, $\mathbf{I}_m$ is the identity matrix of size m, Δ is the equalizer delay, and $L_r = N+L-\Delta-L_s-1$. The solution to the above optimization is well understood [11]. The optimum solution, $\mathbf{c}_{MMSE}$, is the eigenvector that corresponds to the minimum eigenvalue of the matrix $\mathbf{R}_\Delta$. Once the optimum TIR is obtained, the TEQ coefficients can be calculated by solving the corresponding Wiener-Hopf equation or using an adaptive algorithm. We also note that for a given TIR, $\mathbf{c}$, the MSE at the TIR output is given by

$$\xi = \mathbf{c}^T\mathbf{R}_\Delta\mathbf{c}. \tag{4}$$

In [10], based on a thorough study of the MMSE TEQ, it is conclude that to be sure of no loss of any of subcarriers, one shall choose a TEQ whose frequency response does not experience any null over the useful portion of the received signal band. It has also been noted that the TEQ response is directly related to the TIR, $\mathbf{c}$, and accordingly the following general guidelines have been suggested:

- The amplitude response associated with the TIR, $\mathbf{c}$, shall not have any null.
- At the same time, $\mathbf{c}$ should be chosen so that the MSE given by (4) remains relatively low.

Using the above guidelines the following selection of $\mathbf{c}$ is found to be a good choice [10]:

$$\mathbf{c} = \left(\sum_{i=0}^{L_s-1}\frac{1}{\lambda_i}\mathbf{q}_i\right)\Bigg/\sqrt{\sum_{i=0}^{L_s-1}\frac{1}{\lambda_i^2}} \tag{5}$$

where the column vectors $\mathbf{q}_0, \mathbf{q}_1, \cdots, \mathbf{q}_{L_s-1}$ and the scalars $\lambda_0, \lambda_1, \cdots, \lambda_{L_s-1}$, are the unit-norm eigenvectors and the corresponding eigenvalues of $\mathbf{R}_\Delta$, respectively. This choice of $\mathbf{c}$ has been found to result in TEQs comparable with those designed by the much more complexrocedure of [3] and [4].

III. The New Design

An observation in [10] is that in choosing TIR, we need only to comply with the design guidelines that were mentioned in the previous section. Moreover, there are many choices of TIR which satisfy the design guidelines and thus result in TEQs which all perform about the same and probably close to the optimum design (which is not known).

Another observation that we may have here is that the matrix $\mathbf{R}_\Delta$ whose eigenvalues and eigenvectors could be used to select a well behaved TIR according to (5), is directly related to the autocorrelation coefficients of the channel output. Furthermore, we note that the autocorrelation coefficients are related to the power spectral density of the underlying signal which, in turn, is directly related to the channel magnitude response and, for large signal-to-noise ratios, to a lesser extent to the noise power spectral density.

Considering these observations, in a recent patent [9], filed by the first author of this paper, a novel scheme has been proposed for design of TEQ in the DMT based transceivers. In [9], only a general over-view of the proposed scheme is given. Our aim in this section is adopt the general scheme of [9] and give a detailed evaluation of that when applied to the particular case of asymmetric digital subscriber line (ADSL) channels.

The scheme proposed in [9] consists of two separate phases; namely, off-line and on-line.

The off-line phase consists of the following steps:
1. A large set of the members of the class of channels of interest are obtained, through measurements and/or simulations, whichever appropriate.
2. The channels with close magnitude responses are grouped together to make a number of subclasses.
3. A member of each subclass with a magnitude response close to the mean of all members of the subclass is chosen as typical (average) response for that subclass. The magnitude responses of these typical channels are normalized to the length of unity and stored in a look-up table. The time indices of the peak-points[1] of these impulse responses are also stored in the same look-up table. These will be later used for identification of the subclass of the measured channel in the on-line phase of the design.
4. For each of the typical responses, a set of optimum (near-optimum) choices of the TIR and the delay parameter, Δ, are obtained and stored in another look-up table.

The on-line phase of the proposed scheme consists of the following steps:
1. The channel response and the power spectral density of the channel noise are measured.[2]
2. The magnitude response and the location (time index) of peak-point of the impulse response of the measured channel are calculated and compared with the tabulated parameters provided in Step 3 of the off-line phase, and accordingly the subclass of channel is identified, as explained below.
3. Once the subclass of the channel is identified, the corresponding TIR and delay parameter, Δ, are obtained from the look-up table generated in Step 4 of the off-line phase.
4. The TIR and delay parameter along with the channel response and the noise power spectral density are used to generate and solve the Wiener-Hopf equation leading to the equalizer coefficients.

The identification of the subclass of the measured channel is done according to a simple *signature analysis* as explained next. We assume that there are P subclasses. Let $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_P$ denote the column vectors representing the magnitude responses of the averaged channels obtained in Step 3 of the off-line phase. Also, let $\hat{\mathbf{a}}$ denote the column vector consisting of the samples of the magnitude response of the measured channel. We assume that the vectors $\mathbf{a}_1$, $\mathbf{a}_2, \cdots, \mathbf{a}_P$ and $\hat{\mathbf{a}}$ are of the the same dimenssion, M. The closeness of $\hat{\mathbf{a}}$ to $\mathbf{a}_i$, $i = 1, 2, \cdots, P$ is measured by minimizing the cost function

$$\gamma_i(\alpha) = \sum_{k=0}^{M-1} w(k)(\alpha\hat{a}(k) - a_i(k))^2 \tag{6}$$

where $\hat{a}(k)$ and $a_i(k)$ are the kth elements of $\hat{\mathbf{a}}$ and $\mathbf{a}_i$, respectively, and $w(k)$ is a weighting function. Through a large number of numerical tests on typical ADSL channels, we found that to get a better signature and thus a good TEQ, we should give a higher weight to the points where $\hat{a}(j)$ is relatively small. Considering this point and noting that for ADSL channels, generally, the magnitude response decreases over higher range of frequecies, we decided on a number of weighting functions which grow with frequency. After a number of trial and errors we found the following weighting function as a good compromise choice for the CSA loops which we were experimenting on (see next section for the details of the examined loops).

$$w(k) = (1 + (k/M)^2)^2, k = 0, 1, \quad \cdots, M-1 \tag{7}$$

Solving $d\gamma_i(\alpha)/d\alpha = 0$ for α, and using the result in (6), we obtain

$$\gamma_i^{\min} = \mathbf{a}_i^T\mathbf{W}\mathbf{a}_i - \frac{\left(\mathbf{a}_i^T\mathbf{W}\hat{\mathbf{a}}\right)^2}{\hat{\mathbf{a}}^T\mathbf{W}\hat{\mathbf{a}}} \quad \text{for } i = 1, 2, \cdots, P \tag{8}$$

where $\mathbf{W}$ is the diagonal matrix consisting of $w(0)$, $w(1)$, $\cdots$, $w(M-1)$. The subclass of the measured channel is

[1] In [9] only the magnitude responses were considered for channel classification. However, futher tests revealed that the peak-points of the channel responses also play some role in the classification and thus their use helps in improving the results.

[2] In most of the application standards, including the ADSL standard [8], provision for such measurements during the system initialization is provided.

then determined by finding the index j which satisfies the following equation:

$$\gamma_j^{\min} = \min(\gamma_1^{\min}, \gamma_2^{\min}, \cdots, \gamma_P^{\min}). \tag{9}$$

To have a fair comparison of these responses, as was noted earlier, the prestored vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\cdots$, $\mathbf{a}_P$ are all normalized to the unity norm. That is these have been normalized such that

$$\mathbf{a}_i^T \mathbf{a}_i = 1, \quad \text{for } i = 1, 2, \cdots, P. \tag{10}$$

Although the above signature results in satisfactory TEQ designs for most of the channels, it fails to given good results for about 5% of the channels we tested. Through experiments, we found the use of a second signature, along with the above signature, could improve the results significantly. For the clarity of the discussions that follow, we refer to the signature introduced above as *signature 1* and the one intoduced below as *signature 2*.

Numerical tests reveal that a good measure of distinguishing between channels which have similar amplitude responses, but their attachment to a particular subclass is not very strong, is the position of the peak of their time-domain impulse response. We quantify the peak positions, and thus implement the proposed signature 2, as follows. Suppose $\gamma_{j1}^{\min}$ and $\gamma_{j2}^{\min}$ be two extracts of Signature 1 with the smallest and the second smallest value of all the γ_is, respectively. We form the following ratios:

$$R_1 = \frac{\min(\hat{\tau}, \tau_{j_1})}{\max(\hat{\tau}, \tau_{j_1})} \quad \text{and} \quad R_2 = \frac{\min(\hat{\tau}, \tau_{j_2})}{\max(\hat{\tau}, \tau_{j_2})}, \tag{11}$$

and choose j_1 as the subclass of the measured channel if $R_1 \geq R_2$, and j_2, otherwise.

IV. Statistical Evaluation of the Proposed Design

In this section, we present a statistical evaluation of the TEQ design method of the previous section when it is applied to ADSL channels. For this, we start with a study of magnitude responses of digital subscriber lines. Our study has shown that a choice of 20 subclasses gives a good variety of responses which match well with typical DSL loops that are encountered in practice. This choice has been based on a study of a wide range of channels which we generated in simulation using the channel parameters provided in the ADSL standard (T1.413-1998) for 24 and 26 gauge lines [8], and some compromised considerations in terms of system performance and complexity. In generating the frequency responses of the channels, we have used the method discussed in [6].

Fig. 2 presents the details of the 20 loops which we consider as typical channels characterizing various subclasses. For each of these channels we found the best TIR that we could get through a random search based on the equation

$$\mathbf{c} = \left(\sum_{i=0}^{L_s-1} \frac{\beta_i}{\lambda_i} \mathbf{q}_i \right) \Bigg/ \sqrt{\sum_{i=0}^{L_s-1} \frac{1}{\lambda_i^2}} \tag{12}$$

where β_is are a set of independent random numbers taking values of $+1$ and -1. We considered 1000 attempts for each loop and selected the one which results in maximum bit allocation. The TIRs obtained in this way and their corresponding delay parameters, Δ, which were optimized by searching over all possible values that they could take, were then stored in a table. Clearly, all these operations correspond to the off-line phase of TEQ design.

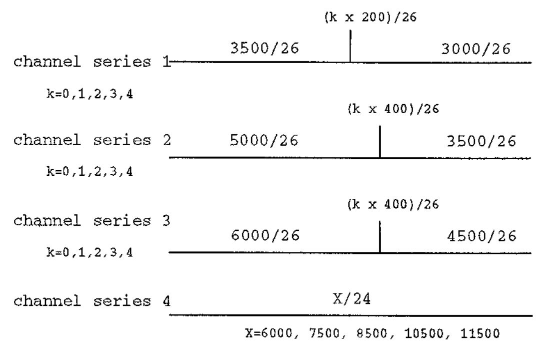

Fig. 2. The CSA loops chosen for channel classification.

To evaluate the on-line performance of the proposed design, we study the results of a large number of randomly generated carrier-serving-area (CSA) loops. Fig. 3 presents the details of the randomly generated loops. The parameters which are indicated in Fig. 3 are chosen randomly in the following order:

- *Principal line length,* l: a constantly distributed random variable in the range 5000-12000 feet.
- *Number of bridged taps* K: takes values of 0, 1, 2, 3 or 4 with equal probability of occurrence.
- *Principal line segments,* l_1, l_2, $\cdots$, l_{K+1}: a set of random variables with similar distribution which satisfy the following conditions:

$$l_1 + l_2 + \cdots + l_{K+1} = l \tag{13}$$

$$l_1,\ l_2,\ \cdots,\ l_{K+1} \geq l/10. \tag{14}$$

- *Length of the bridged tap loops,* l_{b1}, l_{b2}, $\cdots$, l_{bK}: a set of random variables with constant distribution in the range of zero to one tenth of the principal line length, l.

0-7803-6286-1/00/$10.00 (c) 2000 IEEE

All the lines are assumed to be 24 or 26 gauge. The power spectral density of the channel noise is assumed to be fixed and chosen according to the equations provided in Appendix B of [8].

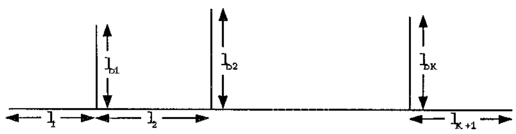

Fig. 3. The model of randomly generated CSA loops. The vertical lines are bridged taps. All lines are either 24 or 26 gauge.

For each loop two TEQs are designed. One based on the eigen-approach method proposed in [10] and another one based on the proposed channel classification method, using the tabulated subclasses. For each case, we calculate the maximum number of bits which could be allocated to each block of DMT based on error probability of 10^{-7} for uncoded data, subject to the maximum of 3000 bits per block.

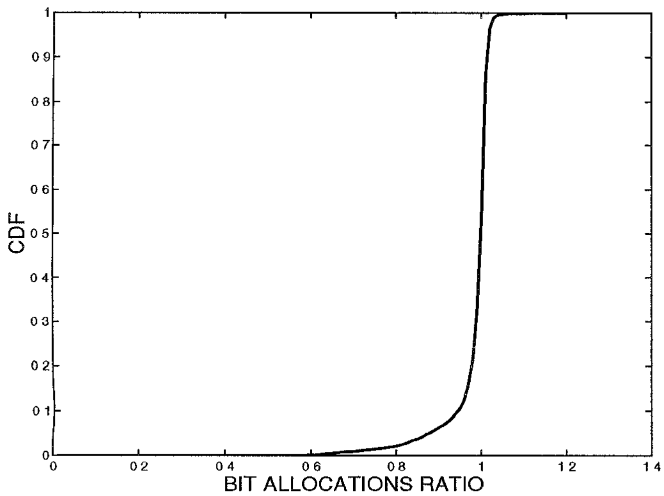

Fig. 4. The cumulative disribution function comparing the proposed method against the eigen-approach of [10]. The horizontal axis shows the ratio of the number of bits per DMT data symbol obtained from the design based on channel classification over the one obtained from the eigen-approach.

Fig. 4 presents a summary of the results that we obtained for 5000 randomly generated CSA loops. The channel identification uses both Signatures 1 and 2, to obtain the best choice of the TIR. This figure presents the cumulative distribution function (CDF) of the ratio of the number of bits per DMT data symbol obtained from the design based on channel classification over the one obtained from the eigen-approach. The results show that in most of the cases the proposed scheme performs satisfactorily and gives results which are comparable with those obtained from the eigen-approach.

V. Conclusions

In this paper, we studied the problem of designing near-optimum time domain equalizers (TEQ) in the application of ADSL. We explored the possibility of storing a number of TIRs in a look-up table and choosing one of these TIRs for the design of TEQ, based on a signature analysis of the measured magnitude response of the channel. A statistical evaluation of this method showed that it works quite well for most of the simulated CSA loops and even about 50% of the cases outperform the eigen-approach [10] which was earlier found to be comparable with the method of [3] and [4]. This look-up table approach which was referred to as channel classification method is in particular a useful engineering approach to the design of TEQ in ADSL modems in real time as it greatly simplifies the design procedure by just using look-up table instead of an eigen-analysis of the correlation matrix of the received signal.

References

[1] E. A. Lee, and D. G. Messerschmitt, *Digital Communication.* 2nd Ed., 1994, Kluwer, Boston.

[2] D. D. Falconer, and F. R. Magee, Jr., "Adaptive channel memory truncation for maximum likelihood sequence estimation", *The Bell Syst. Tech. J.,* vol. 52, no. 9, Nov. 1973, pp. 1541-1562.

[3] N. Al-Dhahir, and J. M. Cioffi, "Optimum finite-length equalization for multicarrier transceivers", *IEEE Trans. Comm.*, vol. 44, no. 1, Jan. 1996, pp. 56-64.

[4] N. Al-Dhahir, and J. M. Cioffi,"A band-optimized reduced-complexity equalized multicarrier transceiver",*IEEE Trans. Comm.*, vol. 45, no. 8, Aug. 1997, pp.948-956.

[5] P. S. Chow, and J. M. Cioffi, "Bandwidth optimization for high speed data transmission over channels with severe intersymbol interference", Globecom'92, Dec. 1992, Orlando, FL, pp. 59-63.

[6] Jean-Jacques Werner,"The HDSL Environment", *IEEE Journal on Select. Areas in Comm.*, vol. 9, no. 6, Aug. 1991, pp.785-800.

[7] *Optimization Toolbox Manual,* MATLAB Software Package Version 4.1. The Mathworks Inc., Natick, MA 1992.

[8] "Network and Customer Installation Interfaces - Asymmetric Digital Subscriber Line (ADSL) Metallic Interface", *T1.413-1998*,American National Standards Institute,NY,1998.

[9] B. Farhang-Boroujeny, B Wang, and M Chakraborty, "A Method of Designing an Equaliser", Case ref. P12498SG, Filing date: Oct. 13, 1998.

[10] B. Farhang-Boroujeny, and Ming Ding, "An eigen-approach to the design of near-optimum time domain equalizer for DMT transceivers", ICC'99, vol. 2, June, 1999, pp. 937 -941 .

[11] B. Farhang-Boroujeny, *Adaptive Filters: Theory and Applications.* John Wiley & Sons, Chichester, England, 1998.