WORDLENGTH OPTIMIZATION WITH COMPLEXITY-AND-DISTORTION MEASURE AND ITS APPLICATION TO BROADBAND WIRELESS DEMODULATOR DESIGN

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ABSTRACT

Many digital signal processing algorithms are first developed in floating point and later mapped into fixed point for digital hardware implementation. During this mapping, wordlengths are searched to minimize total hardware cost and maximize system performance. Complexity and distortion measures have been separately researched for optimum wordlength selection. This paper proposes a complexity-and-distortion measure (CDM) method that combines these two measures. The CDM method trades off these two measures using a weighting factor. The proposed method is applied to wordlength design of a fixed broadband wireless demodulator. For this case study, the proposed method finds the optimal solution in one-third the time that exhaustive search takes. The contributions of this paper are (1) a generalization of search methods based on complexity or distortion measures, (2) a framework of automatic wordlength optimization, and (3) a wireless demodulator case study.

1. INTRODUCTION

Digital signal processing algorithms often rely on long wordlengths for high precision whereas digital hardware implementations of these algorithms need short wordlengths to minimize total hardware costs. The determination of the wordlength is a time consuming job when assignments of wordlengths are performed manually and iteratively. In a complex system, 50% of the design time may be spent on wordlength determination [1]. An approach [2] that analyzes finite wordlength effects has difficulty finding optimum wordlengths for complicated systems. For these reasons, many simulation-based wordlength optimizations have been researched [3–6].

Choi and Burleson [3] showed how a general search-based wordlength optimization can produce optimal or near-optimal solutions for different objective-constraint formulations. Sung and Kim [4] proposed simulation-based wordlength optimization for fixed-point digital signal processing systems. These search algorithms try to find the cost-optimal solution by using either exhaustive search or heuristics.

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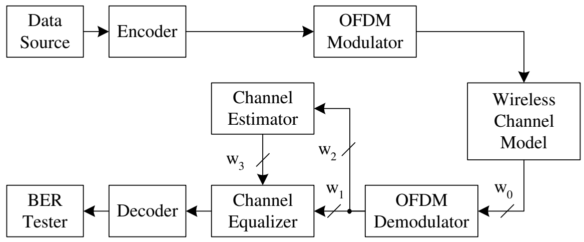

Fig. 1. Wordlength model for a fixed broadband wireless access demodulator.

Han *et al.* [5] proposed a fast search algorithm by measuring performance distortion due to the finite wordlength effects. The fast algorithm compares the sensitivity information of the distortion to find the performance-optimal solution regardless cost or complexity information. However, complex digital systems such as a digital communication modem possess different cost or complexity in the blocks.

In this paper, we propose an algorithm that combines and distortion measures. We apply this algorithm to a design of the digital demodulator for fixed broadband wireless access as shown in Fig.1. This “last mile” broadband access technology uses orthogonal frequency division multiplexing (OFDM). OFDM uses a fast Fourier transform (FFT) to implement multicarrier demodulation.

2. BACKGROUND

Wordlengths in digital signal processing blocks are determined for hardware implementations. Consider a wordlength variable w with cost function c. A set of w is wordlength vector $\mathbf{w}$. We assume that the total cost is a function of wordlengths with $c : \mathbb{I}^n \rightarrow \mathbb{R}$ defined by

$$\forall \mathbf{w} \in \mathbb{I}^n, c(\mathbf{w}) = \sum_{k=1}^{n} c_k(w_k). \tag{1}$$

The total cost is the sum of every wordlength cost. We include a performance function p of wordlength set $\mathbf{w}$ as constraints:

$$p(\mathbf{w}) \geq P_{req} \tag{2}$$

Here, P_{req} is a constant for a required performance. We also consider the minimum wordlength $\underline{w}$ and maximum wordlength $\overline{w}$ constraints for each variable:

$$\underline{w}_k \leq w_k \leq \overline{w}_k \;\forall k = 1, \cdots, n \tag{3}$$

We write the complete wordlength optimization problem as

$$\min_{\mathbf{w} \in \mathbf{I}^n} \{c(\mathbf{w}) | p(\mathbf{w}) \geq P_{req}, \underline{\mathbf{w}} \leq \mathbf{w} \leq \overline{\mathbf{w}}\}. \tag{4}$$

The goal of the wordlength optimization is to search for a minimizer $\mathbf{w}^*$ which minimizes the objective function, $c(\mathbf{w}^*)$. The minimizer can be found with iterative update equation as

$$\mathbf{w}^{(h+1)} = \mathbf{w}^{(h)} + \mathbf{\Delta}^{(h)} \tag{5}$$

where h is a iteration index and $\mathbf{\Delta}$ is an update direction. An adequate update direction reduces the number of iterations to find optimum wordlengths.

3. OPTIMUM WORDLENGTH SEARCH

Three measures such as complexity measure, distortion measure, and complexity-and-distortion measure are considered for the update direction to search for optimum wordlength. The complexity measure is used in [4], the distortion measure is utilized in [5], and the complexity-and-distortion measure is proposed in this paper.

3.1. Complexity measure (CM)

The complexity measure method considers hardware complexity function as the cost function in (4) and use the sensitivity information of the complexity as the direction to search for the optimum wordlengths. The sensitivity information is calculated by gradient of the complexity function. For steepest descent direction, the update direction is

$$\mathbf{\Delta} = -\nabla c(\mathbf{w}) \tag{6}$$

where ∇ is gradient of function.

Sung and Kum's method [4] updated wordlengths from the direction of the lowest sensitive complexity until a system met a required performance. The complexity measure method searches the wordlengths that minimize hardware complexity. However, it demands a large number of iterations since it does not use any distortion sensitivity information that can speed up to find the optimum wordlengths.

3.2. Distortion measure (DM)

The distortion measure method considers distortion function as the cost function in (4) and uses the sensitivity information of the distortion for the direction to search for the optimum wordlengths.

This method assumes that every cost or complexity function is same or equal to 1 and selects wordlengths with the update direction according to the distortion sensitivity information.

The complexity objective function in complexity measure is replaced with the distortion objective function $d(\mathbf{w})$ and the complexity minimizing problem is changed into a distortion minimizing problem as

$$\min_{\mathbf{w} \in \mathbf{I}^n} \{d(\mathbf{w}) | d(\mathbf{w}) \leq D_{req}, c(\mathbf{w}) \leq C, \underline{w} \leq \mathbf{w} \leq \overline{w}\} \tag{7}$$

where D_{req} is required distortion, and C is a complexity constant.

The sensitivity information is also calculated by gradient of the distortion function. For the steepest descent direction, the update direction is

$$\mathbf{\Delta} = -\nabla d(\mathbf{w}) \tag{8}$$

For the distortion, Fiore and Lee [7] computed an error variance, and Han *et al.* [5] measured output SNR.

The distortion measure method reduces the number of iterations for searching the optimum wordlengths, since the search direction depends on the distortion by changing the wordlengths. This method rapidly finds the optimum wordlength satisfying the required performance by a fewer number of iterations compared to complexity measure method. However, the wordlengths do not guarantee the optimum wordlengths in terms of the complexity.

3.3. Complexity-and-Distortion measure (CDM)

The complexity-and-distortion measure combines the complexity measure with the distortion measure by a weighting factor. In the objective function, both complexity and distortion are simultaneously considered. We normalize the complexity and the distortion function and multiply them with complexity and distortion weighting factors, α_c, α_d, respectively. The new objective function is

$$f(\mathbf{w}) = \alpha_c \cdot c_n(\mathbf{w}) + \alpha_d \cdot d_n(\mathbf{w}) \tag{9}$$

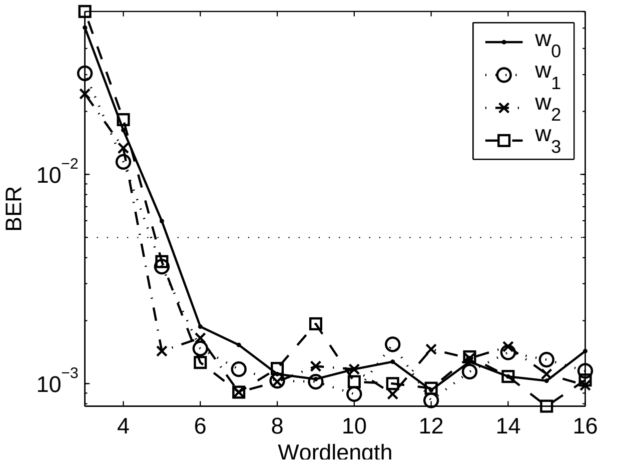

Fig. 2. Wordlength effect for the demodulator in Fig.1, with Stanford University Interim wireless channel model number 3, SNR of 20 dB, FFT length of 256, and least squares comb type channel estimator without error control coding.

where $c_n(\mathbf{w})$ and $d_n(\mathbf{w})$ are normalized complexity function and distortion function, respectively. The relation between the weighting factors is

$$\alpha_c + \alpha_d = 1,\ 0 \le \alpha_c \le 1,\ 0 \le \alpha_d \le 1 \qquad (10)$$

The objective function gives a new optimization problem

$$\min_{\mathbf{w}\in\mathbf{I}^n} \{f(\mathbf{w})|d(\mathbf{w}) \le D_{req}, c(\mathbf{w}) \le C, \underline{w} \le \mathbf{w} \le \overline{w}\} \quad (11)$$

This optimum problem finds wordlengths that minimize complexity and distortion simultaneously according to the weighting factors.

The update direction for the steepest decent direction to find optimum wordlength $\mathbf{w}$ is

$$\mathbf{\Delta} = -\nabla f(\mathbf{w}) \qquad (12)$$

From (9) and (12), the update direction is

$$\mathbf{\Delta} = -[\alpha_c \cdot \nabla c_n(\mathbf{w}) + \alpha_d \cdot \nabla d_n(\mathbf{w})]. \qquad (13)$$

Controlling the complexity and distortion weighting factor, α_c and α_d from 0 to 1 , the proposed method searches for an optimum wordlength with tradeoffs between complexity measure method and distortion measure method.

4. SIMULATION

We simulate the three wordlength selection methods to the design of the fixed broadband wireless access demodulator shown in Fig.1. For the wireless channel, we used Stanford University Interim model [8] [9].

Table 1. Simulation results of several search methods starting from the minimum wordlength for the demodulator arcs in Fig.1: $\{w_0, w_1, w_2, w_3\} = \{5, 4, 4, 4\}$. Full search (FS) is the same method of exhaustive search in [4].

Method	α_c	WL($\mathbf{w}$)	Complexity	Trial
DM	0	{10,9,4,10}	10781	16
CDM	0.5	{7,10,4,6}	7702	15
CM	1	{7,7,4,6}	7699	69
FS [5]	-	{7,7,4,6}	7699	210

For wordlength variables, we choose the wordlengths that have the most significant effect on complexity and distortion in the system. For the OFDM demodulator, we select wordlength variables w_0, w_1, w_2, and w_3 for the FFT, equalizer, and estimator, respectively as shown in Fig.1.

We assume that the input wordlength is propagated in the block and in given blocks the internal wordlengths have already been decided. In simulation, only the inputs to each block are constrained to be in fixed point but the blocks themselves are simulated in floating point precision.

For the complexity, the numbers of complex multiplication are measured assuming that units are not reused. The complexity vector $\mathbf{c}$ of the wordlength per bit is assumed $\{1024, 1, 128, 2\}$ from [4] [10]. We also assume the complexity increases linearly as wordlength increases. For the distortion measurement, bit error rate (BER) is measured.

The minimum wordlength (MWL) is used for the initial wordlength. The MWL is searched by changing one wordlength variable while other variables have high precision (i.e., 16 bits) [4] [5]. The simulation for the MWL is shown in Fig.2. Assuming the first minimum performance of BER is 5×10^{-3}, the MWL is $\{5, 4, 4, 4\}$ from the Fig.2. Starting from the MWL, wordlengths are increased according to the sensitivity information of different measures. We measure the number of iterations until they find their own optimum wordlength satisfying the required performance such as $1.5 \times 10^{-3} \le$ BER $\le 2 \times 10^{-3}$ (without channel decoder). For the optimum wordlength, we follow the Hybird procedure [11] that combines the wordlength increase procedure followed by wordlength decrease procedure. The simulation results of the increasing wordlength are shown in Fig.3.

The complexity measure method ($\alpha_c = 1$) requires a large number of iterations since it explores all low complexity wordlengths blocks such as w_1 or w_3. However, these wordlengths have small effect to the performance even though they are adequately long.

The distortion measure method ($\alpha_c = 0$) and complexity-and-distortion measure method ($0 < \alpha_c < 1$) requires a small number of iterations compared to the complexity measure method. The distortion measure method, however, results in large total complexity. Full search (FS) [5] that ex-

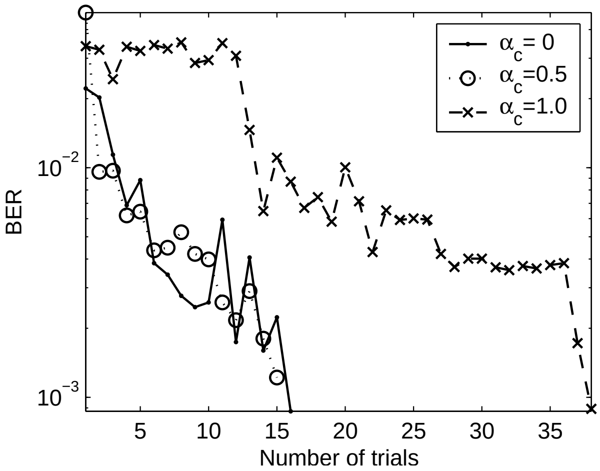

Fig. 3. Number of iterations according to complexity measure weighting factor α_c with increasing wordlengths (maximum wordlength = 16 bits).

haustively searches feasible wordlengths for the low hardware complexity from the MWL is also simulated for comparison. The complexities of four methods are shown in the Table 1. The CDM method leads high complexity compared to the CM and FS, but it has low complexity compared to the DM. The CDM reduces the number of trials compared to the CM and FS.

CDM method has flexibility to search for an optimum wordlength. This method decreases the number of iterations compared to CM and FS method and reduces the hardware cost compared to DM method.

5. CONCLUSION

The proposed CDM model expresses both the DM model and the CM model choosing the weighting factor between 0 to 1. We have demonstrated the application to the wordlength design of the FBWA digital demodulator that has a large complexity difference among digital blocks. The CDM method has flexibility to search for an optimum wordlength. Using adequate value of the weighting factor decreases the number of iterations compared to CM method and reduce the complexity compared to the DM method. Future extensions of this work include the study of the analytic wordlength optimization and eventually a combined wordlength optimization with a simulation-based method. Development of the automatic wordlength optimization is also a future goal to reduce the number of iterations.

6. REFERENCES

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