MIMO Receiver Design in Impulsive Noise

Aditya Chopra and Kapil Gulati

Final Project Report

Advanced Space Time Communications

Prof. Robert Heath

December 7^{th}, 2007

Abstract

Multi-Input Multi-Output (MIMO) receivers have been typically designed and their communication performance studied in the presence of additive white Gaussian noise. While this is an accurate model for the thermal noise present at the receiver, it falls short of modeling the radio frequency interference (RFI) that may affect the transceivers. RFI is a combination of independent radiation events and has predominantly non-Gaussian impulsive statistics. In this paper, MIMO receiver design in the presence of impulsive noise has been considered. Performance degradation of conventional MIMO receivers in impulsive noise is demonstrated to motivate this problem. Different impulsive noise models are discussed and a maximum likelihood (ML) receiver is derived for the two receive antenna case which assumes the knowledge of noise statistics at the receiver. Furthermore, a sub-optimal ML receiver is derived with reduced complexity and estimated noise statistics. Simulations reveal significant communication performance improvement over conventional receivers.

I. Introduction

Multi-Input Multi-Output (MIMO) systems have been typically designed under the assumption of additive white Gaussian noise. However, this assumption may not hold true as impulsive non-Gaussian noise is prevalent in many communication environments generated by external sources as man-made electromagnetic interference and atmospheric noise. In [9], it was shown that co-channel interference can also be modeled as an impulsive noise. Furthermore, it was recently shown that wireless data communication transceivers deployed on a computation platform are greatly affected by the radio frequency interference (RFI), which is also impulsive by nature, generated by the computation platform itself [4].

In [1], performance analysis of typical MIMO systems, such as zero forcing, maximum likelihood and space-time block coding systems, is performed in a mixture of Gaussian and impulsive noise. Results demonstrate that the communication performance of such systems degrade in an impulsive noise environment. This motivates the requirement of accurate noise modeling and MIMO receiver design in presence of impulsive noise.

Modeling of impulsive noise sources, primarily RFI sources, has been actively studied for the single antenna case [4]. The key models are the Middleton's Class A, B and C models [7] that capture the non-Gaussian phenomenon governing electromagnetic interference and also explicitly include a Gaussian component in the noise, which simulates the effect of thermal noise at the receiver. However, extensions of these models for multi-antenna systems have not been fully explored. An extension of the Middleton noise model to a two-receive antenna system was developed in [5] for narrowband noise with the far-field assumptions. MIMO receiver design and space-time block coding in presence of spatially uncorrelated Middleton noise was studied in [3]. However, the assumption of spatially uncorrelated noise is generally not valid in physical systems which limits the application of this work. In [2], an adaptive MIMO receiver was proposed where the impulsive noise is modeled as a mixture of multivariate Gaussian distributions.

In this paper, we consider the problem of impulsive noise modeling and MIMO receiver design in presence impulsive noise modeled as a Middleton noise. A maximum likelihood (ML) receiver is derived and a sub-optimal ML is proposed.

The organization of this paper is as follows. Section II describes the MIMO system model under consideration. Discussion on impulsive noise modeling is provided in Section III. Section IV considers the problem of receiver design in presence of impulsive noise. The results and conclusions for the proposed receiver design are presented in sections V and VI respectively.

II. System Model

Consider a wireless communication system where the transmitter is equipped with N_t antennas and the receiver is equipped with N_r antennas. The baseband MIMO channel model can be expressed as

$$\mathbf{Y} = \sqrt{\frac{E_s}{N_t}}\mathbf{H}\mathbf{S} + \mathbf{N}, \tag{1}$$

where $\mathbf{Y}$ is the $N_r \times T$ matrix of received signals, T is the length of the transmitted data block, $\mathbf{H}$ is an $N_r \times N_t$ channel matrix, with independent and identical distributed (iid) complex Gaussian entries with mean zero and unit variance, E_s is the total transmit power, $\mathbf{S}$ is the $N_t \times T$ transmitted data block, $\mathbf{N}$ is the $N_t \times T$ additive impulsive noise matrix.

In this paper, we consider the problem of receiver design for for the case of two transmit and two receive antennas ($N_t = 2, N_r = 2$) only for mathematical tractability of the noise models as discussed in Section III.

III. Noise Modeling

The primary source of impulsive noise in wireless data communication transceivers is the radio frequency interference (RFI) experienced by the transceiver. Two approaches for modeling radio frequency interference (RFI) are through physical and statistical-physical modeling. In physical modeling, each source of RFI would require a different circuit model. On the other hand, statistical-physical models provide a universal model for accurately modeling RFI from natural and man-made sources. The key statistical-physical models developed for single antenna case are the Middletons Class A, B and C noise models [7]. Middleton model are the most widely accepted model for RFI primarily since these models are canonical, i.e. their mathematical form is independent of the physical environment. However, these models were derived for single antenna systems and their extension to multi-antenna case is not straightforward.

A. Statistical-Physical Models

1) Middleton Class A model: For a two-antenna system, [5] derived an extension to the single antenna Middleton Class A model with the narrowband and far-field assumptions. Let $\mathbf{z} = [z_1 \; z_2]$ denote the vector of the noise observations for the two-antenna system. The joint probability density function of the noise observations is summarized in Equation (2).

$$f_{Z_1(t),Z_2(t)}(z_1, z_2) = e^{-A} \left[\frac{\Phi_\kappa \left(\frac{z_1}{c_{0+0}}, \frac{z_2}{\hat{c}_{0+0}} \right)}{c_{0+0}\hat{c}_{0+0}} \right] + \left(1 - e^{-A}\right) \left[\frac{\Phi_\kappa \left(\frac{z_1}{c_{0+1}}, \frac{z_2}{\hat{c}_{0+1}} \right)}{c_{0+1}\hat{c}_{0+1}} \right] \tag{2}$$

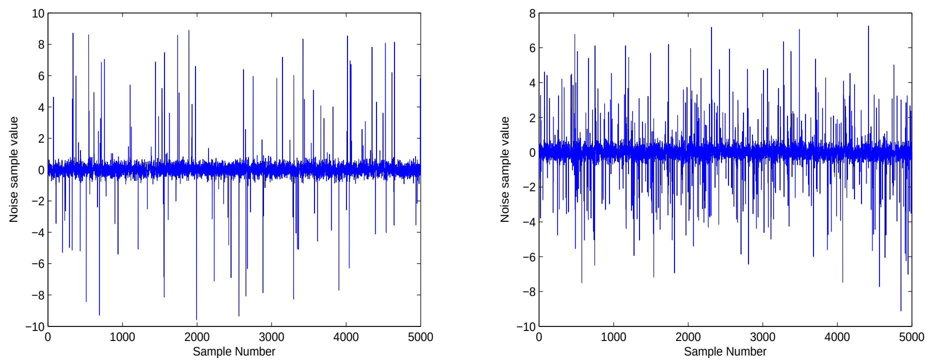
where,

$$c^2_{0+n} = \frac{\frac{n}{A} + \Gamma_1}{1 + \Gamma_1} \;, \qquad \hat{c}^2_{0+n} = \frac{\frac{n}{A} + \Gamma_2}{1 + \Gamma_2} \quad \text{and} \quad \Phi_\kappa = \frac{1}{2\pi\sqrt{1-\kappa^2}} e^{\frac{-\left[z_1^2 + z_2^2 - 2z_1 z_2 \kappa\right]}{2(1-\kappa^2)}} \tag{3}$$

Hence the model is uniquely determined by the following four parameters:

- A is the overlap index. It is the product of the average number of emission events impinging on the receiver per second and mean duration of a typical interfering source emission, and $A \in [10^{-2}; 1]$ in general [10].
- Γ_1, Γ_2 are the ratio of the Gaussian to the non-Gaussian component intensity at each antenna, and $\Gamma_1, \Gamma_2 \in [10^{-6}, 1]$ in general [10].
- κ is the correlation coefficient between z_1 and z_2.

Figure 1 shows the noise sample values for different values of model parameters. The "impulsiveness" of the noise can be controlled through these model parameters.



(a) Less impulsive noise samples ($A = 0.03, \Gamma = 0.1$) (b) Highly impulsive noise samples ($A = 0.12, \Gamma = 0.1$)

Fig. 1: Noise samples at different model parameters

B. Statistical Models

While the Middleton model is known to accurately model RFI sources, their practical applications are limited due to the intractable form of their distributions. Hence, many authors [2],[8] have considered statistical models that capture the ”impulsive” properties of the noise.

1) Weighted Sum of Complex Gaussian: The probability density function (pdf) of the $N_r \times 1$ noise vector $\mathbf{N}$ can be modeled as a L-term mixture of multi-dimensional complex Gaussian pdfs [2].

$$f_W(w) = \sum_{l=1}^{L} \frac{\lambda_l}{\pi^{N_r} |\mathbf{R}_l|} exp\left(-\mathbf{N}^H \mathbf{R}_l^{-1} \mathbf{N}\right), \tag{4}$$

where, $(.)^H$ denotes the Hermitian operation, $\lambda_l \geq 0$, $l = 1, 2, \cdots, L$ and $\sum_{l=1}^{L} \lambda_l = 1$. The weights $\lambda_1, \lambda_2, \cdots, \lambda_L$ and the covariance matrices $\mathbf{R}_1, \mathbf{R}_2, \cdots, \mathbf{R}_L$ determine the ”impulsiveness” of the noise.

2) Multivariate Symmetric Alpha Stable Model: If the $N_r \times 1$ noise vector $\mathbf{N}$ is alpha-stable and elliptically contoured [8], then it has a joint characteristic function [8],

$$\Phi_{\mathbf{N}}(\alpha, \delta, \mathbf{\Sigma}) = exp(-(\mathbf{N}^T \mathbf{\Sigma} \mathbf{N})^{\alpha/2} + j < \mathbf{N}, \delta >) \tag{5}$$

where, $< .,. >$ denotes the inner product of the two vectors and $(.)^T$ represents matrix transpose. No closed form expression of the probability density function exists in general. The multivariate stable distributions are characterized by the following three parameters:

- α is the characteristic exponent which determines the impulsiveness of the stable process, and $\alpha \in [0, 2]$.
- δ is the $N_r \times 1$ localization vector which is analogous to the mean of a Gaussian process
- $\mathbf{\Sigma}$ is the $N_r \times N_r$ is a positive definite dispersion matrix analogous to the covariance matrix for a Gaussian process.

IV. Receiver Design

Consider a 2×2 MIMO system model defined in Equation (1) with the additive impulsive noise modeled as a Middleton noise defined in Section III-A1. We assume throughout that the channel $\mathbf{H}$ is known at the receiver and all transmitted codewords are equally likely. We start with the derivation of the maximum likelihood estimator assuming the knowledge of the noise parameters at the receiver to provide a performance benchmark. Furthermore, the real and imaginary components of the noise are considered are independent and follow the Middleton distribution.

A. Maximum Likelihood Receiver

The Maximum Likelihood (ML) is a class of receivers that performs detection using the following rule:

$$\hat{\mathbf{S}}_{\mathbf{ML}} = \arg \underbrace{\max}_{\mathbf{S} \in \mathcal{S}} \{p(\mathbf{Y}|\mathbf{S})\} \tag{6}$$

where $\mathcal{S}$ is the set of all possible transmit codeword vectors. Using the joint probability density function of the Middleton noise model given in Equation (2), the ML estimate can be expressed as

$$\hat{\mathbf{S}}_{\mathbf{ML}} = \arg\underbrace{\max}_{\mathbf{S}\in\mathcal{S}} \underbrace{\left\{ e^{-A} \left[\frac{\Phi_{\kappa}\left(\frac{(\mathbf{Y}-\mathbf{HS})\mathbf{e_1}}{c_{0+0}}, \frac{(\mathbf{Y}-\mathbf{HS})\mathbf{e_2}}{\hat{c}_{0+0}} \right)}{c_{0+0}\hat{c}_{0+0}} \right] + \left(1-e^{-A}\right) \left[\frac{\Phi_{\kappa}\left(\frac{(\mathbf{Y}-\mathbf{HS})\mathbf{e_1}}{c_{0+1}}, \frac{(\mathbf{Y}-\mathbf{HS})\mathbf{e_2}}{\hat{c}_{0+1}} \right)}{c_{0+1}\hat{c}_{0+1}} \right] \right\}}_{f_{\mathbf{N}}(\mathbf{Y}-\mathbf{HS})} \tag{7}$$

where $\mathbf{e}_1^T = [1\ 0], \mathbf{e}_2^T = [0\ 1]$. Note that since the Middleton noise model is defined only for real sample observations, we can extend the estimator for complex transmitted codewords by forming

$$\hat{\mathbf{S}}_{\mathbf{ML}} = \arg\underbrace{\max}_{\mathbf{S}\in\mathcal{S}} \{ f_{\mathbf{N}}\left(real(\mathbf{Y}-\mathbf{HS})\right)\ \ f_{\mathbf{N}}\left(imag(\mathbf{Y}-\mathbf{HS})\right) \} \tag{8}$$

where, $real(\cdot)$ and $imag(\cdot)$ denote the real and imaginary part of the vector. Further, note that the estimator requires the knowledge of the noise parameters defined in Section III-A1 at the receiver. The computational complexity of this ML receiver is considerably higher than it's Gaussian counterpart since the cost function given by Equation (7) is no longer a minimum distance criterion. The performance of this ML receiver was analyzed for a 2×2 MIMO system utilizing spatial multiplexing and results are shown in Figure 3.

B. Sub-Optimum Maximum Likelihood Receiver

A sub-optimal ML receiver can be derived by considering noise parameter estimation at the receiver and reducing the complexity of the cost function defined in equation (7).

1) Noise Parameter Estimation: We propose a computationally efficient noise parameter estimation technique based on the method of moments discussed by Middleton in [6] for the single antenna case. We assume that sufficient noise samples are available using training symbols. For a single antenna case, the noise statistics are completely determined by the parameters A and Γ as defined in section III-A1. Since the analytical expression for the characteristic equation for Middleton Class A model is known, we can derive the expression for all even-order moments (odd order moments are zero [6]) and relate the parameters to the moments generated. This yields the following result for the parameter estimates:

$$A = \frac{9\left(e_4 - 2e_2^2\right)}{2\left(e_6 + 12e_2^3 - 9e_2e_4\right)^2}, \quad \Gamma = \frac{2e_2\left(e_6 + 12e_2^3 - 9e_2e_4\right)}{3\left(e_4 - 2e_2^3\right)^3} - 1 \tag{9}$$

where, e_2, e_4, e_6 are the second, fourth and sixth order moments of the envelope data. The envelope of the noise can be derived from the noise observation via the *teager operator*, given as $\Psi(x[n]) = x^2[n] - x[n-1]x[n+1]$. The envelope of the signal can be formed as $E[n] = \sqrt{\Psi(z[n])}$, where $z[n]$ are the noise observations.

For the two-antenna case considered, noise parameters listed in section III-A1 can be estimated using independent parameter estimators at each antenna yielding the estimates $\hat{A}_i$ and $\hat{\Gamma}_i$, for $i = 1, 2$ using Equation (9).

- $\hat{A}$ can be estimated as mean of $\hat{A}_1$ and $\hat{A}_2$
- $\hat{\Gamma}_1, \hat{\Gamma}_2$ estimated at each antenna
- $\hat{\kappa}$ is the empirically estimated correlation coefficient based on the noise samples

2) Complexity Reduction: The maximum likelihood estimator derived in Equation (7) can be expanded using Equation (3) as

$$\begin{aligned}\hat{\mathbf{S}}_{\mathbf{ML}} = \arg \underbrace{\max}_{\mathbf{S} \in \mathcal{S}} \Bigg\{ & \frac{e^{-A}}{2\pi \left(\frac{\Gamma_1}{1+\Gamma_1}\right)\left(\frac{\Gamma_2}{1+\Gamma_2}\right)\sqrt{1-\kappa^2}} \, e^{\frac{-\left[\left(\frac{z_1}{c_{0+0}}\right)^2 + \left(\frac{z_2}{\hat{c}_{0+0}}\right)^2 - 2\kappa\left(\frac{z_1}{c_{0+0}}\right)\left(\frac{z_2}{\hat{c}_{0+0}}\right)\right]}{2(1-\kappa^2)}} + \\ & + \frac{1 - e^{-A}}{2\pi \left(\frac{\frac{1}{A}+\Gamma_1}{1+\Gamma_1}\right)\left(\frac{\frac{1}{A}+\Gamma_2}{1+\Gamma_2}\right)\sqrt{1-\kappa^2}} \, e^{\frac{-\left[\left(\frac{z_1}{c_{0+0}}\right)^2 + \left(\frac{z_2}{\hat{c}_{0+1}}\right)^2 - 2\kappa\left(\frac{z_1}{c_{0+1}}\right)\left(\frac{z_2}{\hat{c}_{0+1}}\right)\right]}{2(1-\kappa^2)}} \Bigg\}\end{aligned} \tag{10}$$

where, $z_1 = (\mathbf{Y} - \mathbf{HS})[1\ 0]^T, z_2 = (\mathbf{Y} - \mathbf{HS})[0\ 1]^T$. Based on the typical range of the parameters A, Γ_1 and Γ_2 defined in section III-A1, we notice that the scaling of exponent for the second term is much smaller than the first term. The sub-optimum maximum likelihood estimator can hence be formed based on the estimated noise parameters as

$$\hat{\mathbf{S}}_{\mathbf{ML}} = \arg \underbrace{\min}_{\mathbf{S} \in \mathcal{S}} \left\{ \left(\frac{z_1}{c_{0+0}}\right)^2 + \left(\frac{z_2}{\hat{c}_{0+0}}\right)^2 - 2\kappa\left(\frac{z_1}{c_{0+0}}\right)\left(\frac{z_2}{\hat{c}_{0+0}}\right) \right\} \tag{11}$$

V. RESULTS

The 2×2 MIMO system discussed in section II was simulated to observe the performance of MIMO receivers in impulsive noise. The symbol error rate performance of the optimal and the proposed sub-optimal maximum likelihood (ML) detector for impulsive noise was also simulated and compared to the ML detector for Gaussian Noise. The symbol error rates were calculated as an average for 10^6 symbols transmitted. 4-QAM modulation was used for spatial multiplexing while 16-QAM modulation was used for Alamouti transmission strategy.

A. Performance analysis of conventional MIMO receivers in impulsive noise

Figure 2 shows a communication performance degradation in some of the typical MIMO receivers, in the presence of impulsive noise. At low SNR, these standard receivers show little deterioration in performance, however as SNR increases, their performance deteriorates significantly in the presence of impulsive noise.

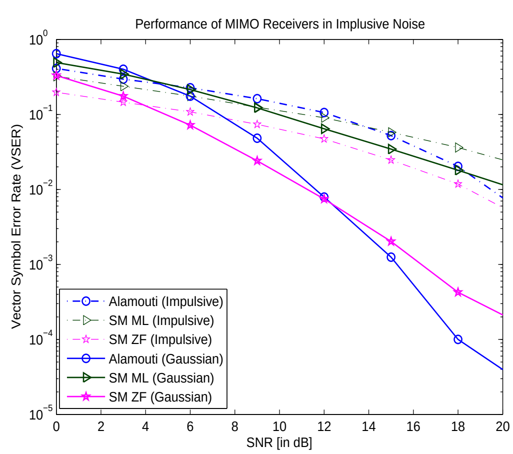
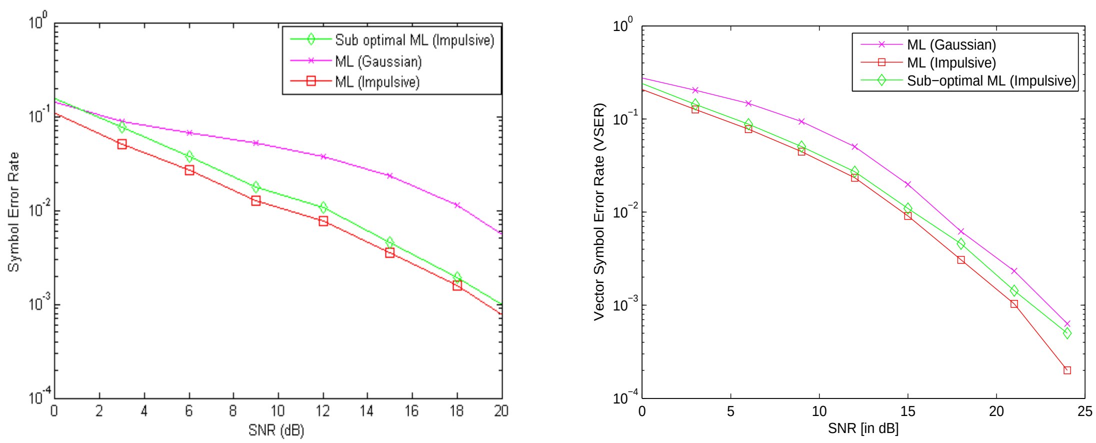


Fig. 2: Performance degradation of typical MIMO receivers in impulsive noise

B. Comparison of Gaussian and Impulsive ML Detector

A 2×2 MIMO system operating in spatial multiplexing mode was simulated to compare the performance of the proposed and traditional receivers. Figure 3 shows the symbol error performance of the ML receiver for Gaussian distributed noise, optimal ML receiver for impulsive noise and the sub-optimal ML receiver. In this analysis, model parameters A, Γ_1 and Γ_2 were assigned values 0.05, 0.1 and 0.1 respectively, resulting in noise with low rate of impulses. Figure 3 also shows the performance of these receivers in highly impulsive noise, with model parameters A, Γ_1 and Γ_2 having values of 0.2, 0.1 and 0.1 respectively. As the rate of impulses in noise increases, it's density function starts approximating the Gaussian distribution with high variance, and as shown in the figures, the optimal and sub-optimal ML receivers designed for impulsive noise, approach the performance of the Gaussian ML detector. Thus, one possible receiver design may include an algorithm to switch to the standard Gaussian ML decoding if the estimated noise parameters suggest that the noise is highly impulsive.



(a) Symbol Error Performance in less impulsive noise $(A = 0.05, \Gamma = 0.1)$

(b) Symbol Error Performance in highly impulsive noise $(A = 0.12, \Gamma = 0.1)$

Fig. 3: Performance comparison of ML detectors in different classes of impulsive noise

VI. Conclusions

In this paper, we evaluated the performance of typical MIMO communication system in the presence of additive impulsive noise and showed that these systems suffer from a bit-error performance degradation compared to receivers with additive Gaussian noise . In the 2x2 MIMO system under consideration, the Middleton model for impulsive noise was assumed across the receivers. An optimal maximum likelihood (ML) receiver was derived assuming the Middleton noise model. A sub-optimal ML receiver was also proposed, with noise parameter estimation and a reduced complexity likelihood function. It was shown that this practical receiver exhibited symbol-error rate performance close to the optimal ML receiver. Both the optimal and the sub-optimal ML receivers for the impulsive noise model showed significant improvement in communication performance over the ML receiver which assumed a Gaussian density function for the additive noise.

REFERENCES

[1] Weiyu Xu Anxin Li, Youzheng Wang and Zucheng Zhou. Performance evaluation of mimo systems in a mixture of gaussian noise and impulsive noise. *Proc. 10th Asia Pacific Conference on Communications and 5t h International Symposium on Multi-Dimensional Mobile- Communications*, 2004.

[2] R.S. Blum, R.J. Kozick, and B.M. Sadler. An adaptive spatial diversity receiver for non-gaussian interference and noise. In *1997 First IEEE Signal Processing Workshop on Signal Processing Advances in Wireless Communications*, pages 385–388, 16-18 April 1997.

[3] Ping Gao and C. Tepedelenlioglu. Space-time coding over mimo channels with impulsive noise. *IEEE Transactions on Wireless Communications*, 6(1):220–229, January 2007.

[4] Marcel Nassar Kapil Gulati and Brian L. Evans. In-platform radio frequency interference mitigation for wireless communications. Technical report, The University of Texas at Austin, http://users.ece.utexas.edu/ bevans/projects/rfi/reports/RFIReportSpring2007.doc, 2007.

[5] K.F. McDonald and R.S. Blum. A physically-based impulsive noise model for array observations. In *Signals, Systems & Computers, 1997. Conference Record of the Thirty-First Asilomar Conference on*, volume 1, pages 448–452vol.1, 2-5 Nov. 1997.

[6] D. Middleton. Procedures for determining the properties of the first-order canonical models of class a and class b electromagnetic interference. *IEEE Trans. on Electromagnetic Compatibility*, 22:190–208, August 1979.

[7] D. Middleton. Non-gaussian noise models in signal processing for telecommunications: New methods and results for class a and class b noise models. *IEEE Transactions on Information Theory*, 45(4):1129 – 1149, May 1999.

[8] John P. Nolan. Multivariate stable densities and distribution functions: general and elliptical case. In *Deutsche Bundesbank's 2005 Annual Fall Conference*, 2005.

[9] Xueshi Yang and A.P. Petropulu. Co-channel interference modeling and analysis in a poisson field of interferers in wireless communications. *IEEE Transactions on Signal Processing*, 51(1), 2003.

[10] S. M. Zabin and H. V. Poor. Efficient estimation of class a noise parameters via the em [expectation-maximization] algorithms. *IEEE Transaction on Information Theory*, 37(1):60–72, Jan 1991.