Modeling and Mitigation of Interference in Wireless Receivers with Multiple Antennae

Aditya Chopra

PhD Committee:

Prof. Jeffrey Andrews

Prof. Brian L. Evans **(Supervisor)**

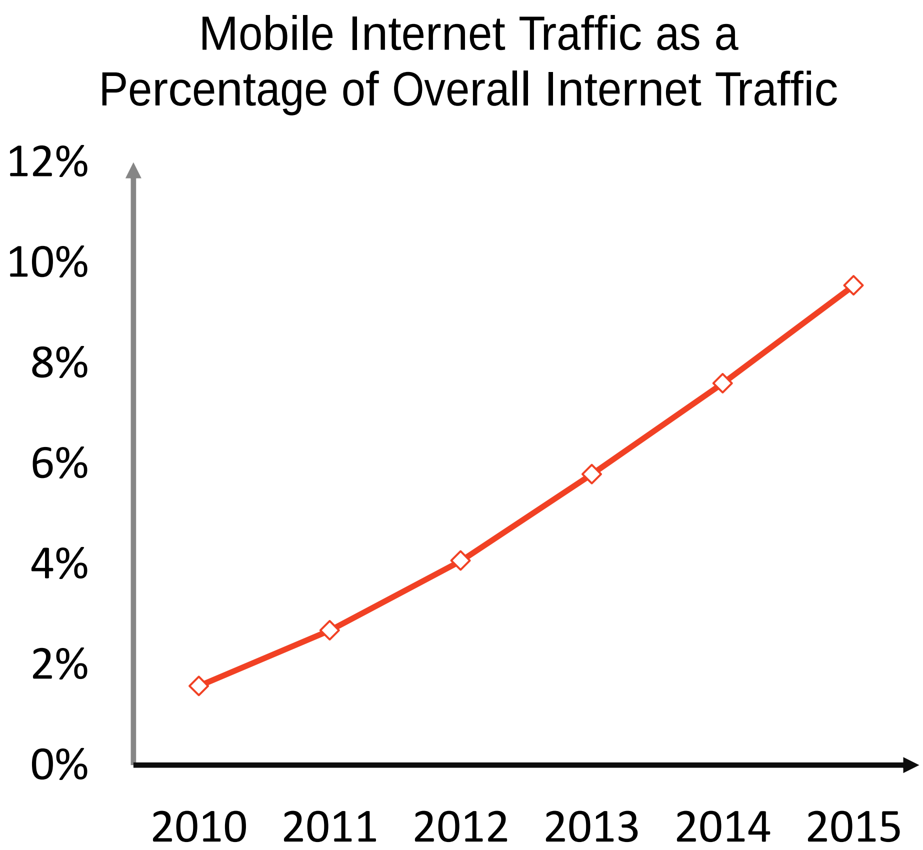
Prof. Robert W. Heath, Jr.

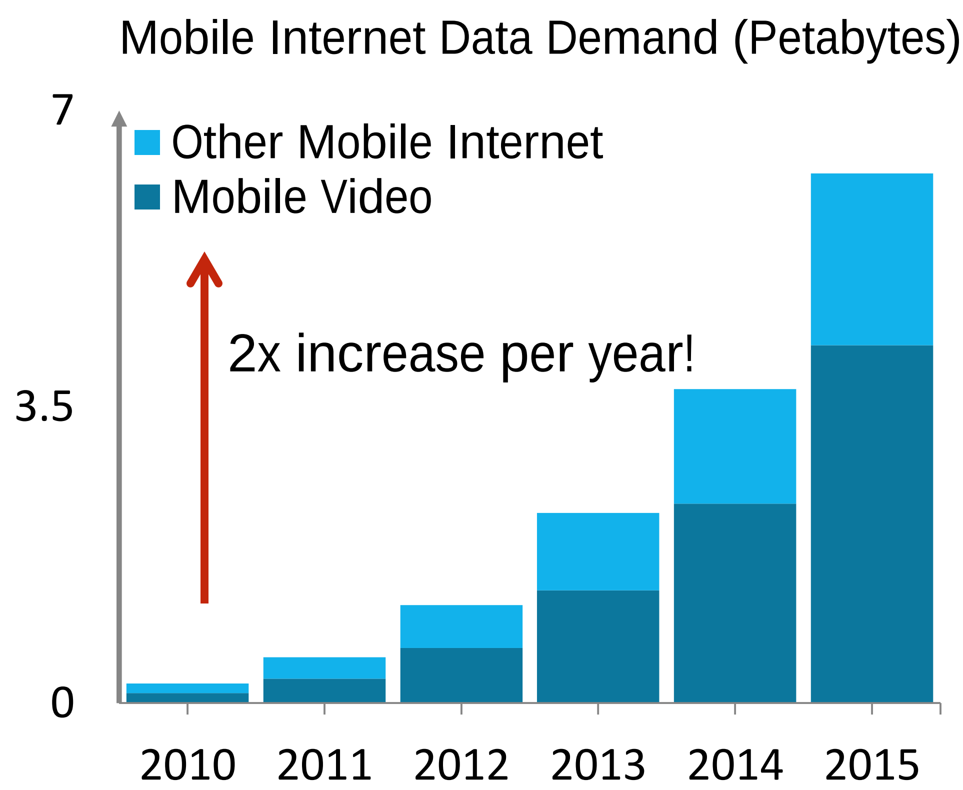
Prof. Elmira Popova

Prof. Haris Vikalo

November 18, 2011

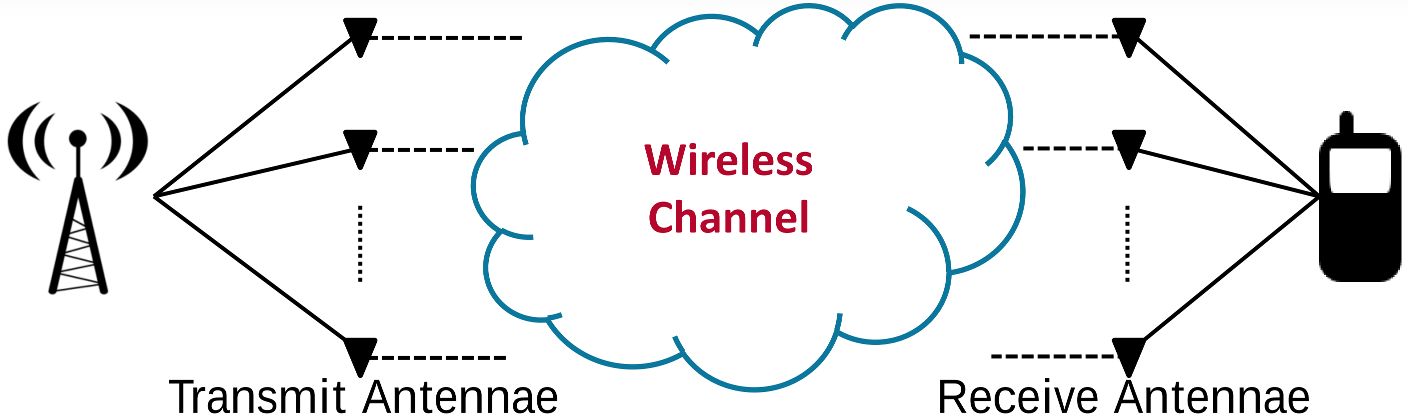
The demand for wireless Internet data is predicted to increase 1000× over the next decade





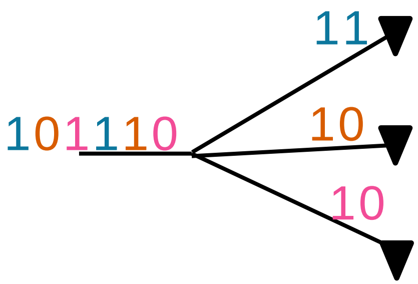
Source: Cisco Visual Networking Index Forecast

Wireless communication systems are increasingly using multiple antennae to meet demand



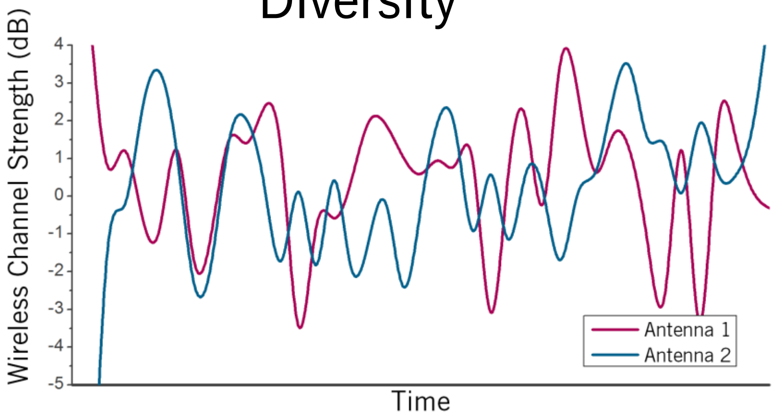
Benefits

Multiplexing



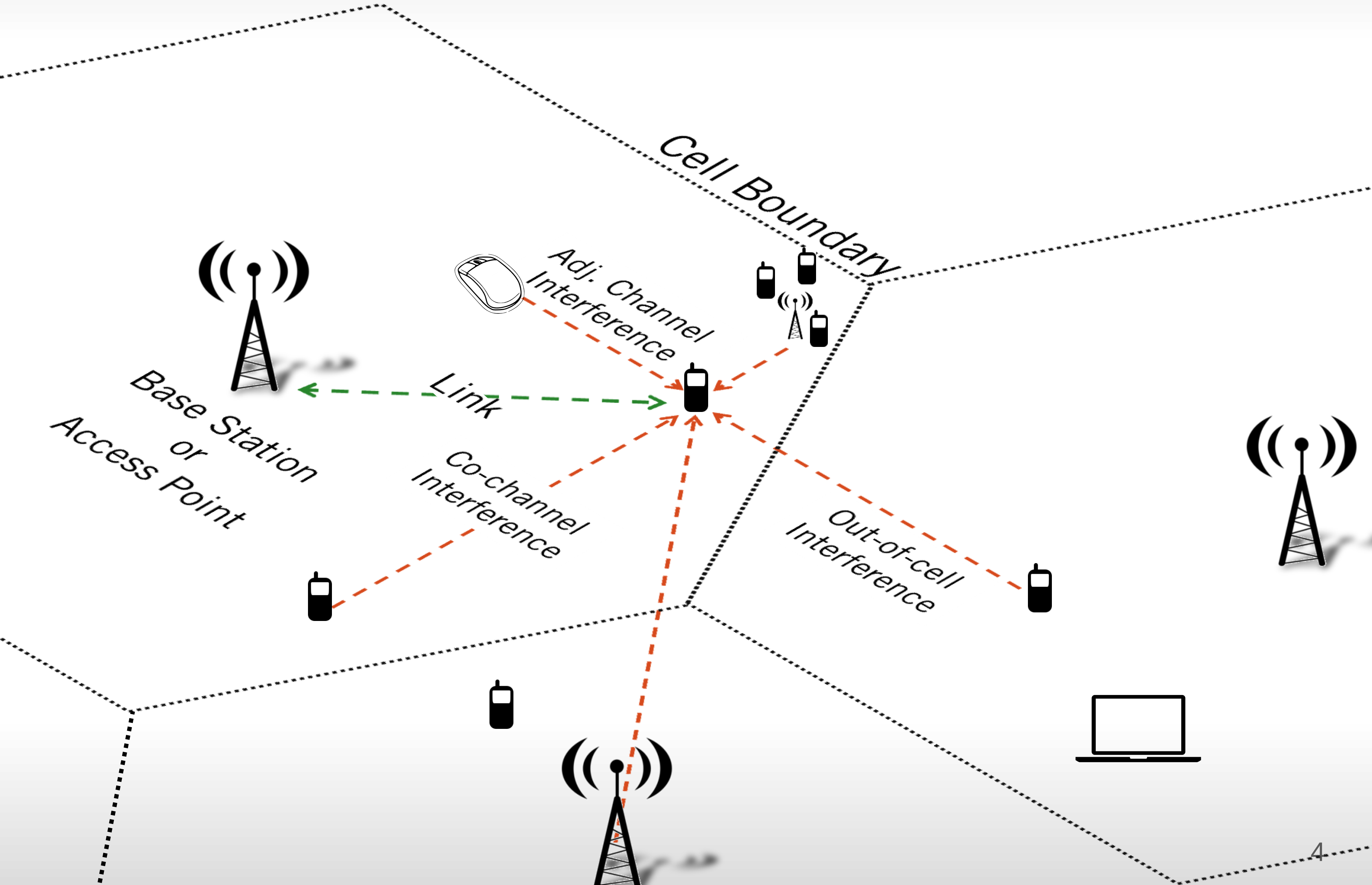
Multiple data streams are transmitted simultaneously to increase data rate

Diversity



Antennae with strong channels compensate for antennae with weak channels to increase reliability

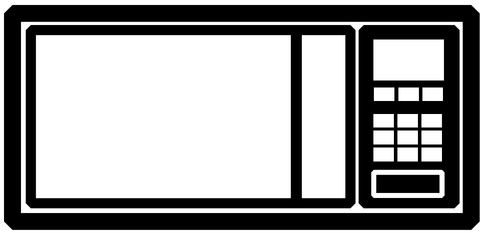
A growing mobile user population with increasing wireless data demand leads to interference



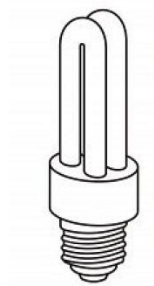
Interference is also caused by non-communicating source emissions …

Non-communicating devices

Microwave ovens



Fluorescent bulbs

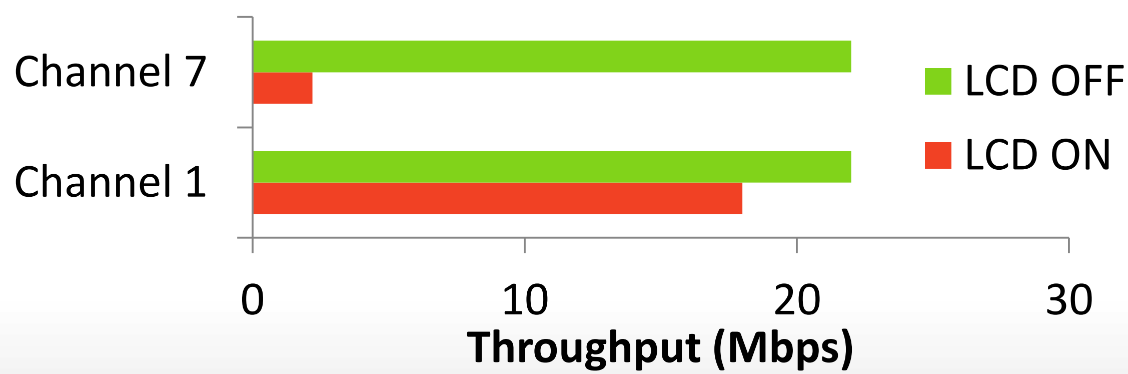


Computational Platform

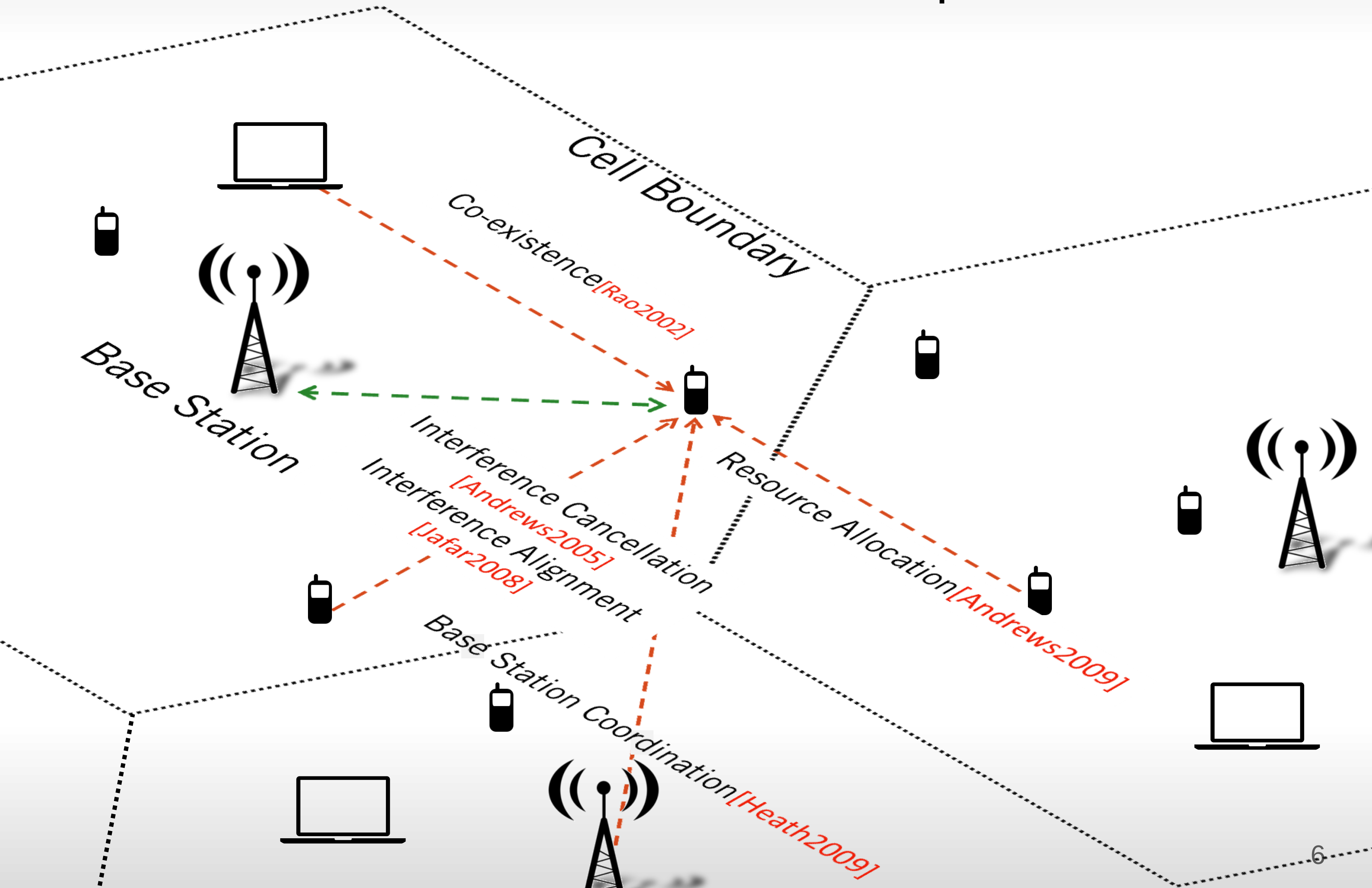
Clocks, amplifiers, busses

… and impairs wireless communication performance

Impact of platform interference from a laptop LCD on wireless throughput (IEEE 802.11g) [Slattery06]



Interference mitigation has been an active area of research over the past decade



I employ a statistical approach to the interference modeling and mitigation problem

Thesis statement

Accurate statistical modeling of interference observed by multi-antenna wireless receivers facilitates design of wireless systems with significant improvement in communication performance in interference-limited networks.

Proposed solution

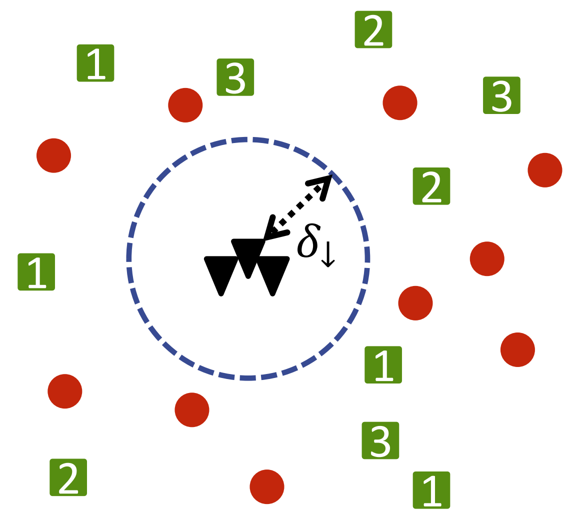
1. Model statistics of interference in multi-antenna receivers
2. Analyze performance of conventional multi-antenna receivers
3. Develop multi-antenna receiver algorithms using statistical models of interference

A statistical-physical model of interference generation and propagation

Key Features

- Co-located receiver antennae (▼)
- Interferers are common to all antennae (●) or exclusive to n^th antenna (n)
- Interferers are stochastically distributed in space as a 2D Poisson point process with intensity λ_0 (●), or λ_n (n) (per unit area)
- Interferer free guard-zone (----) of radius $\delta_{\uparrow}$
- Power law propagation and fast fading

A 3-antenna receiver within a Poisson field of interferers



Non-Gaussian distributions have been used in prior work to model single antenna interference statistics

Guard Zone Radius ($\delta_{\downarrow}$)	Single Antenna Statistics	Characteristic function	Parameters	Density Distribution
0	Symmetric Alpha Stable (SAS) [Sousa92]	$\Phi(\omega) = e^{\sigma\omega^{\alpha}}$	σ: Dispersion, > 0 α: Index, $\in (0,2]$	Not known except α=2$^{\sharp}$,1$^{\vee}$,0.5°
> 0	Middleton Class A (MCA) [Middleton99]	$\Phi(\omega) = e^{Ae^{-\frac{\omega^2\Omega}{2}}}$	A: Impulsive index > 0 Ω: Variance > 0	$f(x) = \sum_{m=0}^{\infty} \frac{A^m}{m!\sqrt{2\pi\Omega m}} e^{-\frac{x^2}{m\Omega}}$

$^{\sharp}$ Gaussian distribution
$^{\vee}$ Cauchy distribution
$^{\circ}$ Levy distribution

I derive joint statistics of interference observed by multi-antenna receivers

1. Wireless networks with guard zones (Centralized Networks)

2. Wireless networks without guard zones (De-centralized Networks)

Using the system model, the sum interference at the nth antenna is expressed as

$$Z_n = \sum_{i_0 \in \mathcal{S}_0} A_{i_0} e^{j\phi_{i_0}} H_{i_0,n} e^{j\theta_{i_0,n}} \|r_{i_0}\|^{-\frac{\gamma}{2}} + \sum_{i_n \in \mathcal{S}_n} A_{i_n} e^{j\phi_{i_n}} H_{i_n} e^{j\theta_{i_n}} \|r_{i_n}\|^{-\frac{\gamma}{2}}$$

SOURCE EMISSION

FADING CHANNEL

PATHLOSS

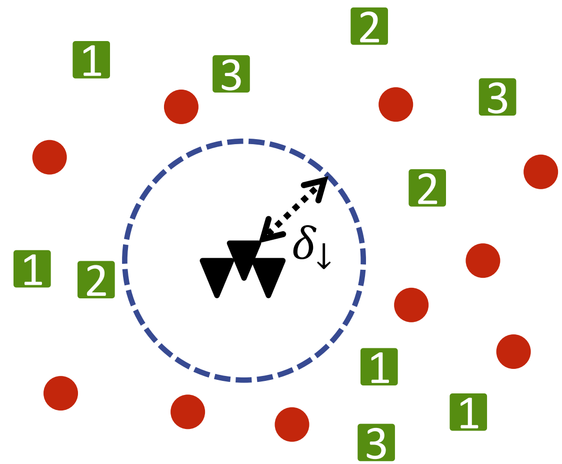
COMMON INTERFERERS

EXCLUSIVE INTERFERERS

Network model	Single Ant. Statistics	Multi antenna joint statistics	
		Common interferers	Independent interferers
Decentralized	Symmetric Alpha Stable (SAS)	Isotropic SAS [Ilow98] $\Phi(\mathbf{w}) = e^{\sigma_0 \|\mathbf{w}\|^{\alpha}}$	Independent SAS $\Phi(\mathbf{w}) = \prod_{n=1}^{N} e^{\sigma_n \lvert\omega_n\rvert^{\alpha}}$
Centralized	Middleton Class A (MCA)	$\times$	Independent MCA $\Phi(\mathbf{w}) = \prod_{n=1}^{N} e^{A_n e^{-\frac{\lVert w \rVert^2 \Omega_n}{2}}}$

In networks with guard zones, interference from common interferers exhibits isotropic Middleton Class A statistics

A 3-antenna receiver within a Poisson field of interferers



Interference in decentralized networks

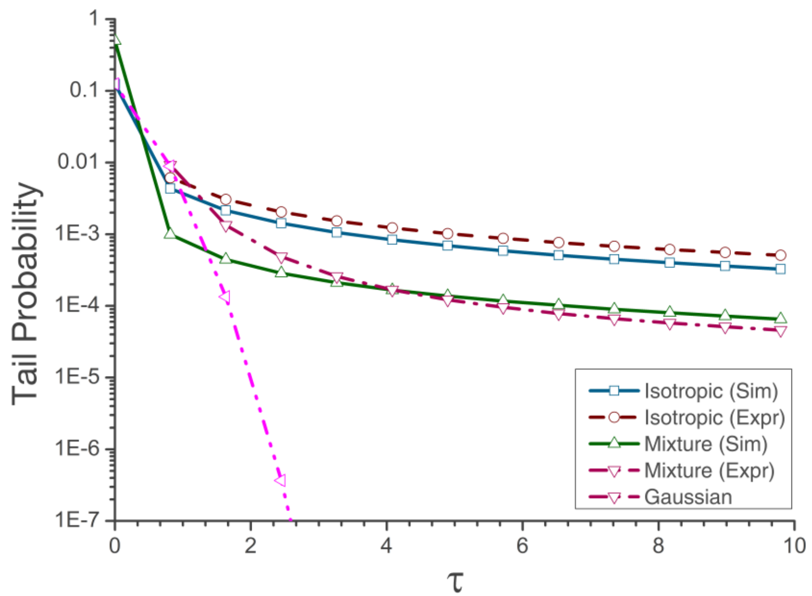
Joint characteristic function	Parameters
$\Phi(\mathbf{w}) = e^{\sigma_0 \lVert \mathbf{w} \rVert^\alpha} \times \prod_{n=1}^{N} e^{\sigma_n \lvert \omega_n \rvert^\alpha}$	$\alpha = \frac{4}{\gamma}$, $\sigma_n \propto \lambda_n$

Interference in centralized networks

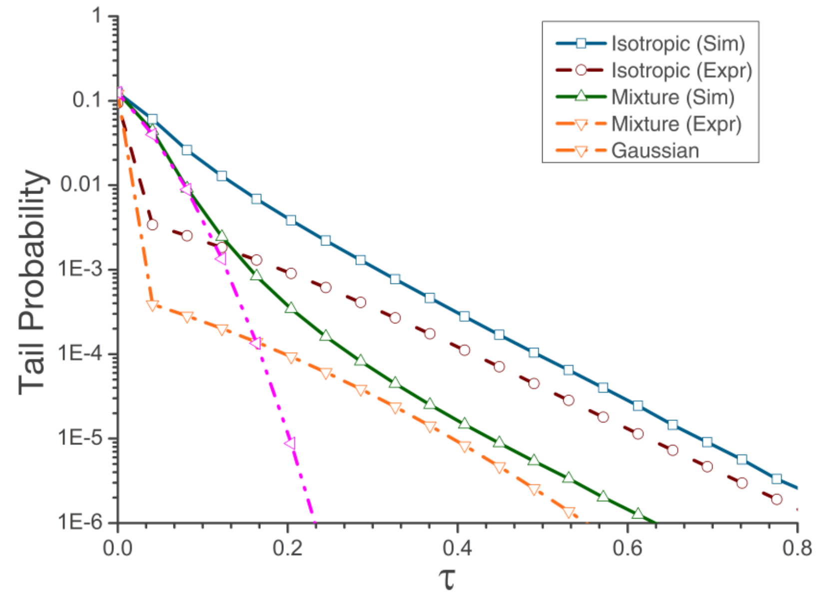
Joint characteristic function	Parameters
$\Phi(\mathbf{w}) = e^{A_0 e^{-\frac{\lVert \mathbf{w} \rVert^2 \Omega_0}{2}}} \times \prod_{n=1}^{N} e^{A_n e^{-\frac{\lvert w_n \rvert^2 \Omega_n}{2}}}$	$A_n \propto \lambda_n \delta_\downarrow^2$, $\Omega_n \propto A_n \delta_\downarrow^{-\gamma}$

Simulation results indicate a close match between proposed statistical models and simulated interference

Tail probability of simulated interference in networks without guard zones



Tail probability of simulated interference in networks with guard zones



Tail Probability: $\mathbb{P}\{\ |Z_1| > \tau, |Z_2| > \tau\ \ldots |Z_n| > \tau\}$

PARAMETER VALUES

γ	4	'Isotropic'	$\lambda_0 = 10^{-3}$, $\lambda_n = 0$ (per unit area)
$\delta_{\downarrow}$	1.2 (Distance Units) (w/ GZ)	'Mixture'	$\lambda_0 = 9.5 \times 10^{-4}, \lambda_n = 5 \times 10^{-5}$ (per unit area)

My framework for multi-antenna interference across co-located antennae results in joint statistics that are

1. Spatially isotropic (common interferers)
2. Spatially independent (exclusive interferers)
3. In a continuum between isotropic and independent (mixture)

for two impulsive distributions

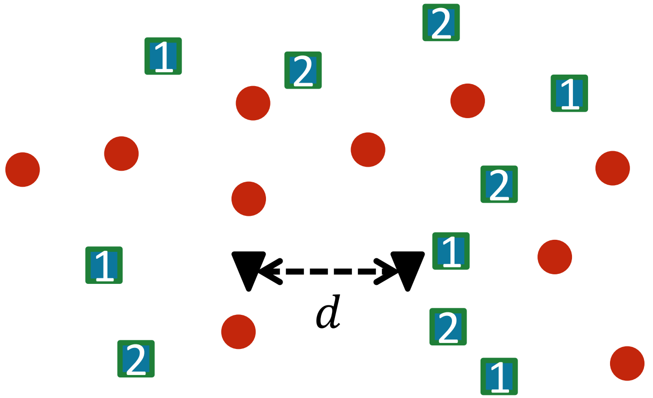
1. Middleton Class A (networks with guard zones)
2. Symmetric alpha stable (networks without guard zones)

In networks without guard zones, antenna separation is incorporated into the system model

Applications

- Cooperative MIMO
- Two-hop communication
- Temporal modeling of interference in mobile receivers

Two antennae (▼) and interferers (●) in a decentralized network



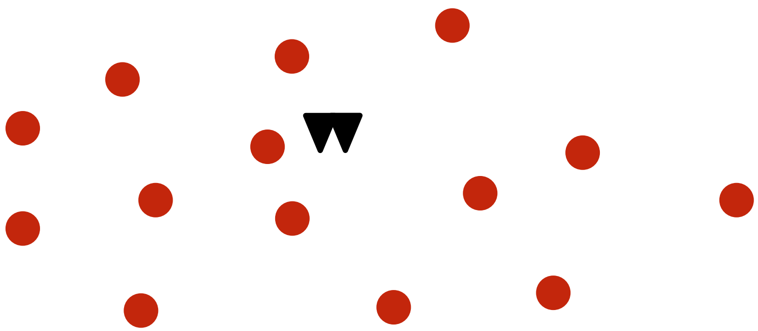
Sum interference expression

$$Z_1 = \sum_{i_0 \in \mathcal{S}_0} A_{i_0} e^{j\phi_{i_0}} H_{i_0,1} e^{j\theta_{i_0,1}} \|r_{i_0}\|^{-\frac{\gamma}{2}} + \sum_{i_1 \in \mathcal{S}_1} A_{i_1} e^{j\phi_{i_1}} H_{i_1} e^{j\theta_{i_1}} \|r_{i_1}\|^{-\frac{\gamma}{2}}$$

$$Z_2 = \sum_{i_0 \in \mathcal{S}_0} A_{i_0} e^{j\phi_{i_0}} H_{i_0,2} e^{j\theta_{i_0,2}} \|r_{i_0} - d\|^{-\frac{\gamma}{2}} + \sum_{i_2 \in \mathcal{S}_2} A_{i_2} e^{j\phi_{i_2}} H_{i_2} e^{j\theta_{i_2}} \|r_{i_2} - d\|^{-\frac{\gamma}{2}}$$

The extreme scenarios of antenna colocation ($d = 0$) and antenna isolation ($d \to \infty$) are readily resolved

Colocated antennae ($d = 0$)

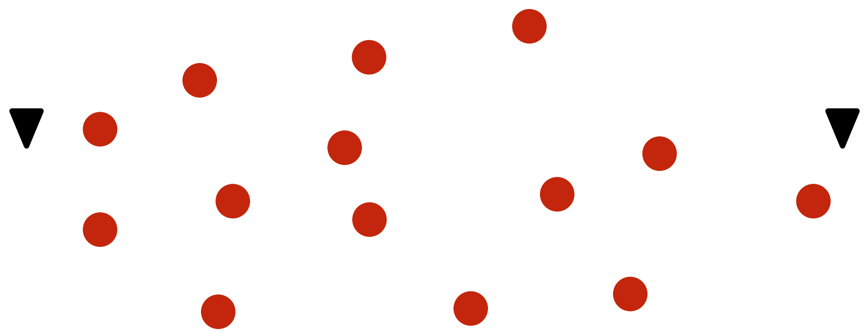


Characteristic function of interference:

$$\Phi(\omega_1, \omega_2) = e^{\sigma(\omega_1^2 + \omega_2^2)^{\frac{\alpha}{2}}}$$

Interference exhibits spatial isotropy

Remote antennae ($d \to \infty$)



Characteristic function of interference:

$$\Phi(\omega_1, \omega_2) = e^{\sigma(\omega_1^\alpha + \omega_2^\alpha)}$$

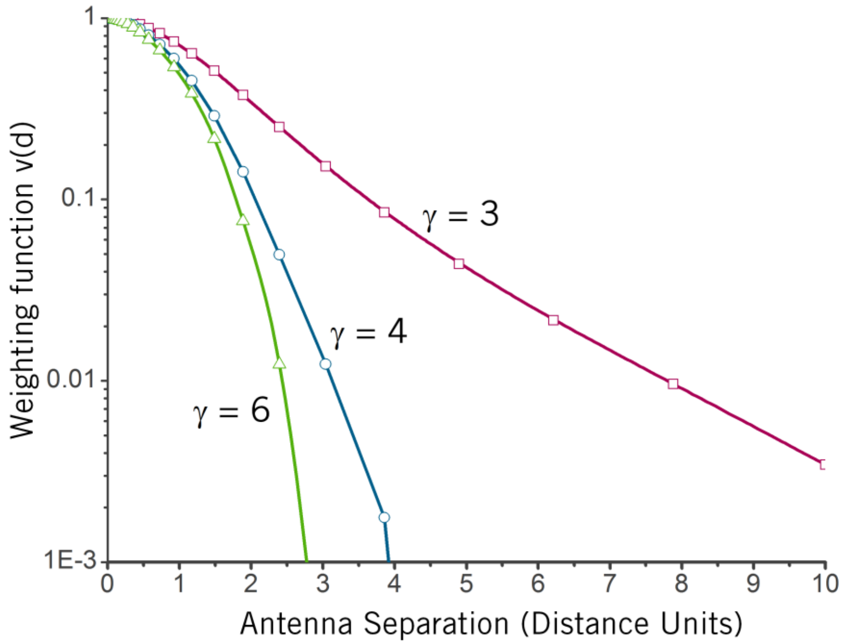
Interference exhibits spatial independence

Interference statistics move in a continuum from spatially isotropy to spatial independence as antenna separation increases!

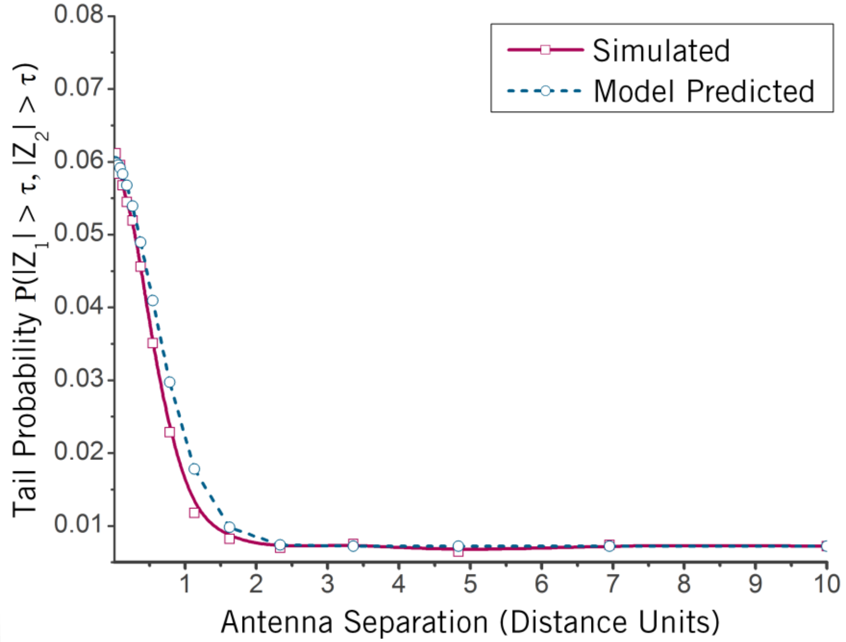
Interference statistics are approximated using the isotropic-independent statistical mixture framework

$$\Phi(\omega_1, \omega_2) \approx e^{\nu(d)\sigma(\omega_1^2+\omega_2^2)^{\frac{\alpha}{2}}+(1-\nu(d))\sigma(\omega_1^\alpha+\omega_2^\alpha)}$$

Weighting function $v(d)$ for different pathloss exponents (γ)



Joint tail probability vs. antenna separation for $\gamma = 4$, $\lambda_0 = 10^{-3}, \tau = 3$



The framework is used to evaluate communication performance of conventional multi-antenna receivers

Prior Work

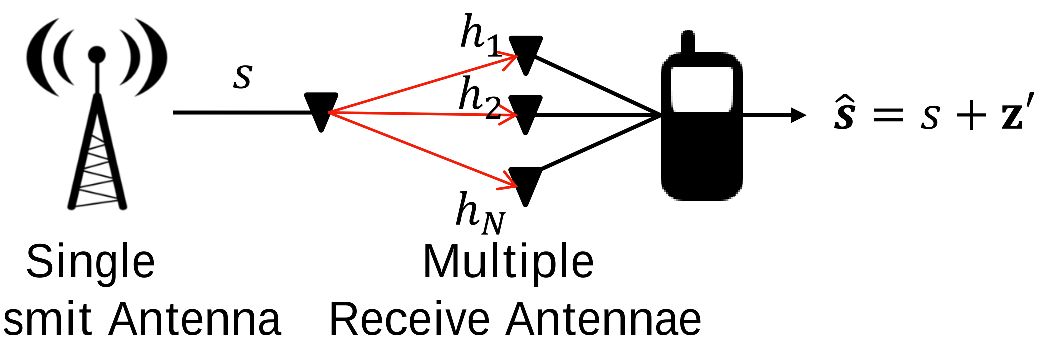
Article	Wireless System	Interference	Joint Statistics	Performance Metric
[Rajan2011]	SIMO	SAS	Independent	Bit Error Rate (BPSK)
[Gao2005]	SIMO	MCA	Indp. / Isotropic	Bit Error Rate (BPSK)
[Gao2007]	MIMO	MCA	Independent	Bit Error Rate (BPSK)

System Model

Received signal vector $\mathbf{y} = \mathbf{h}s + \mathbf{z}$

$\mathbf{h} = [h_1\ h_2\ h_3\ \cdots h_N]^T \sim$ Rayleigh(σ)

$\mathbf{z} \sim$ Isotropic + Independent SAS

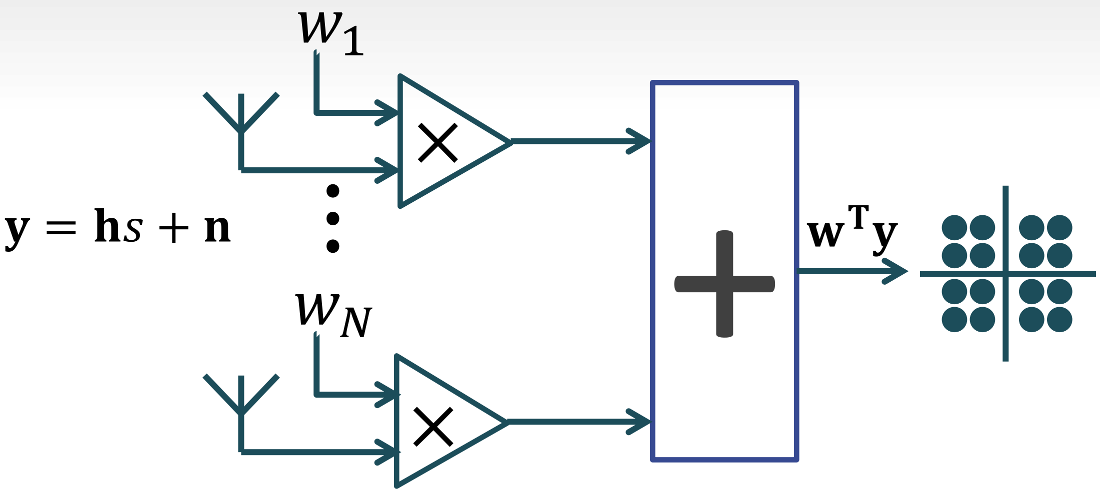


Communication performance is evaluated using outage probability

$$P^{out}(\theta) = \mathbb{P}\{\,\mathrm{SIR} < \theta\,\} = \mathbb{P}\left\{\frac{|s|^2}{|z'|^2} < \theta\right\}$$

SIR: Signal-to-Interference Ratio

Outage probability of linear combiners



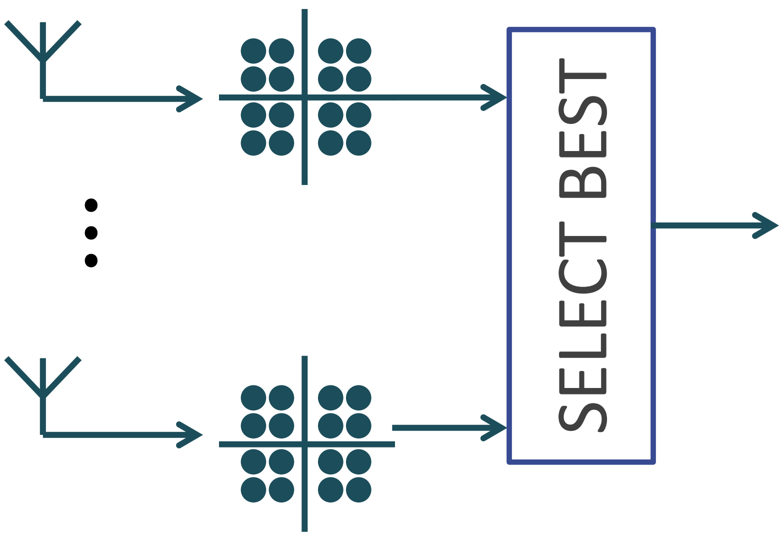
$$\mathrm{SIR} = \frac{|\mathbf{w}^T\mathbf{h}|^2|s|^2}{|\mathbf{w}^T\mathbf{z}|^2}$$

SIR: Signal-to-Interference Ratio

Receiver algorithm	Weight vector	Outage probability ($\mathbb{P}\{SIR < \theta\}$)
Equal Gain Combiner	$\mathbf{w} = \mathbf{1}_N$	$C_0\theta^{\frac{\alpha}{2}}(\lambda_0 + \lambda_{\mathrm{n}}N^{1-\frac{\alpha}{2}})$
Maximum Ratio Combiner	$\mathbf{w} = \mathbf{h}^*$	$C_0\theta^{\frac{\alpha}{2}}\mathbb{E}_{\mathbf{h}}\left[\frac{\lambda_n\|\mathbf{h}\|_\alpha^\alpha}{\|\mathbf{h}\|_2^\alpha} + \frac{\lambda_0}{\|\mathbf{h}\|_2^{2\alpha}}\right]$
Selection Combiner	$w_n = \mathcal{J}_{h_n=\max\{\mathbf{h}\}}$	$C_0(\lambda_0 + \lambda_n)\theta^{\frac{\alpha}{2}}\sum_{n=1}^{N}(-1)^{n+1}\frac{\binom{N}{n}}{n!}$

$$C_0 = \frac{4\,\Gamma\left(\frac{1+\alpha}{2}\right)\Gamma\left(1-\frac{\alpha}{2}\right)\mathbb{E}[A^\alpha]}{\sqrt{\pi}\cos\left(\frac{\pi\alpha}{4}\right)\,\mathrm{E}_{\mathrm{s}}^{\alpha}\sigma_s^\alpha\sigma_I^\alpha}$$

Outage probability of a genie-aided non-linear combiner



Select antenna stream with best detection SIR

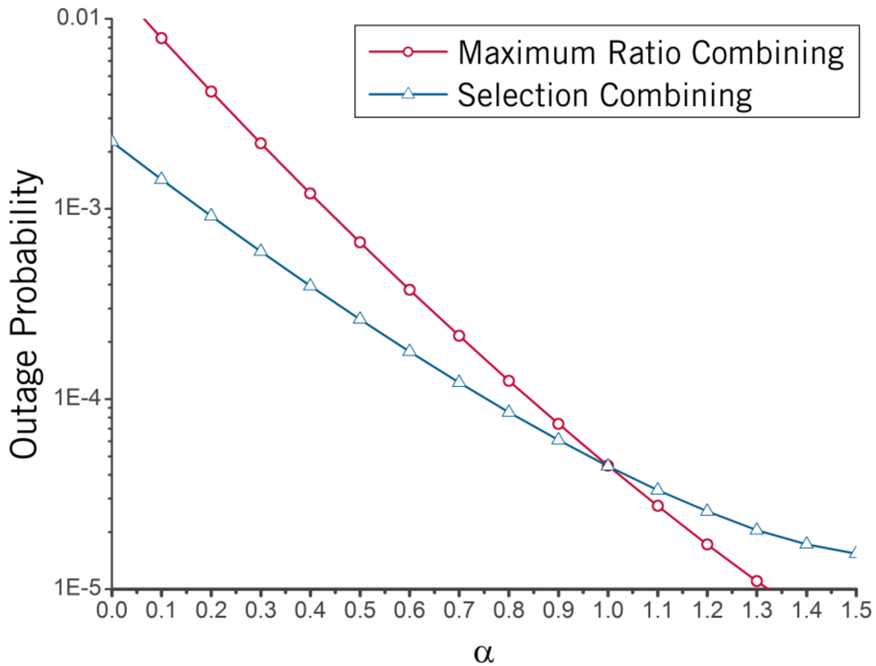
Receiver assumes knowledge of SIR at each antenna

$$\mathrm{SIR}_n = \frac{|h_n|^2 |s|^2}{|z_n|^2}$$

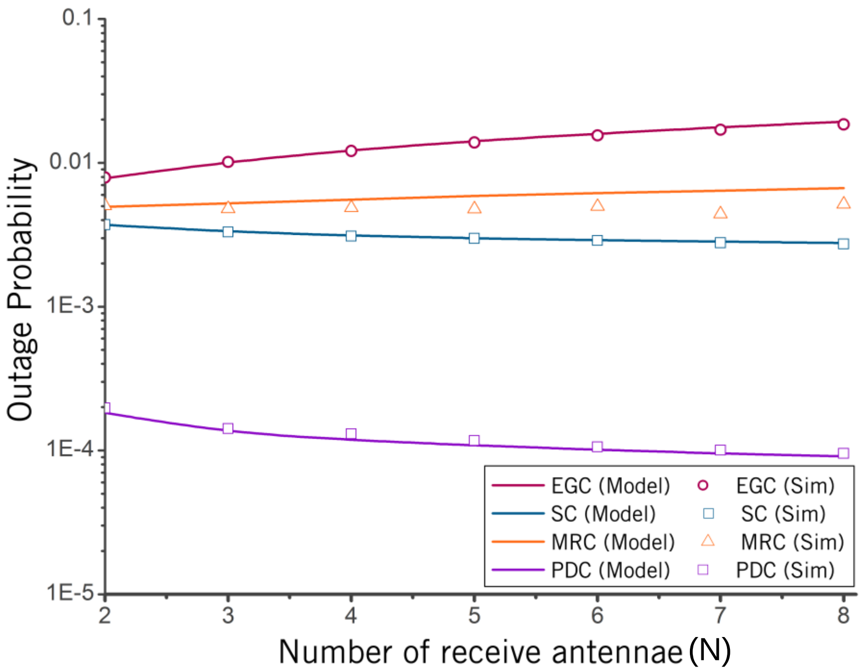
Receiver algorithm	Outage probability ($\mathbb{P}[SIR_1 < \theta, SIR_2 < \theta, \cdots, SIR_N < \theta]$)
Post Detection Combining	$C_0 \sum_{m=1}^{N} (-1)^{m+1} \binom{N}{m} \frac{(m+1+\frac{2}{\gamma})!}{(m-1)! \sin\frac{2\pi}{\gamma}} \theta^{\frac{\alpha}{2}} + \left(C_0 \frac{\pi^2}{\gamma \sin\frac{2\pi}{\gamma}}\right)^N \theta^{\frac{N\alpha}{2}}$

Derived expressions ('Model') match simulated outage ('Sim') for a variety of spatial dependence scenarios

Maximum ratio combining and selection combining receiver performance vs. α (N=4)



Outage performance of different combiners vs. number of antennae (γ =6)



PARAMETER VALUES

Common interferer density(λ_0) (per unit area)	0.0005
Excl. interferer density(λ_n) (per unit area)	0.0095

Using communication performance analysis, I design algorithms that outperform conventional receivers

Prior Work

Receiver Type	Interference Model	Joint Statistics	Fading Channel
Filtering	Symmetric alpha stable [Gonzales98][Ambike94]	Independent	No
Sequence detection, Decision feedback	Gaussian Mixture [Blum00][Bhatia94]	Independent	No
Detection	Symmetric alpha stable[Rajan10]	Independent	Yes

Proposed Receiver Structures

Receiver Type	Interference Model	Joint Statistics	Fading Channel
Linear filtering	SAS	Independent/Isotropic	Yes
Non-linear filtering	SAS	Independent/Isotropic	Yes

I investigate linear receivers in the presence of alpha stable interference

Linear receivers without channel knowledge

Select antenna with strongest mean channel to interference power ratio

Optimal linear receivers with channel knowledge

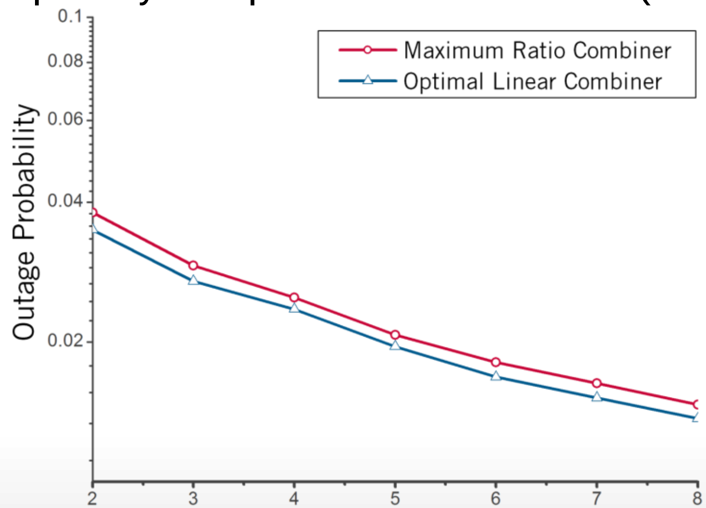
1. Independent SAS interference

Outage optimal $w_n = \begin{cases} \dfrac{h_n^*}{|h_n|^{\frac{\alpha-2}{\alpha-1}}}, & \alpha > 1 \\ ?, & \alpha \leq 1 \end{cases}$

2. Isotropic SAS interference

Maximum ratio combining is outage optimal

Outage of optimal linear combiner in spatially independent interference (α=1.3)



I propose sub-optimal non-linear receivers for impulsive interference

'Deviation' in an antenna output y_n is defined as

$$\Delta_n = \left|\, |y_n| - \text{median}\{|\mathbf{y}|\} \,\right|$$

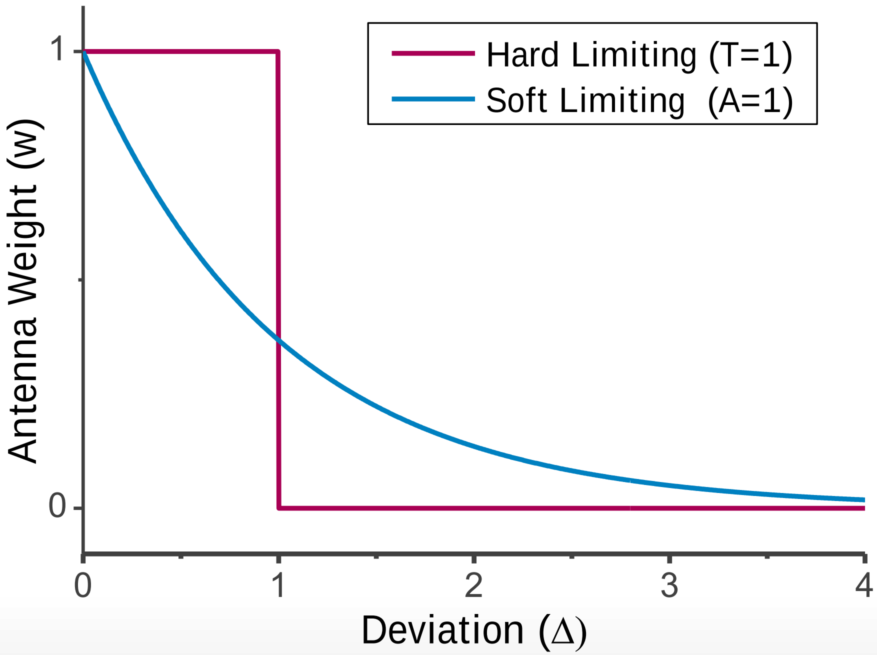
Proposed diversity combiners

1. Hard-limiting combiner

$$w_n = \mathbf{1}_{\Delta_n < T} h_n^*$$

2. Soft-limiting combiner

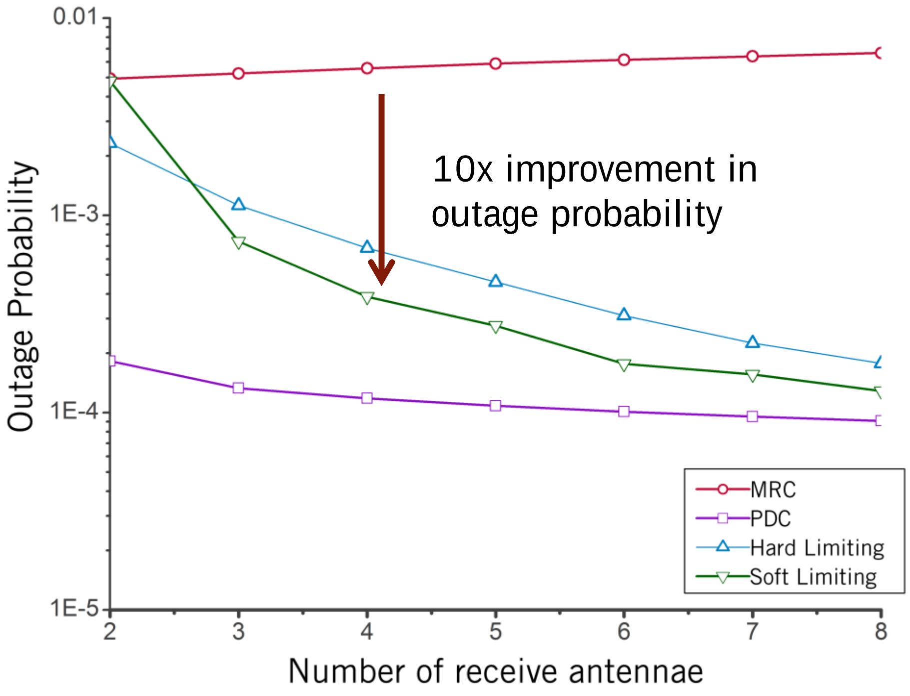
$$w_n = e^{-A\Delta_n} h_n^*$$



Proposed diversity combiners exhibit better outage performance compared to conventional combiners

Parameter values

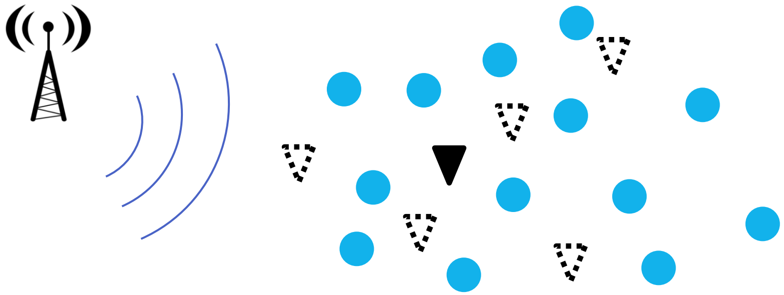
Pathloss coefficient (γ)	4
Guard- zone radius ($\delta_{\downarrow}$) (Unit Distance)	0
Common interferer density(λ_0) (per unit area)	0.0005
Exclusive interferer density(λ_n) (per unit area)	0.0095
HL combiner parameter (T)	1
SL combiner parameter (A)	2



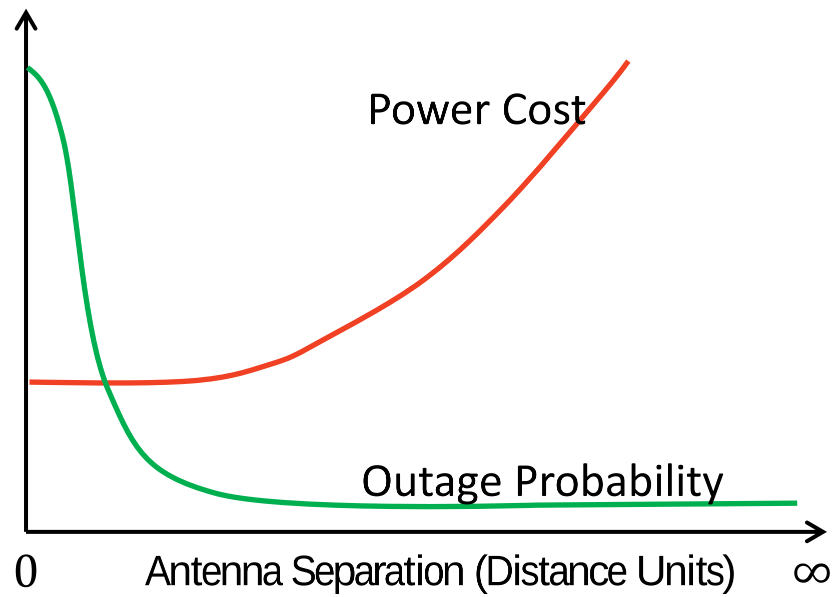
Joint interference statistics across separate antennae can also improve cooperative reception strategies

System Model

A distant base-station transmits a signal to the destination receiver (▼) surrounded by interferers (●) and cooperative receivers (▽)



Performance-Cost tradeoff



Which cooperative receiver should be selected to assist in signal reception?

Total cost is evaluated using a re-transmission based model

Optimal Antenna Separation

$$d^* = \arg\min_{d>0} \left\{ C(d) \times \frac{P^{out}(d)}{1 - P^{out}(d)} \right\}$$

Optimal cooperative antenna location — Cost per re-transmission — Expected re-transmissions

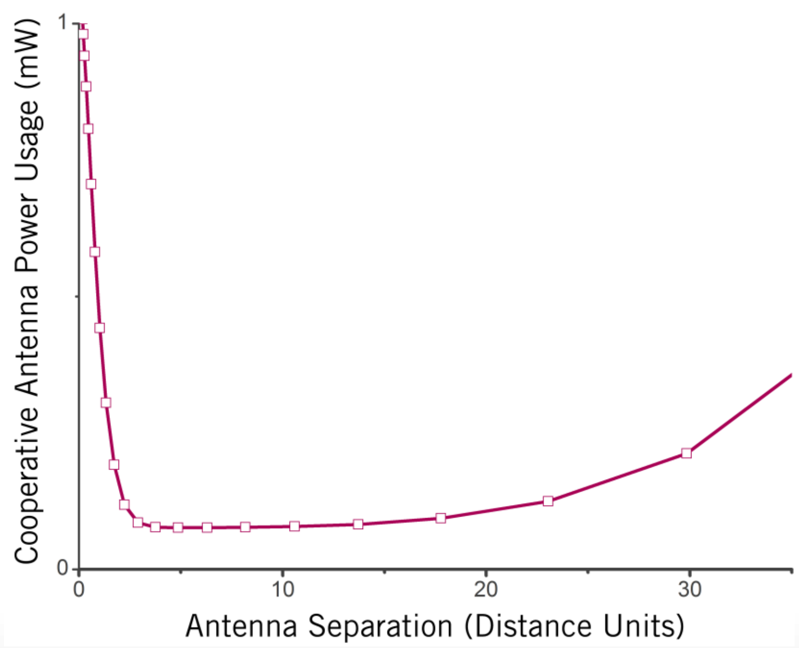
k^{th}-Nearest Neighbor Selection

$d_k \sim D_k$ is the random variable describing the location of the k-th nearest neighbor

$$k^* = \arg\min_{d>0} \left\{ \mathbb{E}_{d_k} \left[C(d_k) \times \frac{P^{out}(d_k)}{1 - P^{out}(d_k)} \right] \right\}$$

Optimal k-th nearest neighbor — Expected Cost of k-th re-transmission

Cooperative antenna power usage vs. separation. Power usage increases as d^γ ($\gamma = 6$) with 10mW fixed overhead and usage of 150mW at 50 distance units. 10% Outage probability per individual antenna.



In conclusion, the contributions of my dissertation are

1. A framework for modeling multi-antenna interference
   – Interference statistics are mix of spatial isotropy and spatial independence

2. Statistical modeling of multi-antenna interference
   – Co-located antennae in networks without guard zones
   – Two geographically separate antennae in networks with guard zones

3. Outage performance analysis of conventional receivers in networks without guard zones
   – Accurate outage probability expressions inform receiver design

4. Design of receiver algorithms with improved performance in impulsive interference
   – Order of magnitude reduction in outage probability compared to linear receivers
   – 80% reduction in power by using physically separate antennae

Future work

Statistical Modeling

- Non-Poisson distribution of interferer locations
- >2 physically separate antennae in a field of interferers
- Physically separate antennae in a centralized network
- Temporal modeling of interference statistics with correlated fields of randomly distributed interferers

Performance Analysis

- Performance analysis of multi-antenna wireless networks

Receiver Design

- Closed form expressions and bounds on performance of non-linear receivers
- Incorporate interference modeling into conventional relaying strategies

Journal Papers

1. A. Chopra and B. L. Evans, ``Outage Probability for Diversity Combining in Interference-Limited Channels", *IEEE Transactions on Wireless Communications*, submitted Sep. 14, 2011
2. A. Chopra and B. L. Evans, ``Joint Statistics of Radio Frequency Interference in Multi-Antenna Receivers", *IEEE Transactions on Signal Processing*, accepted with minor mandatory changes.
3. A. Chopra and B. L. Evans, ``Design of Sparse Filters for Channel Shortening", *Journal of Signal Processing Systems*, May 2011, 14 pages, DOI 10.1007/s11265-011-0591-0

Conference Papers

1. A. Chopra and B. L. Evans, ``Design of Sparse Filters for Channel Shortening", *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Proc.*, Mar. 14-19, 2010, Dallas, Texas USA.
2. A. Chopra, K. Gulati, B. L. Evans, K. R. Tinsley, and C. Sreerama, ``Performance Bounds of MIMO Receivers in the Presence of Radio Frequency Interference", *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Proc.*, Apr. 19-24, 2009, Taipei, Taiwan.
3. K. Gulati, A. Chopra, B. L. Evans, and K. R. Tinsley, ``Statistical Modeling of Co-Channel Interference", *Proc. IEEE Int. Global Communications Conf.*, Nov. 30-Dec. 4, 2009, Honolulu, Hawaii.
4. K. Gulati, A. Chopra, R. W. Heath, Jr., B. L. Evans, K. R. Tinsley, and X. E. Lin, ``MIMO Receiver Design in the Presence of Radio Frequency Interference", *Proc. IEEE Int. Global Communications Conf.*, Nov. 30-Dec. 4th, 2008, New Orleans, LA USA.
5. A. G. Olson, A. Chopra, Y. Mortazavi, I. C. Wong, and B. L. Evans, ``Real-Time MIMO Discrete Multitone Transceiver Testbed", *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Nov. 4-7, 2007, Pacific Grove, CA USA.

In preparation

1. A. Chopra and B. L. Evans, ``Joint Statistics of Interference Across Two Separate Antennae"

RFI Modeling

1. D. Middleton, "Non-Gaussian noise models in signal processing for telecommunications: New methods and results for Class A and Class B noise models", *IEEE Trans. Info. Theory*, vol. 45, no. 4, pp. 1129-1149, May 1999.
2. J. Ilow and D . Hatzinakos, "Analytic alpha-stable noise modeling in a Poisson field of interferers or scatterers", *IEEE Trans. on Signal Proc.,* vol. 46, no. 6, pp. 1601-1611, Jun. 1998.
3. E. S. Sousa, "Performance of a spread spectrum packet radio network link in a Poisson field of interferers," *IEEE Trans. on Info. Theory*, vol. 38, no. 6, pp. 1743–1754, Nov. 1992.
4. X. Yang and A. Petropulu, "Co-channel interference modeling and analysis in a Poisson field of interferers in wireless communications," *IEEE Trans. on Signal Proc.*, vol. 51, no. 1, pp. 64–76, Jan. 2003.
5. Cisco visual networking index: Global mobile data traffic forecast update, 2010 - 2015. Technical report, Feb. 2011.
6. John P. Nolan. Multivariate stable densities and distribution functions: general and elliptical case. Deutsche Bundesbank's Annual Fall Conference, 2005.

1. Ping Gao and C. Tepedelenlioglu. Space-time coding over fading channels with impulsive noise. IEEE Transactions on Wireless Communications, 6(1):220–229, Jan. 2007.
2. A. Rajan and C. Tepedelenlioglu. Diversity combining over Rayleigh fading channels with symmetric alpha-stable noise. IEEE Transactions on Wireless Communications, 9(9):2968–2976, 2010.
3. S. Niranjayan and N. C. Beaulieu. The BER optimal linear rake receiver for signal detection in symmetric alpha-stable noise. IEEE Transactions on Communications, 57(12):3585–3588, 2009.
4. C. Tepedelenlioglu and Ping Gao. On diversity reception over fading channels with impulsive noise. IEEE Transactions on Vehicular Technology, 54(6):2037–2047, Nov. 2005.
5. G. A. Tsihrintzis and C. L. Nikias. Performance of optimum and suboptimum receivers in the presence of impulsive noise modeled as an alpha stable process. IEEE Transactions on Communications, 43(234):904–914, 1995.

Receiver Design

1. A. Spaulding and D. Middleton, “Optimum Reception in an Impulsive Interference Environment-Part I: Coherent Detection”, *IEEE Trans. Comm*., vol. 25, no. 9, Sep. 1977
2. J.G. Gonzalez and G.R. Arce, “Optimality of the Myriad Filter in Practical Impulsive-Noise Environments”, *IEEE Trans. on Signal Proc.*, vol. 49, no. 2, Feb 2001
3. S. Ambike, J. Ilow, and D. Hatzinakos, “Detection for binary transmission in a mixture of Gaussian noise and impulsive noise modelled as an alpha-stable process,” *IEEE Signal Proc. Letters*, vol. 1, pp. 55–57, Mar. 1994.
4. O. B. S. Ali, C. Cardinal, and F. Gagnon. Performance of optimum combining in a Poisson field of interferers and Rayleigh fading channels. IEEE Transactions on Wireless Communications, 9(8):2461–2467, 2010.
5. Andres Altieri, Leonardo Rey Vega, Cecilia G. Galarza, and Pablo Piantanida. Cooperative strategies for interference-limited wireless networks. In Proc. IEEE International Symposium on Information Theory, pages 1623–1627, 2011.
6. Y. Chen and R. S. Blum. Efficient algorithms for sequence detection in non-Gaussian noise with intersymbol interference. IEEE Transactions on Communications, 48(8):1249–1252, Aug. 2000.

about me

Member of the Wireless Networking and Communications Group at The University of Texas at Austin since 2006.

Completed projects

ADSL testbed (Oil & Gas)	2 x 2 wired multicarrier communications testbed using PXI hardware, x86 processor, real-time operating system and LabVIEW
Spur modeling/mitigation (NI)	Detect and classify spurious signals; fixed and floating-point algorithms to mitigate spurs

Currently active projects

Interference modeling and mitigation (Intel)	Statistical models of interference; receiver algorithms to mitigate interference; MATLAB toolbox
Impulsive noise mitigation in OFDM (NI)	Non-parametric interference mitigiation for wireless OFDM receivers using PXI hardware, FPGAs, and LabVIEW
Powerline communications (TI, Freescale, SRC)	Modeling and mitigating impulsive noise; building multichannel multicarrier communications testbed using PXI hardware, x86 processor, real-time operating system, LabVIEW

Interference mitigation has been an active area of research over the past decade

INTERFERENCE MITIGATION STRATEGY	LIMITATIONS
Hardware design - Receiver shielding	Does not mitigate interference from devices using same spectrum
Network planning - Resource allocation - Basestation coordination - Partial frequency re-use	Requires user coordination Slow updates
Receiver algorithms - Interference cancellation - Interference alignment - Statistical interference mitigation	Require user coordination and channel state information Statistical methods require accurate interference models

Interference Mitigation Techniques

- RFI Mitigation Methods
  - Static
    - Shielding
    - Fractional Frequency Reuse
    - Sectored Antennas
  - Dynamic
    - Orthogonal Multiple Access Schemes
      e.g., TDMA, FDMA, CDMA, OFDMA
    - MAC Layer Channel Access Protocols
      e.g., ALOHA, CSMA
    - Interference Cancellation
      e.g., MUD, SIC
    - Interference Alignment
    - Robust Transceivers

Interference alignment

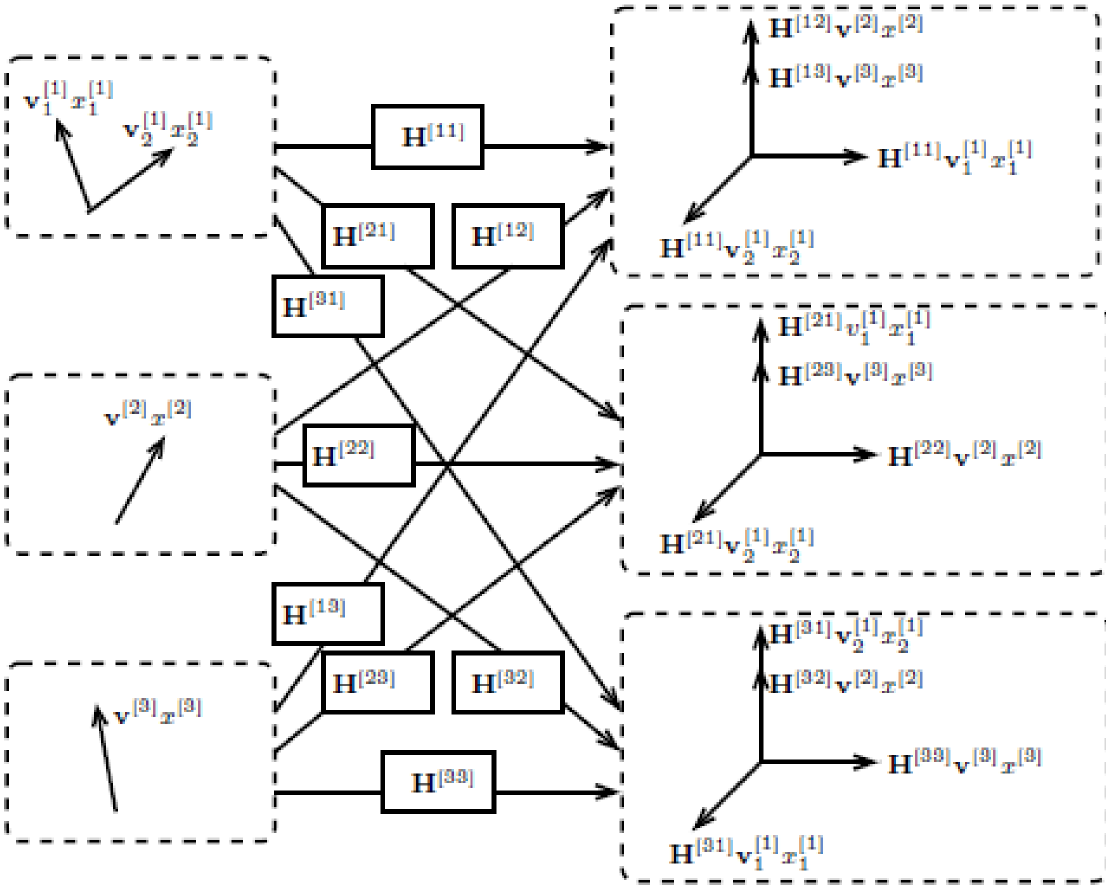


Fig. 1. Interference alignment on the 3 user interference channel to achieve $4/3$ degrees of freedom

Interference cancellation

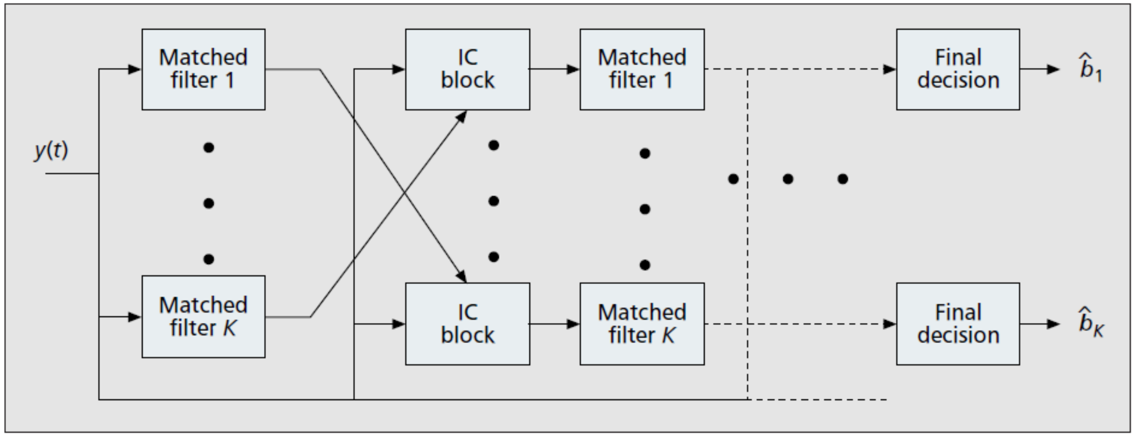


Figure 3. *Parallel interference cancellation.*

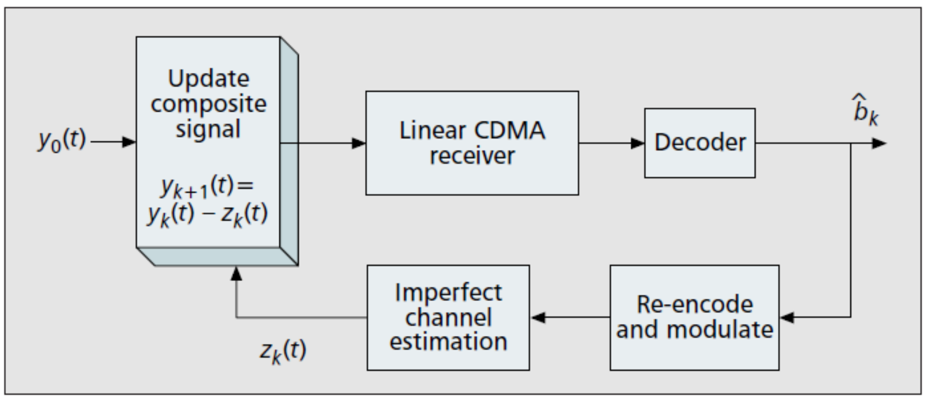
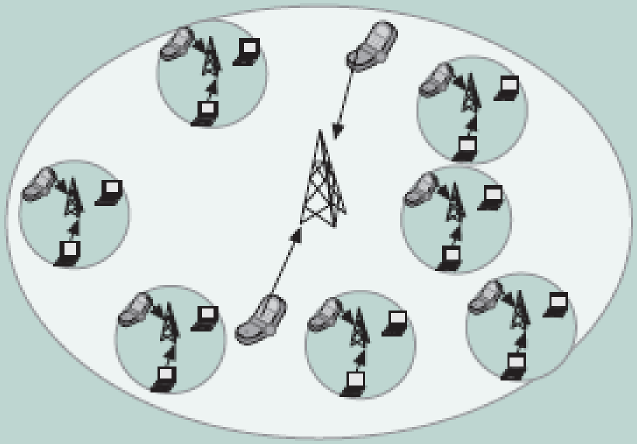


Figure 4. *Successive interference cancellation.*

J. G. Andrews, ”Interference Cancellation for Cellular Systems: A Contemporary Overview”, *IEEE Wireless Communications Magazine*, Vol. 12, No. 2, pp. 19-29, April 2005

Femtocell Networks

Infrastructure	Expenses	Features
Femtocell: Consumer installed wireless data access point inside homes, which backhauls data through a broadband gateway (DSL/cable/Ethernet/WiMAX) over the Internet to the cellular operator network. 	**Capital expenditure.** Subsidized femtocell hardware. **Operating expenditure.** a) Providing a scalable architecture to transport data over IP; b) upgrading femtocells to newer standards.	**Benefits.** a) Lower cost, better coverage and prolonged handset battery life from shrinking cell-size; b) capacity gain from higher SINR and dedicated BS to home subscribers ; c) reduced subscriber churn **Shortcomings.** a) Interference from nearby macrocell and femtocell transmissions limits capacity; b) increased strain on backhaul from data traffic may affect throughput.

V. Chandrasekhar, J. G. Andrews and A. Gatherer, "Femtocell Networks: a Survey", *IEEE Communications Magazine*, Vol. 46, No. 9, pp. 59-67, September 2008

Spectrum Occupied by Typical Standards

Standard	Carrier (GHz)	Wireless Networking	Interfering Clocks and Busses
Bluetooth	2.4	Personal Area Network	Gigabit Ethernet, PCI Express Bus, LCD clock harmonics
IEEE 802. 11 b/g/n	2.4	Wireless LAN (Wi-Fi)	Gigabit Ethernet, PCI Express Bus, LCD clock harmonics
IEEE 802.16e	2.5–2.69 3.3–3.8 5.725–5.85	Mobile Broadband (Wi-Max)	PCI Express Bus, LCD clock harmonics
IEEE 802.11a	5.2	Wireless LAN (Wi-Fi)	PCI Express Bus, LCD clock harmonics

Impact of LCD on 802.11g

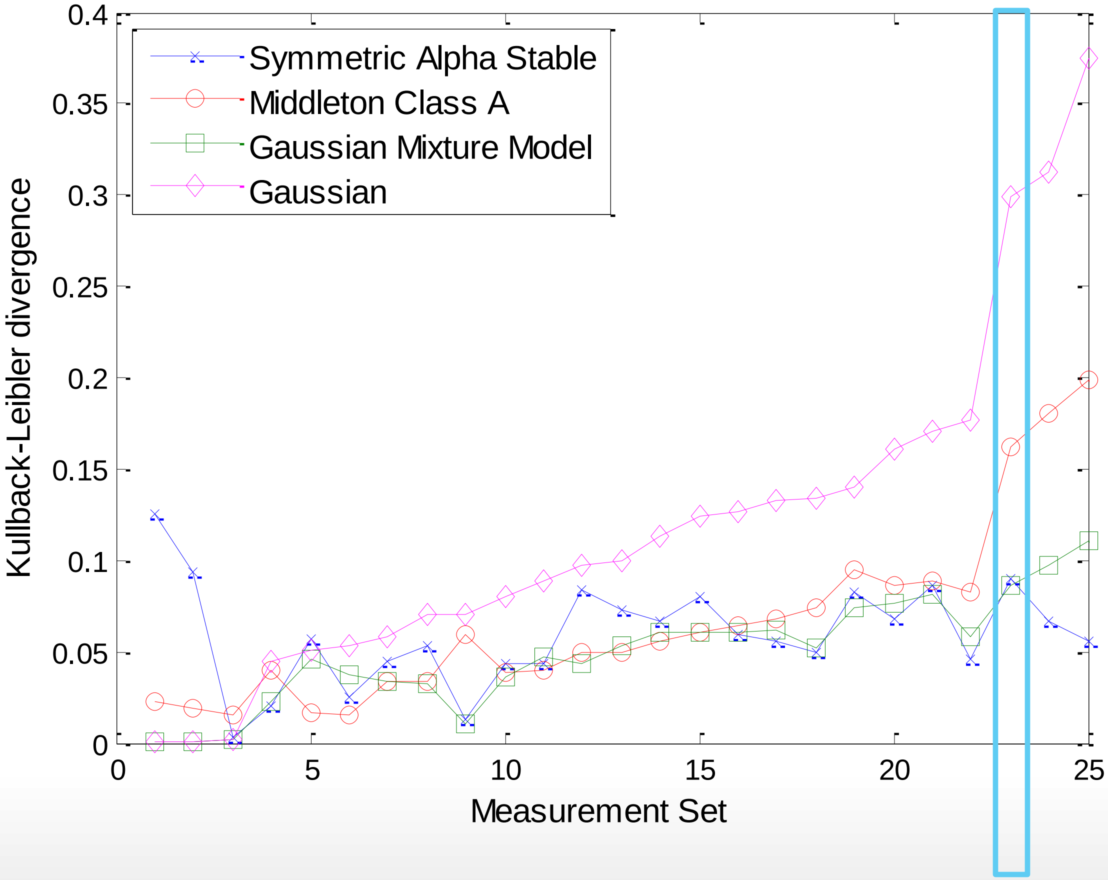
Pixel clock 65 MHz

LCD Interferers and 802.11g center frequencies

LCD Interferers	802.11g Channel	Center Frequency	Difference of Interference from Center Frequencies	Impact
2.410 GHz	Channel 1	2.412 GHz	~2 MHz	Significant
2.442 GHz	Channel 7	2.442 GHz	~0 MHz	Severe
2.475 GHz	Channel 11	2.462 GHz	~13 MHz	Just outside Ch. 11. Impact minor

Measured Data

25 radiated computer platform RFI data sets from Intel each with 50,000 samples taken at 100 MSPS



Single Antenna RFI Models

Model Name	Key Features
Symmetric alpha stable [Sousa,1992] [Ilow & Hatzinakos,1998]	• Models wireless ad hoc networks, computational platform noise • No closed form distribution function (except $\alpha = 1,2$) • Unbounded variance (generally $E[X^{\alpha}] \rightarrow \infty$)
Middleton Class A [Middleton, 1979, 1999]	• Models wireless networks with guard zones and interferers in a finite area around receiver [Gulati, Chopra, Evans & Tinsley, 2009] • Model incorporates thermal noise present at receiver • Special case of the Gaussian mixture distribution
Gaussian mixture distribution	• Models wireless networks with hotspots, femtocell networks [Gulati, Evans, Andrews & Tinsley, 2009]

Single Antenna RFI Models

- Symmetric alpha stable distribution [Sousa,1992]
  - Characteristic function:

$$\Phi(\omega) = e^{-\sigma|\omega|^{\alpha}}$$

Parameter	Range
α	[0,2]
σ	$(0,\infty)$

- Middleton Class A distribution [Middleton, 1977, 1999]
  Amplitude distribution:

$$f_{\mathbf{Y}}(y) = e^{-A} \sum_{k=1}^{\infty} \frac{A^k}{k!} \frac{1}{\sqrt{2\pi\sigma^2 \frac{k/A+\Gamma}{1+\Gamma}}} e^{-\frac{y^2}{2\frac{k/A+\Gamma}{1+\Gamma}}}$$

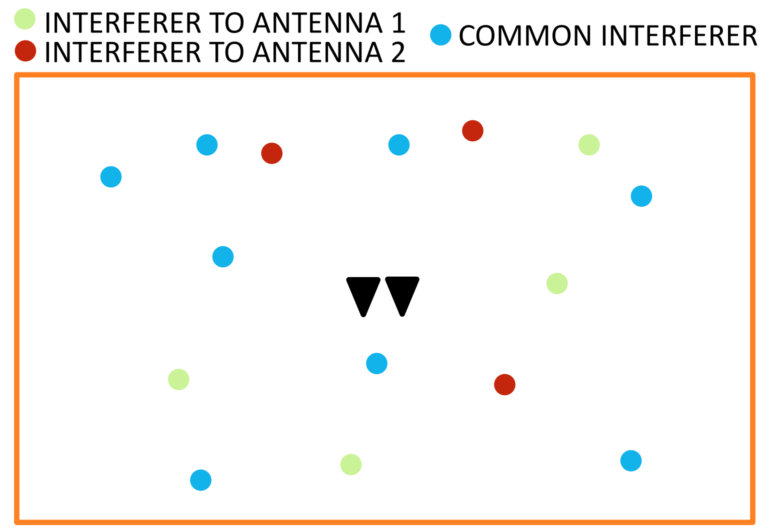
Parameter	Range
A	[0,2]
Γ	$(0,\infty)$
σ	$(0,\infty)$

- Gaussian mixture distribution
  - Amplitude distribution:

$$f_{\mathbf{Y}_I,\mathbf{Y}_Q}(y_I, y_Q) = p_0\delta(y_I)\delta(y_Q) + \sum_{l=1}^{\infty} p_l \frac{1}{\sigma_l\sqrt{2\pi}} e^{-\frac{y_I^2+y_Q^2}{2\sigma_l^2}}$$

Parameter	Range
$p_1, p_2, \cdots$	[0,1]
$\sigma_1, \sigma_2, \cdots$	$(0,\infty)$

Two-Antenna RFI Generation Model



- Key model characteristics
  - Correlated interferer field observed by receive antennas
  - Inter-antenna distances insignificant compared to antenna-interferer distances

- Sum interference in **two-antenna** receiver

$$\mathbf{Y}_1 = \sum_{i \in \Pi_0} B_i e^{j\theta_i} r_i^{-\gamma/2} h_i e^{j\phi_i} + \sum_{i' \in \Pi_1} B_{i'} e^{j\theta_{i'}} r_{i'}^{-\gamma/2} h_{i'} e^{j\phi_{i'}}$$

$$\mathbf{Y}_2 = \sum_{i \in \Pi_0} B_i e^{j\theta_i} r_i^{-\gamma/2} h_i e^{j\phi_i} + \sum_{i' \in \Pi_2} B_{i'} e^{j\theta_{i'}} r_{i'}^{-\gamma/2} h_{i'} e^{j\phi_{i'}}$$

  - Π_0 denotes set of interferers observed by both antennas (intensity λ_0)
  - Π_1, Π_2 denote interferers observed at antenna 1 and 2 respectively (intensity λ_1, λ_2)

Multi-Antenna RFI Generation Model

- Spatially correlated interferer fields in N_R-antenna receiver
  - $2^{N_R} - 1$ *i.i.d.* interferer sets
  - Sum interference from $2^{N_R - 1}$ sets at each antenna
- Proposed model extension to N_R-antenna receiver
  - Two categories of interferers

Emissions lead to RFI in ***all*** antennas	Emissions lead to RFI in ***one*** antenna

  - Sum interference from 2 sets at each antenna
- Sum interference in N_R-antenna receiver

$$\mathbf{Y}_k = \sum_{i \in \Pi_0} B_i e^{j\theta_i} r_i^{-\gamma/2} h_i e^{j\phi_i} + \sum_{i' \in \Pi_k} B_{i'} e^{j\theta_{i'}} r_{i'}^{-\gamma/2} h_{i'} e^{j\phi_{i'}}$$

  - Π_0 is set of interferers common to all receive antennas (intensity λ_0)
  - Π_k is set of interferers observed by receive antenna k (intensity λ_k)

Existing Models of Multi-Antenna RFI

Model Name	Key Features
Symmetric alpha stable (isotropic) [Ilow & Hatzinakos,1998]	• Models spatially dependent RFI generated from single set of interferers observed by all receive antennas • No closed form distribution function (except $\alpha = 1,2$) • Unbounded variance (generally $E[X^{\alpha}] \to \infty$)
Multidimensional Class A Models $\mathrm{I} - \mathrm{III}$ [Delaney, 1995]	• Multidimensional extension of Middleton class A distribution, no statistical derivation • Different statistical distributions required to reflect spatial dependence/independence in RFI
Bivariate class A distribution [McDonald & Blum, 1997]	• Approximate distribution based on statistical-physical derivation • Models RFI observed at two receive antennas only • Spatially dependent RFI
Temporal second-order alpha stable model [Yang & Petropulu, 2003]	• Models second-order temporal statistics of co-channel interference • Assumes temporal correlation in interferer fields

Statistical Models for Multi-Antenna RFI

- Multidimensional symmetric alpha stable distribution [Ilow & Hatzinakos, 1998]

Extension type	Characteristic function
Spatially independent	$\Phi(\omega) = e^{-\sum_{n=1}^{N_R} \sigma_n \lvert\omega_n\rvert^{\alpha}}$
Isotropic	$\Phi(\omega) = e^{-\sigma \lvert\lvert \mathbf{w} \rvert\rvert^{\alpha}}$

- Multidimensional Class A distribution [Delaney, 1995]

Extension type	Amplitude distribution
Spatially independent	$f_{\mathbf{Y}}(\mathbf{y}) = \prod_{n=1}^{N_R} \sum_{k=0}^{\infty} \frac{e^{-A_n} {A_n}^k}{k! \sqrt{2\pi \frac{k/A_n + \Gamma_n}{1+\Gamma_n} \sigma_n^2}} e^{-\frac{y_n^2}{2 \frac{k/A_n+\Gamma_n}{1+\Gamma_n} \sigma_n^2}}$
Isotropic	$f_{\mathbf{Y}}(\mathbf{y}) = \sum_{k=0}^{\infty} \frac{e^{-A} A^k}{k!} \frac{1}{\left(2\pi^N \left\lvert \frac{k/A+\Gamma}{1+\Gamma} \mathbf{\Sigma} \right\rvert\right)^{\frac{1}{2}}} e^{-\frac{\mathbf{y}^T \mathbf{\Sigma}^{-1} \mathbf{y}}{2 \frac{k/A+\Gamma}{1+\Gamma}}}$

Statistical Models for Multi Antenna RFI

- Physical model of RFI for 2 antenna systems
  - Amplitude distribution **[McDonald & Blum, 1997]**

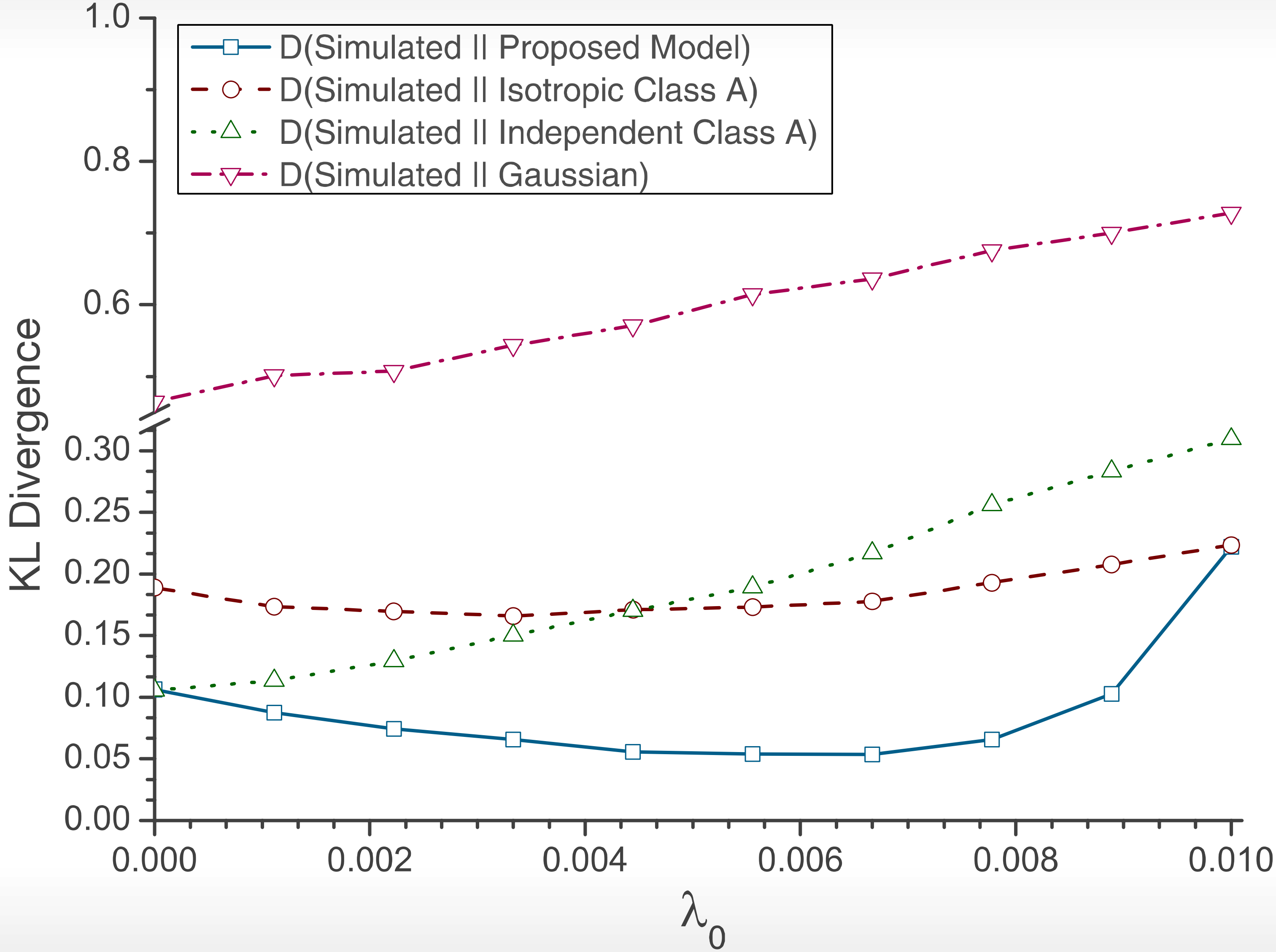
$$f_{\mathbf{n}}(n_1, n_2) = \frac{e^{-A}}{2\pi|\mathbf{K}_0|^{\frac{1}{2}}} e^{\frac{-\mathbf{n}^T \mathbf{K}_0^{-1} \mathbf{n}}{2}} + \frac{\left(1 - e^{-A}\right)}{2\pi|\mathbf{K}_1|^{\frac{1}{2}}} e^{\frac{-\mathbf{n}^T \mathbf{K}_1^{-1} \mathbf{n}}{2}}$$

for $m = 0, 1$

$$\mathbf{K}_m = \begin{bmatrix} (c_m)^2 & \kappa c_m \hat{c}_m \\ \kappa c_m \hat{c}_m & (\hat{c}_m)^2 \end{bmatrix}, \; (c_m)^2 = \frac{\frac{m}{A} + \Gamma_1}{1 + \Gamma_1} \; , \; (\hat{c}_m)^2 = \frac{\frac{m}{A} + \Gamma_2}{1 + \Gamma_2}.$$

Parameter	Range
A	$[0,2]$
Γ_1, Γ_2	$(0, \infty)$
κ	$[0,1]$

KL divergence



Interference in separate antennae

$$\Phi_{\mathcal{S}_0,\mathbf{d}}(\mathbf{w}) = \mathbb{E}\left\{\prod_{i\in\mathcal{S}_0} e^{-\omega_{1,I}^2 (B_i^0)^2 \sigma_H^2 \|\mathbf{R}_i\|^{-\gamma} - \omega_{1,Q}^2 (B_i^0)^2 \sigma_H^2 \|\mathbf{R}_i\|^{-\gamma} - \omega_{2,I}^2 (B_i^0)^2 \sigma_H^2 \|\mathbf{R}_i - \mathbf{d}\|^{-\gamma} - \omega_{2,Q}^2 (B_i^0)^2 \sigma_H^2 \|\mathbf{R}_i - \mathbf{d}\|^{-\gamma}}\right\} \tag{4.23}$$

$$= \mathbb{E}\left\{\prod_{i\in\mathcal{S}_0} e^{-|\omega_1|^2 (B_i^0)^2 \sigma_H^2 \|\mathbf{R}_i\|^{-\gamma} - |\omega_2|^2 (B_i^0)^2 \sigma_H^2 \|\mathbf{R}_i - \mathbf{d}\|^{-\gamma}}\right\} \tag{4.24}$$

$$\Psi_{\mathcal{S}_0,\mathbf{d}}(\mathbf{w}) = \log\left(\Phi_{\mathcal{S}_0,\mathbf{d}}(\mathbf{w})\right)$$

$$= -\lambda_0 \int_{\mathbb{R}^2} \left\{1 - \frac{1}{1 + |\omega_1^2| \sigma_H^2 \sigma_B^2 \|\mathbf{r}\|^{-\gamma} + |\omega_2^2| \sigma_H^2 \sigma_B^2 \|\mathbf{r} - \mathbf{d}\|^{-\gamma}} d\mathbf{r}\right\}$$

Homogeneous Spatial Poisson Point Process

The spatial Poisson point process with uniform intensity $\lambda > 0$ is a point process in $\mathbb{R}^2$ such that

a. for every bounded closed set B, the count $N(B)$ has a Poisson distribution with mean $\lambda\mathcal{L}(B)$, where $\mathcal{L}(B)$ denotes the area of B;

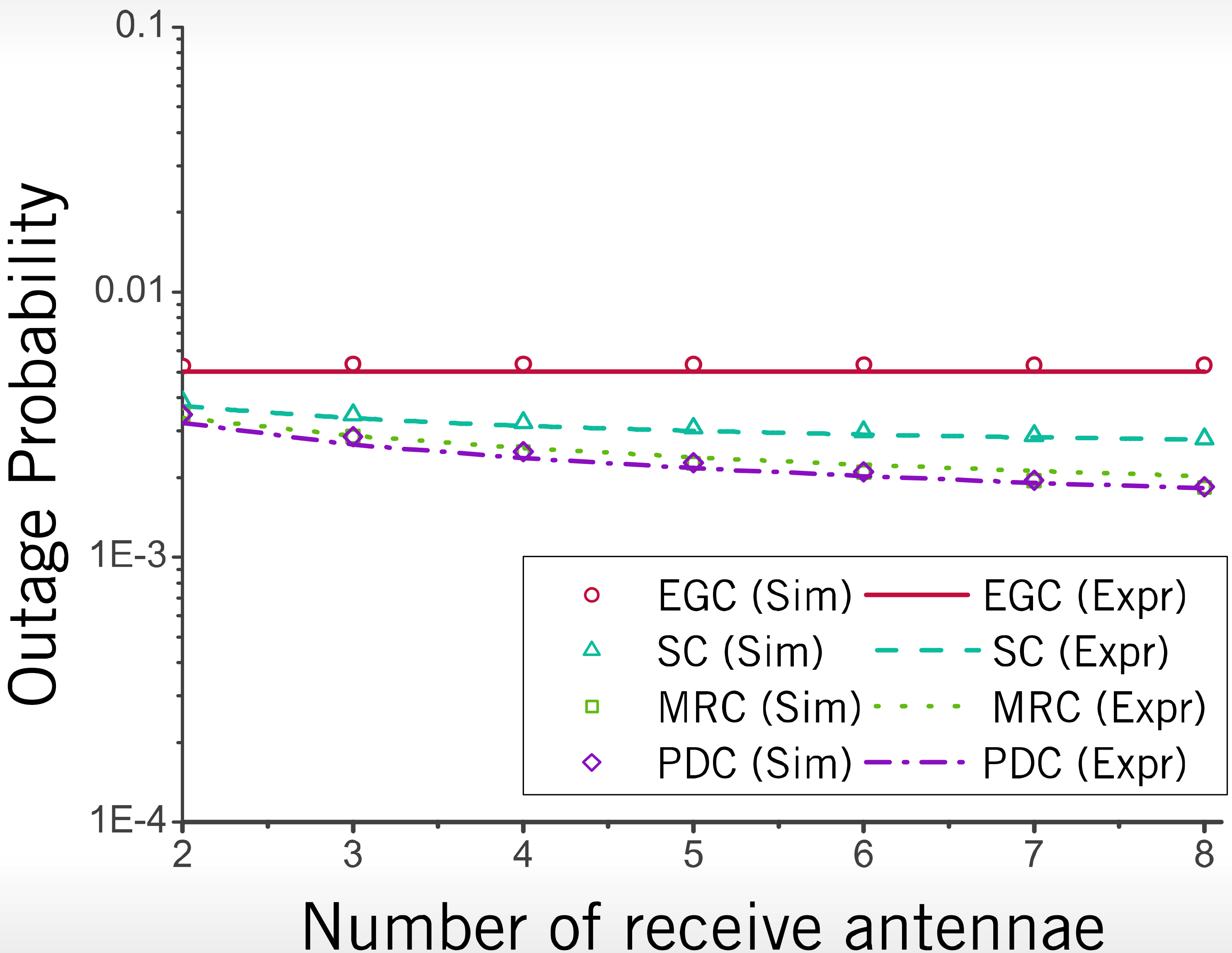
b. if $B_1, \cdots, B_m$ are disjoint regions, then $N(B_1), \cdots, N(B_m)$ are independent.

Poisson Field of Interferers

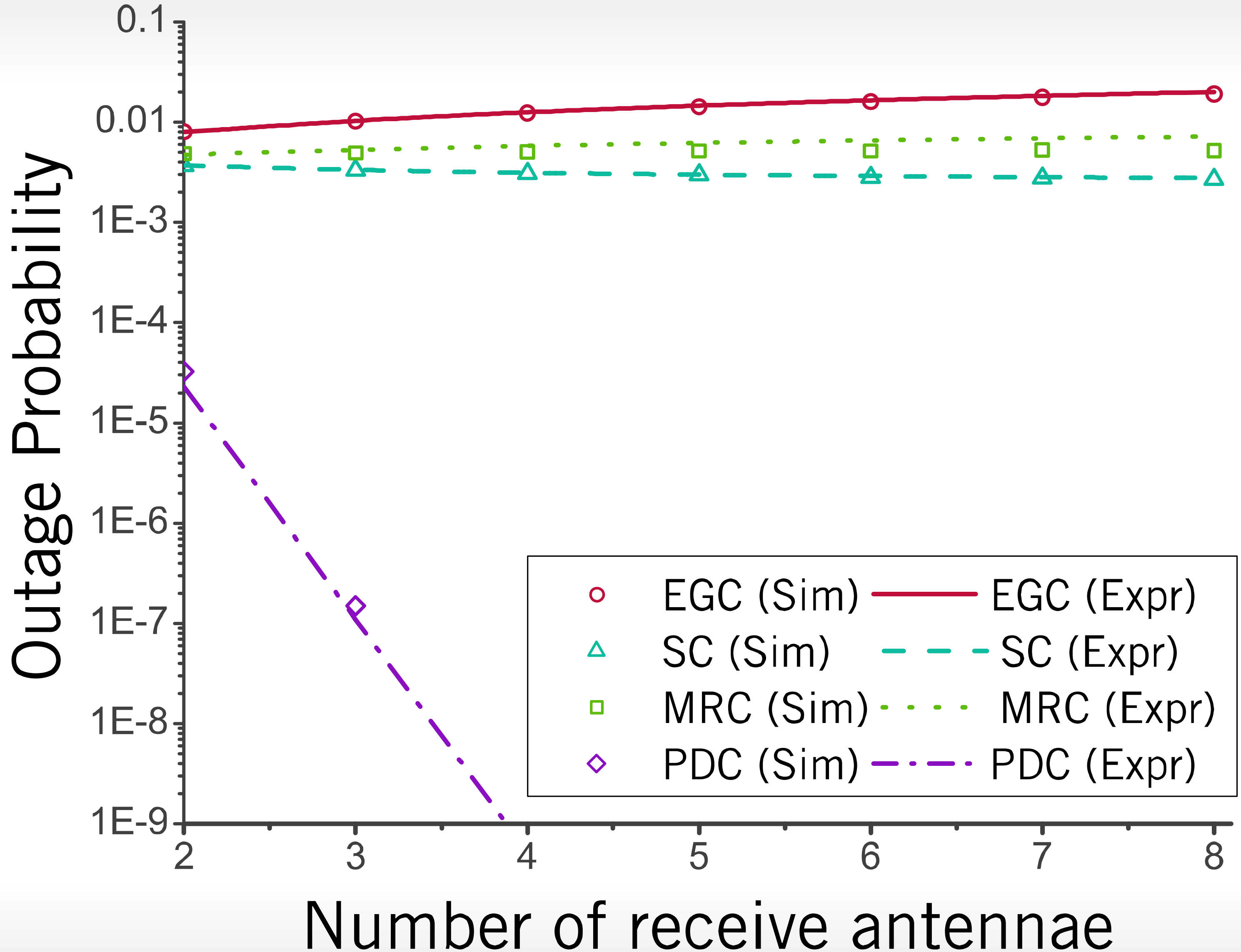
Applied to wireless *ad hoc* networks, cellular networks

Closed Form Amplitude Distribution			
Model	Interference	Region	Key Prior Work
Symmetric Alpha Stable	Spatial	Entire plane	[Sousa, 1992] [Ilow & Hatzinakos, 1998] [Yang & Petropulu, 2003]
Middleton Class A	Spatio-temporal	Finite area	[Middleton, 1977, 1999]
Other Interference Statistics – closed form amplitude distribution not derived			
Statistics	Interference	Region	Key Prior Work
Moments	Spatial	Finite area	[Salbaroli & Zanella, 2009]
Characteristic Function	Spatial	Finite area	[Win, Pinto & Shepp,2009]

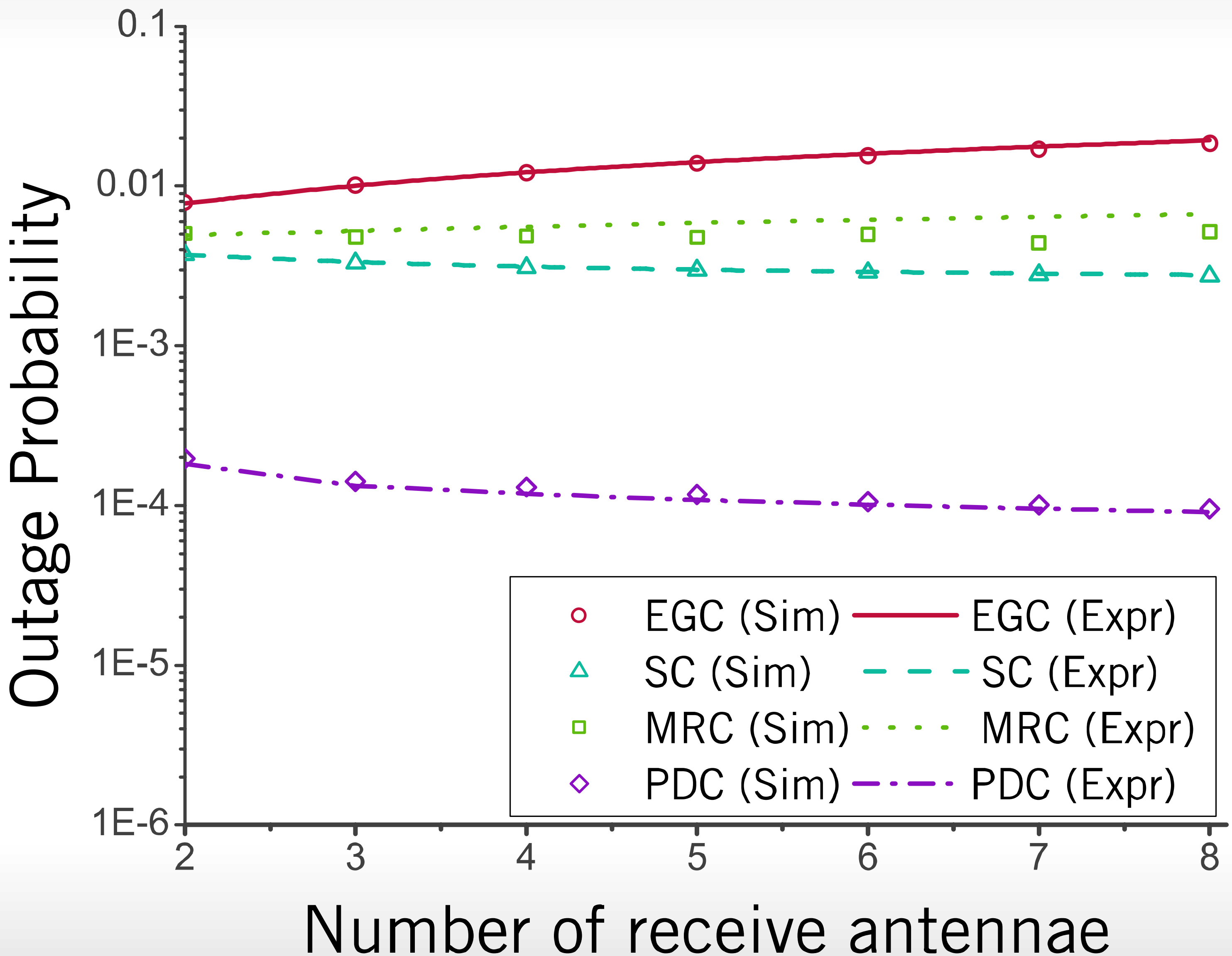
Isotropic SAS



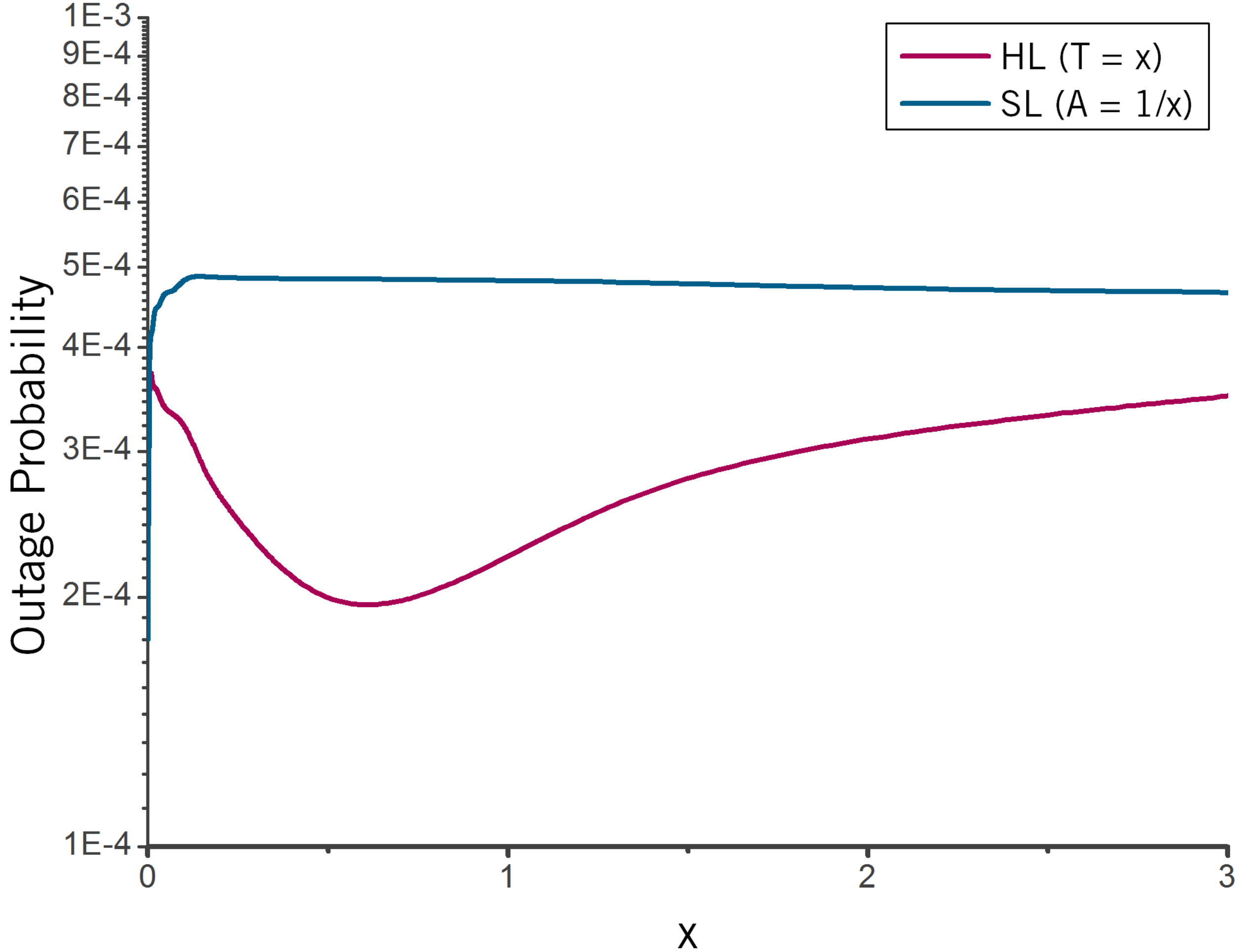
Independent SAS



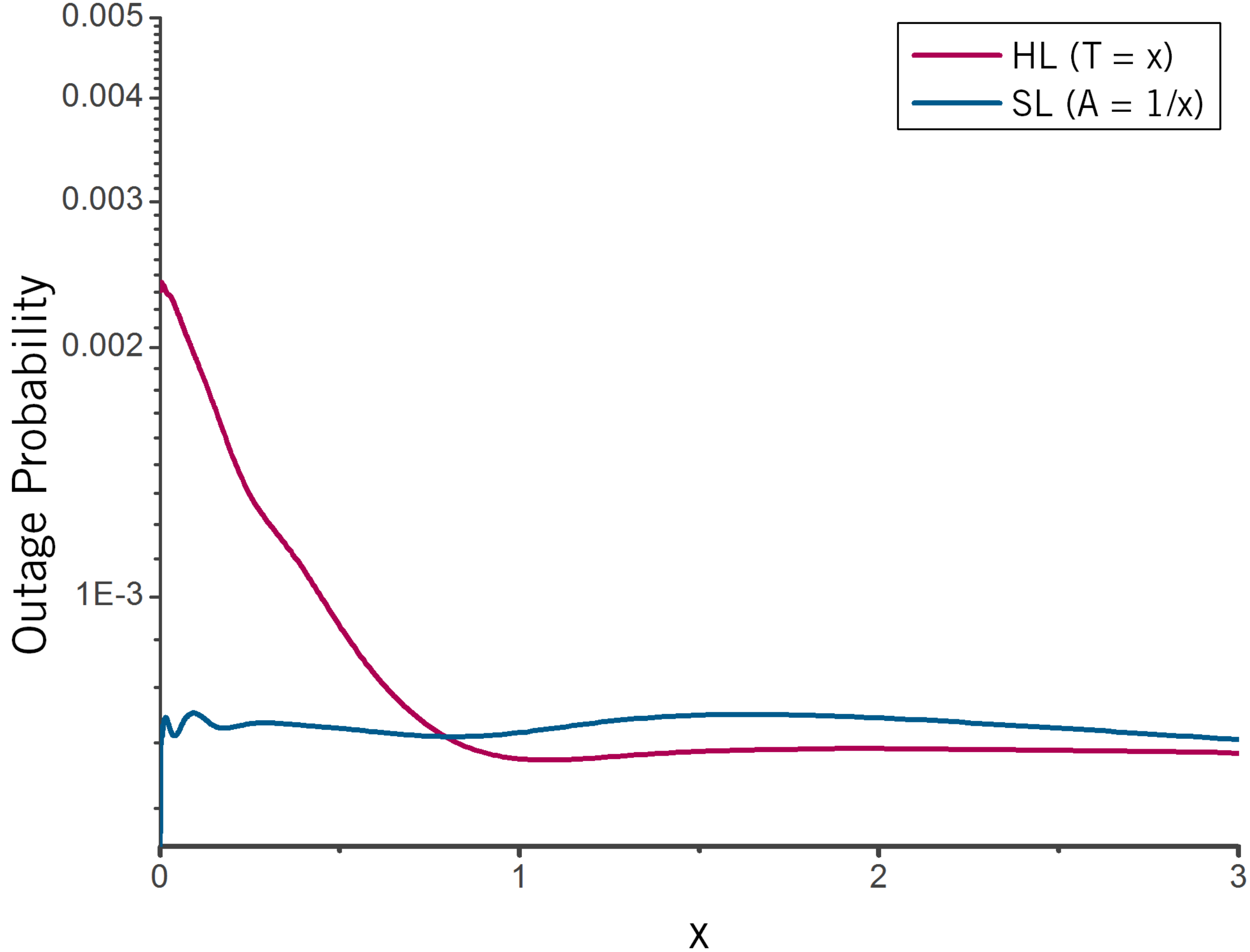
Mixture SAS


0.1
0.01
1E-3
1E-4
1E-5
1E-6
Outage Probability
2
3
4
5
6
7
8
Number of receive antennae
EGC (Sim)
EGC (Expr)
SC (Sim)
SC (Expr)
MRC (Sim)
MRC (Expr)
PDC (Sim)
PDC (Expr)

Threshold selection with $\alpha = \frac{2}{3}$



Threshold selection with $\alpha = \frac{4}{3}$



Parameter Estimators for Alpha Stable

$$\hat{\delta} = \text{median}\,\{X_1,\, X_2,\, \cdots,\, X_N\}$$

Return

L nonoverlapping segments,

Let $\overline{X}_l$ and $\underline{X}_l$ be the maximum and the minimum of the data segment $\mathbf{X}(l)$. We then define

$$\overline{X}_l = \log \overline{X}_l \qquad \text{(2-4)}$$

$$\underline{X}_l = -\log(-\underline{X}_l) \qquad \text{(2-5)}$$

$$\overline{s} = \sqrt{\frac{1}{L-1}\sum_{l=1}^{L}(\overline{X}_l - \overline{X})^2}; \quad \overline{X} = \frac{1}{L}\sum_{l=1}^{L}\overline{X}_l \qquad \underline{s} = \sqrt{\frac{1}{L-1}\sum_{l=1}^{L}(\underline{X}_l - \underline{X})^2}; \quad \underline{X} = \frac{1}{L}\sum_{l=1}^{L}\underline{X}_l$$

$$\hat{\alpha} = \frac{\pi}{2\sqrt{6}}\left(\frac{1}{\overline{s}} + \frac{1}{\underline{s}}\right).$$

$$\hat{\gamma} = \left[\frac{\frac{1}{N}\sum_{k=1}^{N}|X_k - \hat{\delta}|^p}{C(p,\, \hat{\alpha})}\right]^{\hat{\alpha}/p}$$

$$C(p,\, \hat{\alpha}) = \frac{1}{\cos\left(\frac{\pi}{2}p\right)}\,\frac{\Gamma\left(1 - \frac{p}{\hat{\alpha}}\right)}{\Gamma(1-p)}$$

Gaussian Mixture vs. Alpha Stable

- Gaussian Mixture vs. Symmetric Alpha Stable

	Gaussian Mixture	Symmetric Alpha Stable
Modeling	Interferers distributed with Guard zone around receiver (actual or virtual due to PL)	Interferers distributed over entire plane
Pathloss Function	With GZ: singular / non-singular Entire plane: non-singular	Singular form
Thermal Noise	Easily extended (sum is Gaussian mixture)	Not easily extended (sum is Middleton Class B)
Outliers	Easily extended to include outliers	Difficult to include outliers

RFI Mitigation in SISO Systems

Return

Mitigation of computational platform noise in single carrier, single antenna systems [Nassar, Gulati, DeYoung, Evans & Tinsley, ICASSP 2008, JSPS 2009]

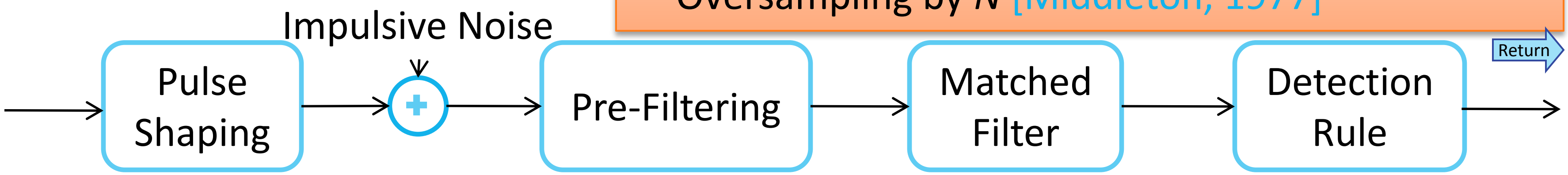
Computer Platform Noise Modelling	Evaluate fit of measured RFI data to noise models • Middleton Class A model • Symmetric Alpha Stable
Parameter Estimation	Evaluate estimation accuracy vs complexity tradeoffs
Filtering / Detection	Evaluate communication performance vs complexity tradeoffs • Middleton Class A: Correlation receiver, Wiener filtering, and Bayesian detector • Symmetric Alpha Stable: Myriad filtering, hole punching, and Bayesian detector

Filterin

Assumption

Multiple samples of the received signal are available

- *N* Path Diversity [Miller, 1972]
- Oversampling by *N* [Middleton, 1977]



Middleton Class A noise

Filtering

- Wiener Filtering (Linear)

Detection

- Correlation Receiver (Linear)
- Bayesian Detector [Spaulding & Middleton, 1977]
- Small Signal Approximation to Bayesian detector [Spaulding & Middleton, 1977]

Symmetric Alpha Stable noise

Filtering

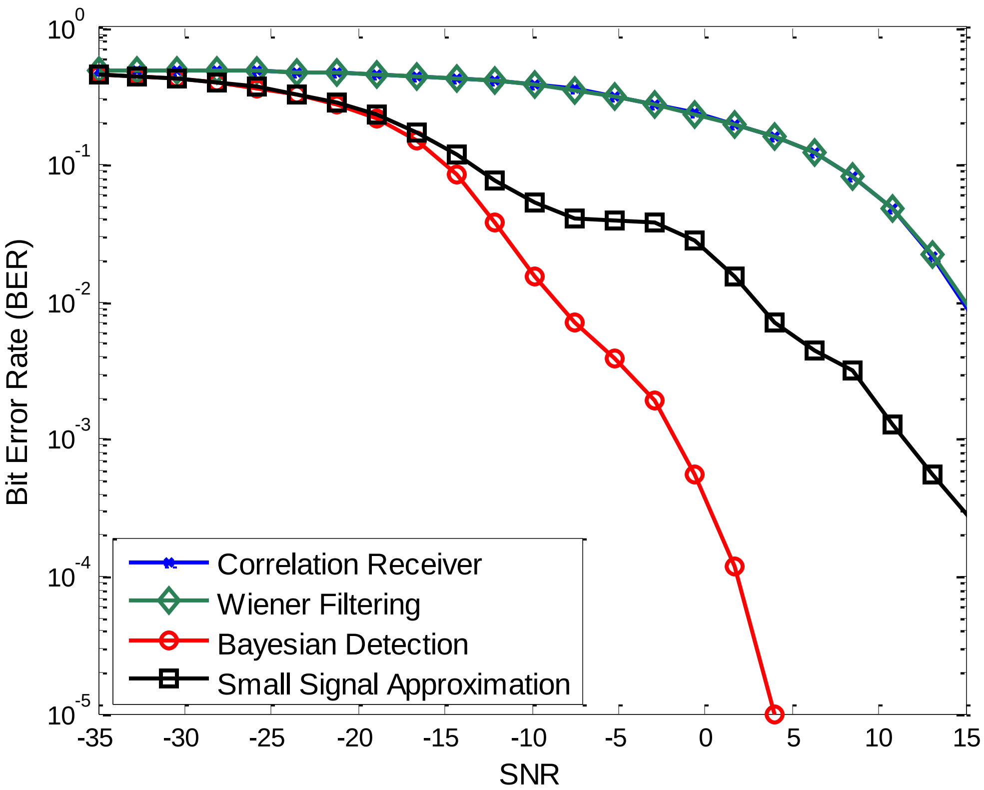
- Myriad Filtering
  - Optimal Myriad [Gonzalez & Arce, 2001]
  - Selection Myriad
- Hole Punching [Ambike *et al*., 1994]

Detection

- Correlation Receiver (Linear)
- MAP approximation [Kuruoglu, 1998]

Results: Class A Detection

Communication Performance



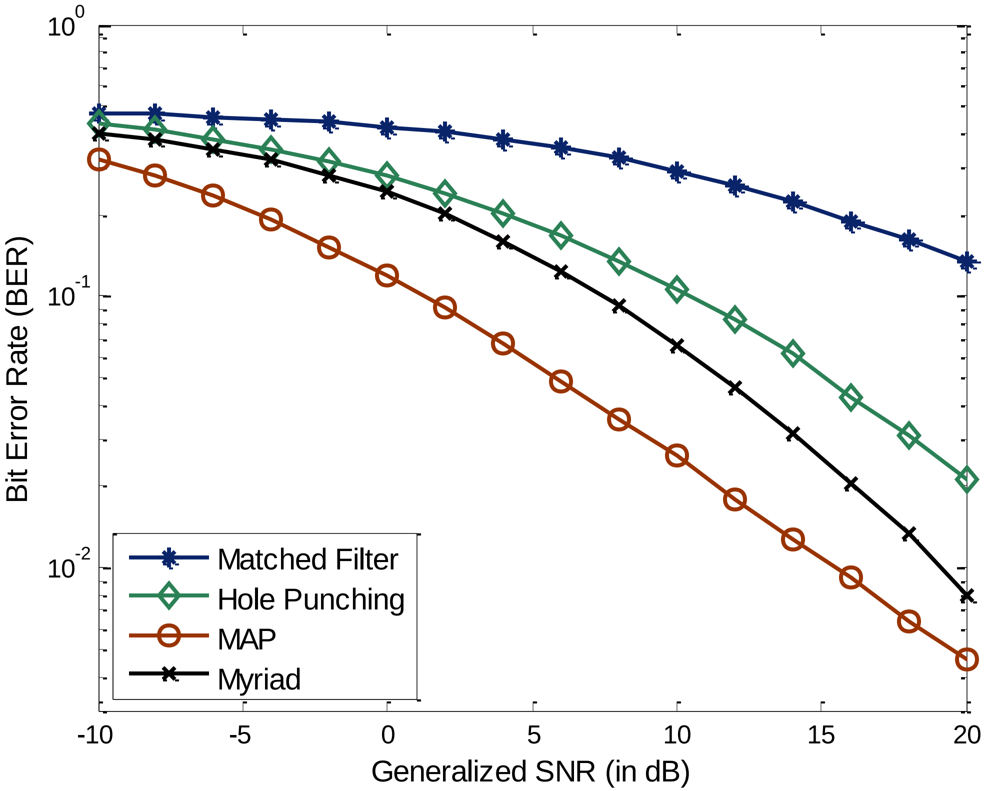
Binary Phase Shift Keying

Pulse shape	Channel
Raised cosine	A = 0.35
10 samples per symbol	$\Gamma = 0.5 \times 10^{-3}$
10 symbols per pulse	Memoryless

Method	Comp. Complexity	Detection Perform.
Correl.	Low	Low
Wiener	Medium	Low
Bayesian S.S. Approx.	Medium	High
Bayesian	High	High

Results: Alpha Stable Detection

Communication Performance



Return

Same transmitter settings as previous slide

Method	Comp. Complexity	Detection Perform.
Hole Punching	Low	Medium
Selection Myriad	Low	Medium
MAP Approx.	Medium	High
Optimal Myriad	High	Medium

Use dispersion parameter γ in place of noise variance to generalize SNR

RFI Mitigation in 2x2 MIMO Systems

2 x *2* MIMO receiver design in the presence of RFI

[Gulati, Chopra, Heath, Evans, Tinsley & Lin, Globecom 2008]

Return

RFI Modeling	• Evaluated fit of measured RFI data to the bivariate Middleton Class A model [McDonald & Blum, 1997] • Includes noise correlation between two antennas
Parameter Estimation	• Derived parameter estimation algorithm based on the method of moments (sixth order moments)
Performance Analysis	• Demonstrated communication performance degradation of conventional receivers in presence of RFI • Bounds on communication performance [Chopra , Gulati, Evans, Tinsley, and Sreerama, ICASSP 2009]
Receiver Design	• Derived Maximum Likelihood (ML) receiver • Derived two sub-optimal ML receivers with reduced complexity

Bivariate Middleton Class A Model

Return

- Joint spatial distribution

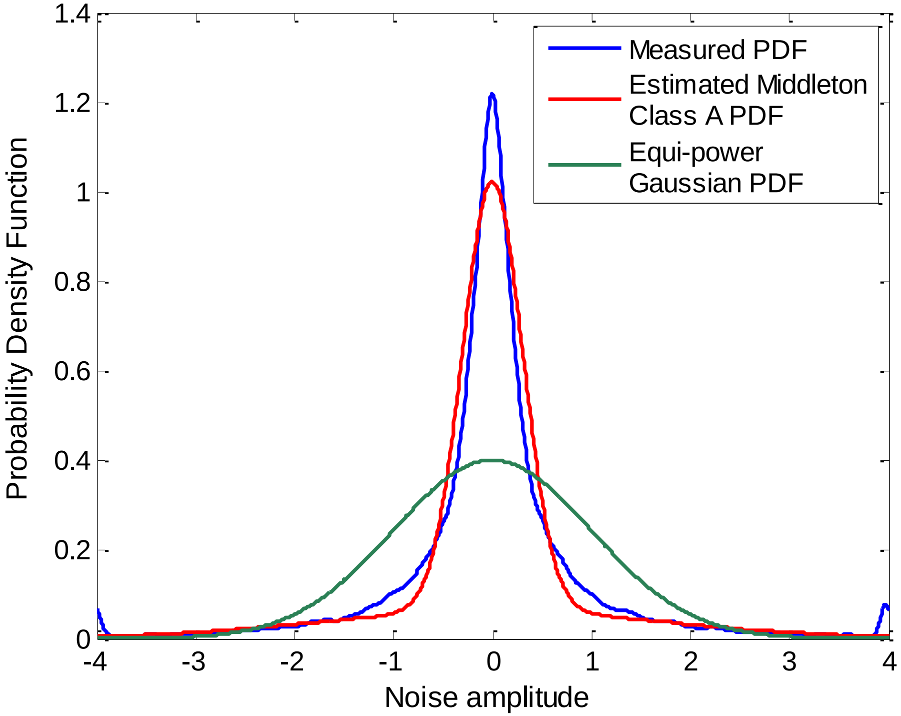
$$f_{\mathbf{n}}(\mathbf{n}) = \frac{e^{-A}}{2\pi|\mathbf{K}_0|^{\frac{1}{2}}} e^{\frac{-\mathbf{n}^T\mathbf{K}_0^{-1}\mathbf{n}}{2}} + \frac{\left(1-e^{-A}\right)}{2\pi|\mathbf{K}_1|^{\frac{1}{2}}} e^{\frac{-\mathbf{n}^T\mathbf{K}_1^{-1}\mathbf{n}}{2}}$$

$$\mathbf{K}_m = \begin{bmatrix} (c_m)^2 & \kappa c_m \hat{c}_m \\ \kappa c_m \hat{c}_m & (\hat{c}_m)^2 \end{bmatrix}, \quad (c_m)^2 = \frac{\frac{m}{A}+\Gamma_1}{1+\Gamma_1}, \; (\hat{c}_m)^2 = \frac{\frac{m}{A}+\Gamma_2}{1+\Gamma_2}.$$

Parameter	Description	Typical Range
A	Overlap Index. Product of average number of emissions per second and mean duration of typical emission	$A \in \left[10^{-2},\ 1\right]$
Γ_1, Γ_2	Ratio of Gaussian to non-Gaussian component intensity at each of the two antennas	$\Gamma \in \left[10^{-6},\ 1\right]$
κ	Correlation coefficient between antenna observations	$\kappa \in \left[-1,\ 1\right]$

Results on Measured RFI Data

- 50,000 baseband noise samples represent broadband interference



Marginal PDFs of measured data compared with estimated model densities

Estimated Parameters		
Bivariate Middleton Class A		
Overlap Index (*A*)	0.313	2D- KL Divergence **1.004**
Gaussian Factor (Γ_1)	0.105	
Gaussian Factor (Γ_2)	0.101	
Correlation (κ)	-0.085	
Bivariate Gaussian		
Mean (μ)	0	2D- KL Divergence **1.6682**
Variance (σ_1)	1	
Variance (σ_2)	1	
Correlation (κ)	-0.085	

System Model

Return

- 2 x 2 MIMO System

$$\mathbf{Y} = \sqrt{\frac{E_s}{2}}\mathbf{H}\mathbf{S} + \mathbf{N}$$

T: Length of transmitted data block
E_s: Total transmit energy
$\mathbf{Y}$: $2 \times T$ received signals
$\mathbf{H}$: 2×2 channel matrix. $\mathbf{H} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$
$\mathbf{S}$: $2 \times T$ transmitted data block
$\mathbf{N}$: $2 \times T$ additive noise matrix $(\mathbf{N} = \mathbf{n}_R + j\mathbf{n}_I)$
Spatial Multiplexing transmission mode

- Maximum Likelihood (ML) receiver

$$\hat{\mathbf{c}}_{ML} = \arg\underbrace{\max}_{\mathbf{s}\in\mathcal{C}}\{L(\mathbf{s}|\mathbf{y})\}$$

Sub-optimal ML Receivers approximate $\phi(\cdot)$

- Log-likelihood function

$$L(\mathbf{s}|\mathbf{y}) = \frac{-\mathbf{n}^T\mathbf{K}_0^{-1}\mathbf{n}}{2} + \ln(\Lambda_0) + \phi\left(\frac{-\mathbf{n}^T\left(\mathbf{K}_1^{-1} - \mathbf{K}_0^{-1}\right)\mathbf{n}}{2} + \ln\left(\frac{\Lambda_1}{\Lambda_0}\right)\right)$$

$$\Lambda_0 = \frac{\left(e^{-A}\right)}{2\pi|\mathbf{K}_0|^{\frac{1}{2}}}, \quad \Lambda_1 = \frac{\left(1 - e^{-A}\right)}{2\pi|\mathbf{K}_1|^{\frac{1}{2}}}, \quad \phi(z) = \ln\left(1 + e^z\right) \quad \forall z \in \mathcal{R}.$$

Sub-Optimal ML Receivers

Return

- Two-piece linear approximation

$$\phi_1(z) = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{if } z \geq 0 \end{cases}$$

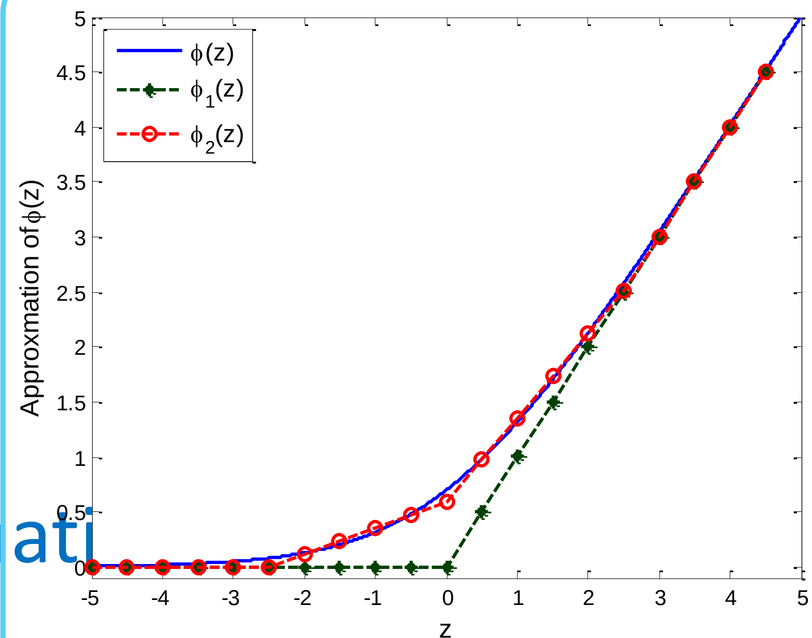
$$L(\mathbf{s}|\mathbf{y}) \approx \begin{cases} \frac{-\mathbf{n}^T\mathbf{K}_0^{-1}\mathbf{n}}{2} + \ln(\Lambda_0) & \frac{\mathbf{n}^T(\mathbf{K}_1^{-1}-\mathbf{K}_0^{-1})\mathbf{n}}{2} > \ln\left(\frac{\Lambda_1}{\Lambda_0}\right) \\ \frac{-\mathbf{n}^T\mathbf{K}_1^{-1}\mathbf{n}}{2} + \ln(\Lambda_1) & \frac{\mathbf{n}^T(\mathbf{K}_1^{-1}-\mathbf{K}_0^{-1})\mathbf{n}}{2} < \ln\left(\frac{\Lambda_1}{\Lambda_0}\right) \end{cases}$$

- Four-piece linear approximati

$$\phi_2(z) = \begin{cases} 0 & \text{if } z < -\gamma \\ \alpha z + \alpha\gamma & \text{if } -\gamma \leq z < 0 \\ (1-\alpha)z + \alpha\gamma & \text{if } 0 \leq z < \gamma \\ z & \text{if } z \geq \gamma \end{cases}$$

$\alpha = 0.236$ and $\gamma = 2.507$ chosen to minimize

$$\int_{-\infty}^{\infty} |\phi_2(z) - \phi(z)|^2 dz$$

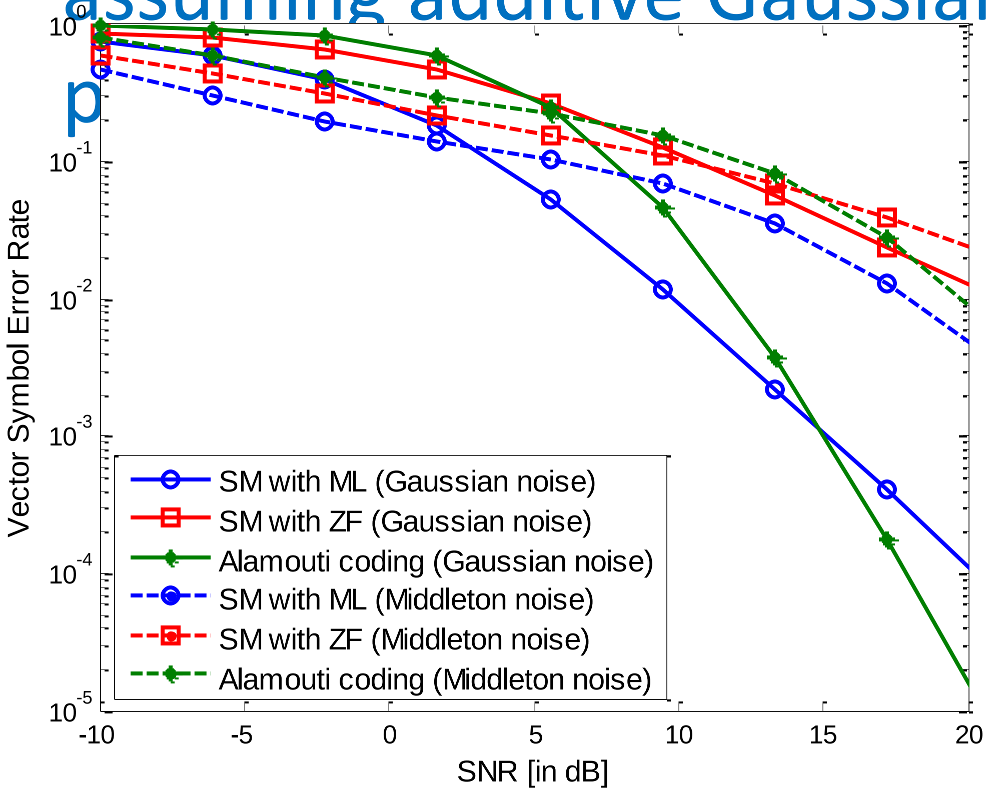


Approximation of $\phi(z) = \ln(1 + e^z)$

Results: Performance Degradation

Return

- Performance degradation in receivers designed assuming additive Gaussian noise in the p

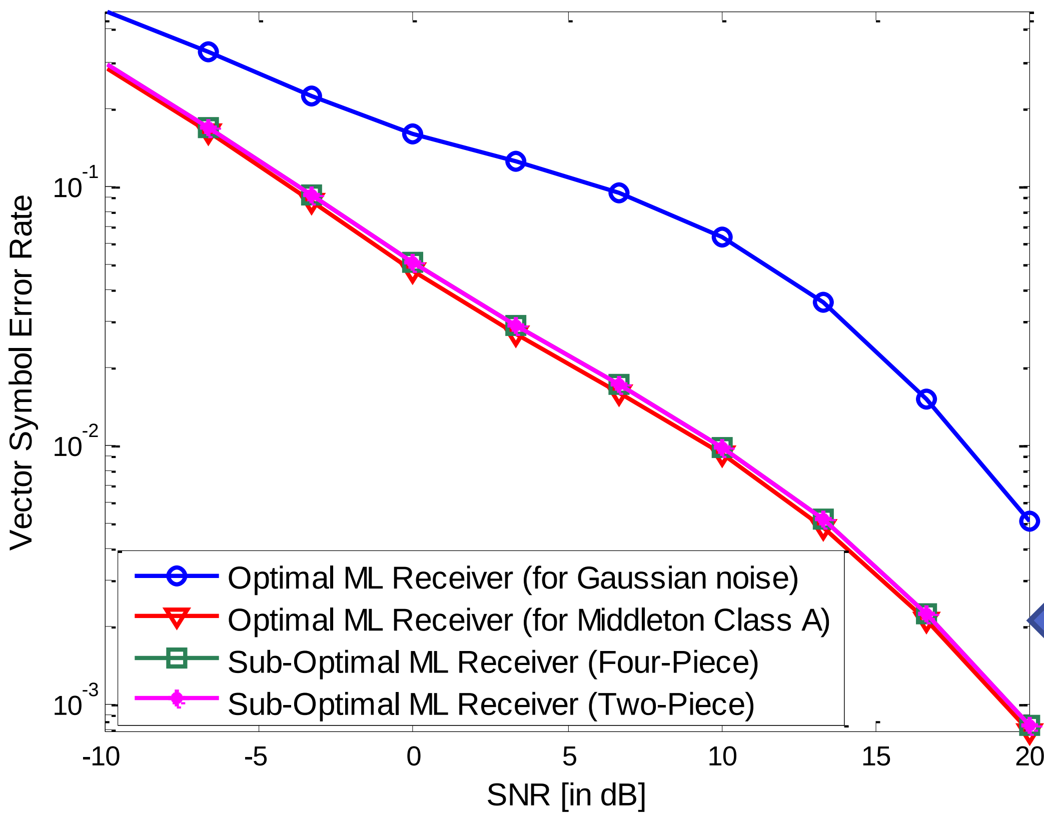


Simulation Parameters

- 4-QAM for Spatial Multiplexing (SM) transmission mode
- 16-QAM for Alamouti transmission strategy
- Noise Parameters: $A = 0.1$, $\Gamma_1 = 0.01$, $\Gamma_2 = 0.1$, $\kappa = 0.4$

Severe degradation in communication performance in high-SNR regimes

Results: RFI Mitigation in 2 x 2 MIMO



Communication Performance
(A = 0.1, Γ_1= 0.01, Γ_2= 0.1, κ = 0.4)

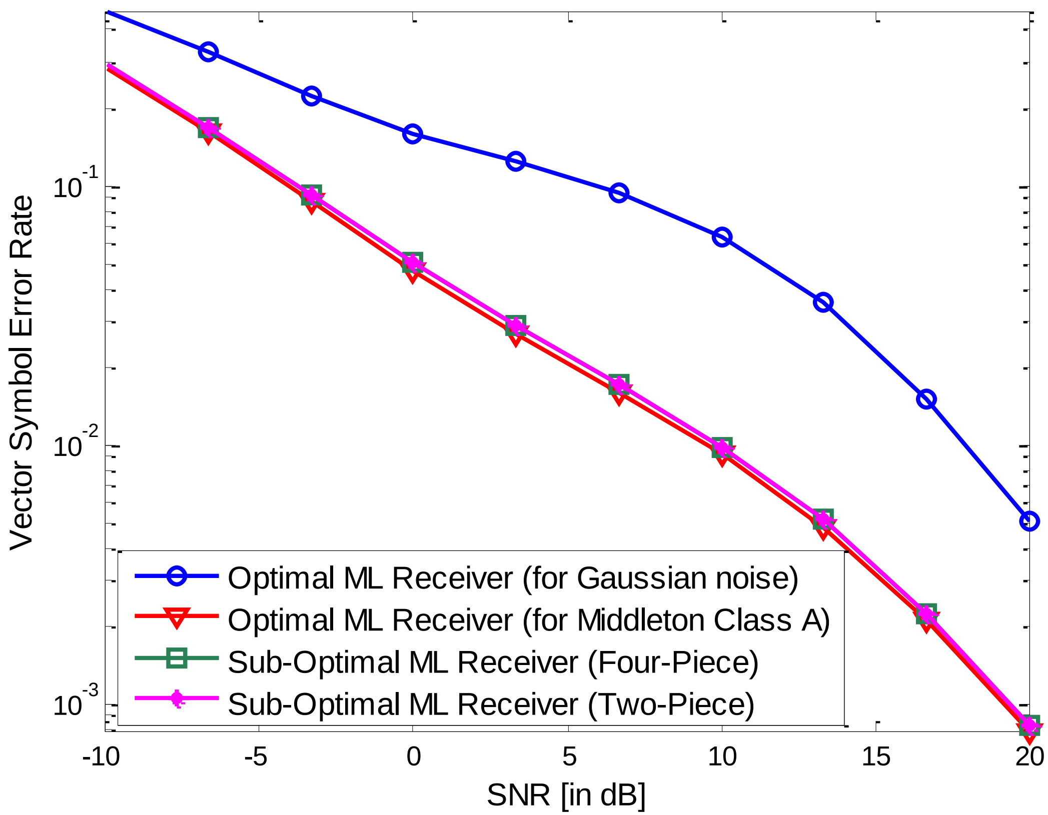
Return

Improvement in communication performance over conventional Gaussian ML receiver at symbol error rate of 10^{-2}

A	Noise Characteristic	Improve -ment
0.01	Highly Impulsive	~15 dB
0.1	Moderately Impulsive	~8 dB
1	Nearly Gaussian	~0.5 dB

Results: RFI Mitigation in 2 x 2 MIMO

Return

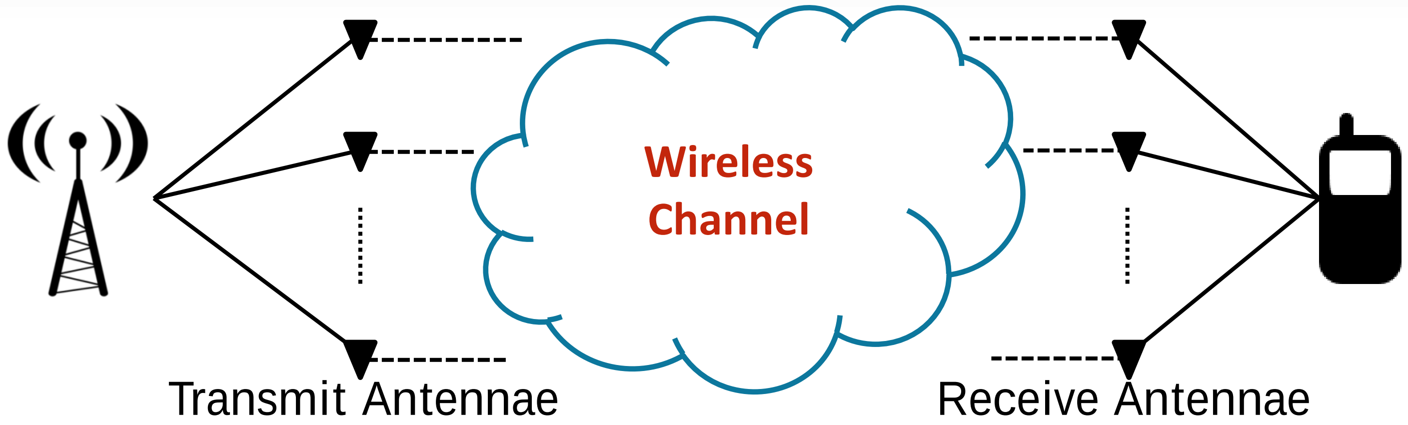


Communication Performance
($A = 0.1$, $\Gamma_1 = 0.01$, $\Gamma_2 = 0.1$, $\kappa = 0.4$)

Complexity Analysis for decoding M-level QAM modulated signal

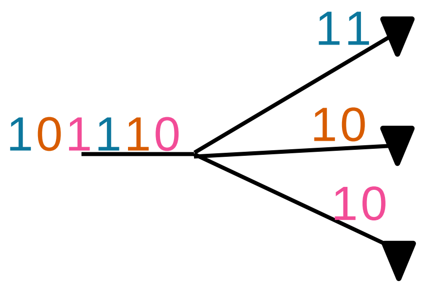
Receiver	$\mathbf{n}^T\mathbf{K}\mathbf{n}$	e^x	a $\gtrless$ b
Gaussian ML	M^2	0	0
Optimal ML	$2M^2$	$2M^2$	0
Sub-optimal ML (Four-Piece)	$2M^2$	0	$2M^2$
Sub-optimal ML (Two-Piece)	$2M^2$	0	M^2

Wireless communication systems are increasingly using multiple antennae



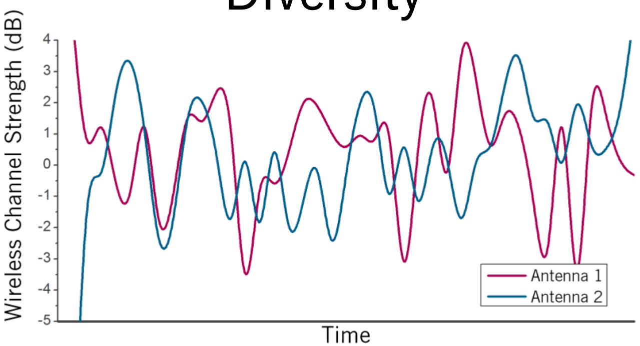
Benefits

Multiplexing



Multiple data streams transmitted simultaneously

Diversity



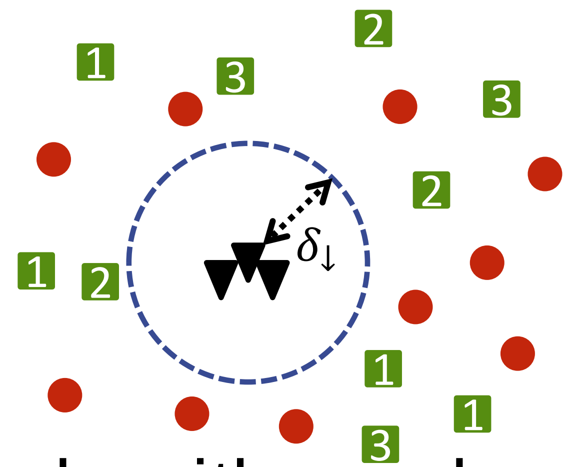
Antennae with strong channels can compensate for antennae with weak channels

Interference Removal

Interference cancellation and alignment

Interference statistics in networks without guard zones are a mix of isotropic and i.i.d. alpha stable …

A 3-antenna receiver within a Poisson field of interferers



Joint characteristic function

$$\Phi(w) = e^{\sigma_0 \|\mathbf{w}\|^\alpha} \times \prod_{n=1}^{N} e^{\sigma_n |\omega_n|^\alpha}$$

$$\alpha = \frac{4}{\gamma}, \sigma_n \propto \lambda_n$$

… and interference statistics in networks with guard zones are a mix of isotropic and i.i.d. Middleton Class A

Joint characteristic function

$$\Phi(w) = e^{A_0 e^{-\frac{\|\mathbf{w}\|^2 \Omega_0}{2}}} \times \prod_{n=1}^{N} e^{A_n e^{-\frac{|w_n|^2 \Omega_n}{2}}}$$

$$A_n \propto \lambda_n \delta_\downarrow^2, \Omega_n \propto A_n \delta_\downarrow^{-\gamma}$$