Equalizer Design to Maximize Bit Rate in ADSL Transceivers

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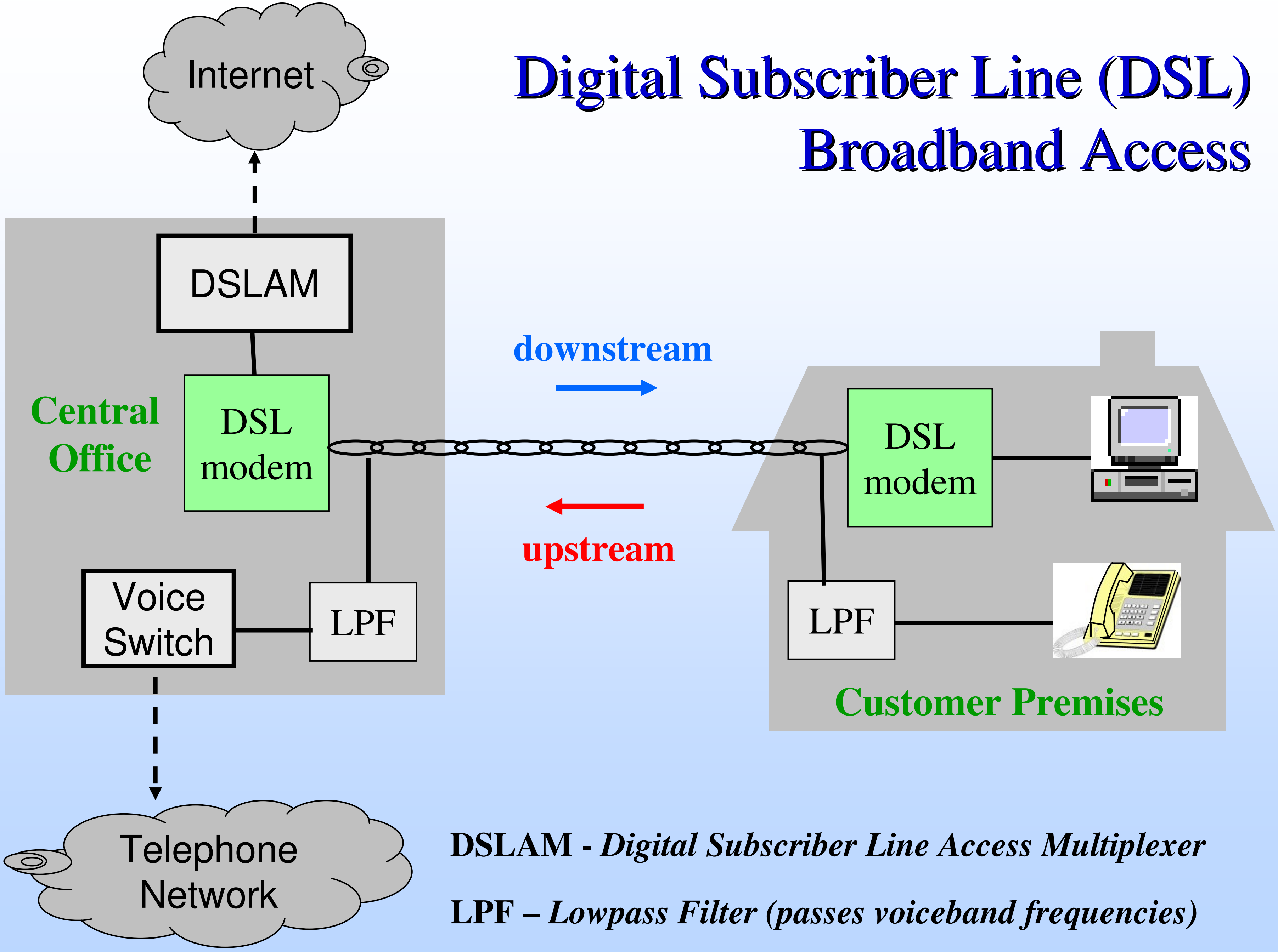
Digital Subscriber Line (DSL)
Broadband Access



DSLAM - *Digital Subscriber Line Access Multiplexer*

LPF – *Lowpass Filter (passes voiceband frequencies)*

Discrete Multitone (DMT) DSL Standards

ADSL – Asymmetric DSL

1997

- Maximum data rates supported in G.DMT standard (*ideal case*)
  - Echo cancelled: 14.94 Mbps downstream, 1.56 Mbps upstream
  - Frequency division multiplexing (FDM): 13.38 Mbps downstream, 1.56 Mbps upstream
- Widespread deployment in US, Canada, Western Europe, and Hong Kong
  - Central office providers only installing frequency-division multiplexed (FDM)
  - ADSL:cable modem market
    1:2 in US & 2:1 worldwide

2003

- ADSL+ 8 Mbps downstream min.

2003

- ADSL2 doubles analog bandwidth

VDSL – Very High Rate DSL

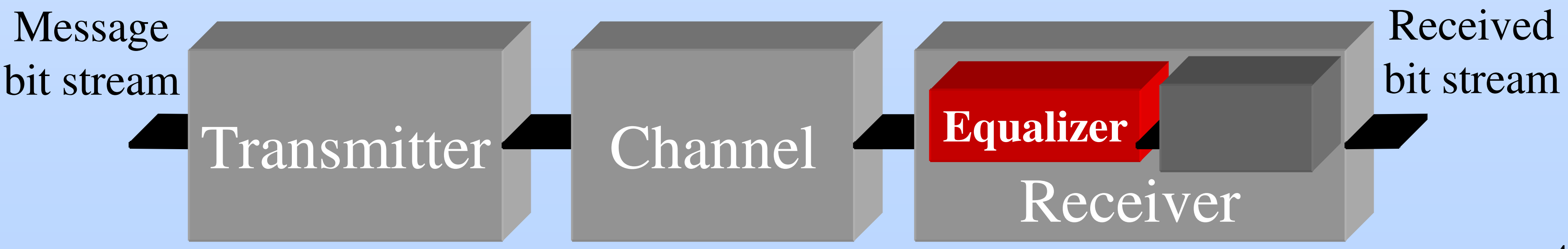
2003

- Asymmetric
  - Faster G.DMT FDM ADSL
  - 2^m subcarriers $m \in [8, 12]$
- Symmetric: 13, 9, or 6 Mbps
- Optional 12-17 MHz band

	G.DMT ADSL	***Asymmetric DMT VDSL***
Data band	0.025 – 1.1 MHz	0.138 – 12 MHz
Upstream subcarriers	32	256
Downstream subcarriers	256	2048/4096
Target up-stream rate	1 Mbps	3 Mbps
Target down-stream rate	8 Mbps	13/22 Mbps

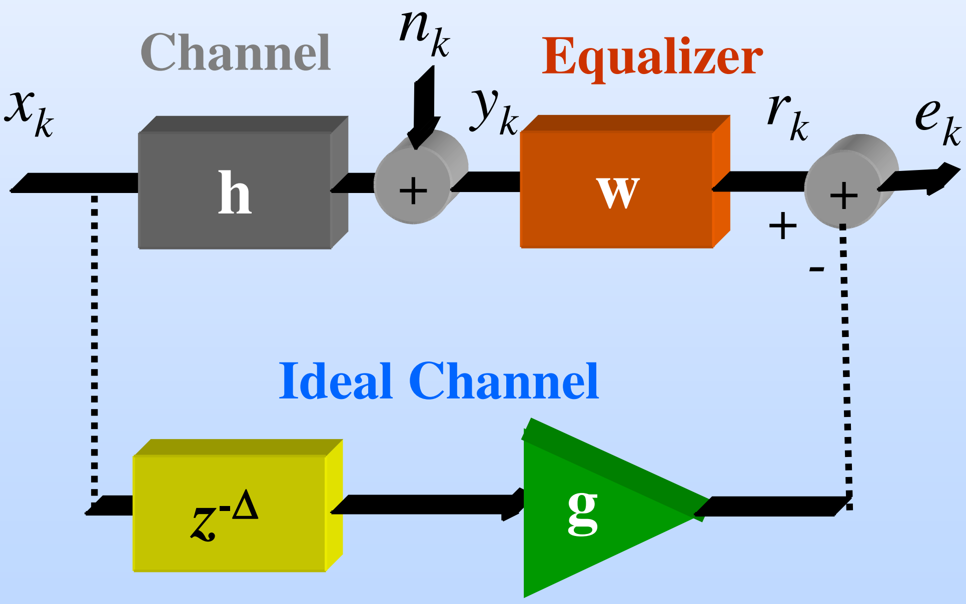
Outline

- **Multicarrier modulation**
- **Conventional equalizer training methods**
  - Minimum Mean Squared Error design [Stanford]
  - Maximum Shortening Signal-to-Noise Ratio design [Tellabs]
  - Maximum Bit Rate design (*optimal*) [UT Austin]
  - Minimum Inter-symbol Interference design (*near-optimal*) [UT Austin]
- **Per-tone equalizer** [Catholic University, Leuven, Belgium]
- **Dual-path equalizer** [UT Austin]
- **Conclusion**



Single Carrier Modulation

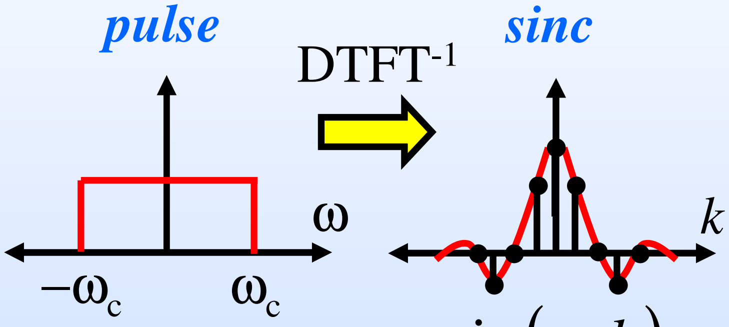
- **Ideal (non-distorting) channel over transmission band**
  - Flat magnitude response
  - Linear phase response: delay is constant for all spectral components
  - No intersymbol interference
- **Impulse response for ideal channel over all frequencies**
  - Continuous time: $g\,\delta(t-T)$
  - Discrete time: $g\,\delta[k-\Delta]$
- **Equalizer**
  - Shortens channel impulse response (*time domain*)
  - Compensates for frequency distortion (*frequency domain*)



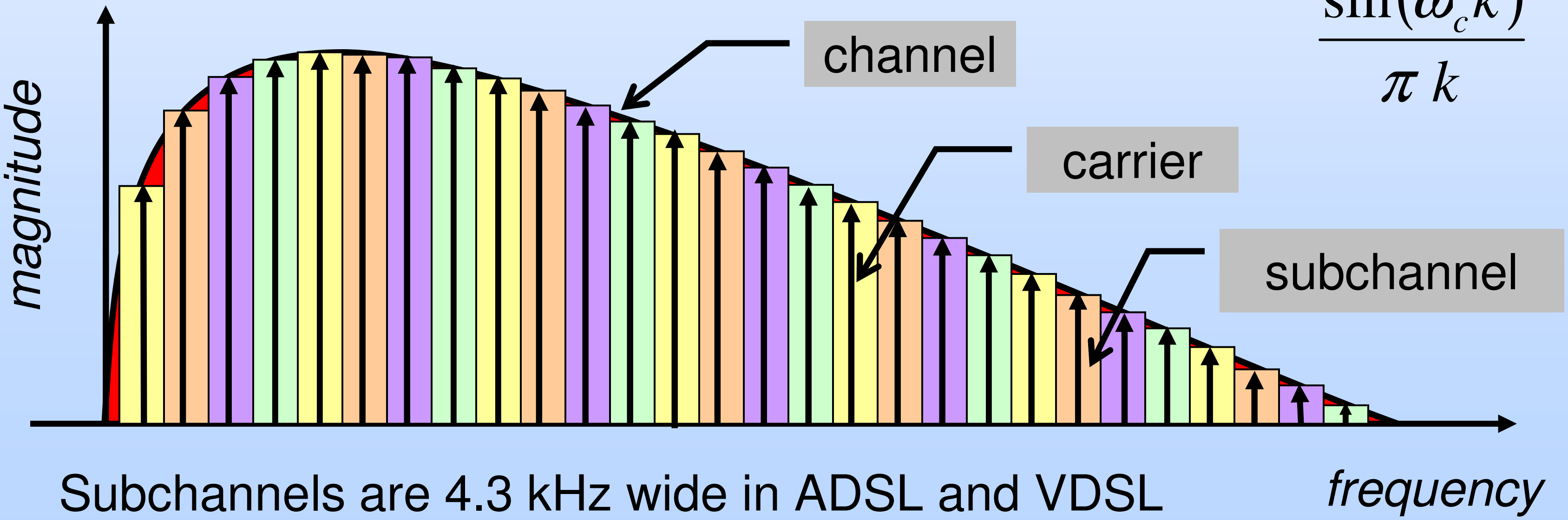
Discretized Baseband System

Multicarrier Modulation

- **Divide channel into narrowband subchannels**
  - No inter-symbol interference (ISI) in subchannels if constant gain within every subchannel and if ideal sampling
- **Discrete multitone modulation**
  - Baseband transmission
  - Based on fast Fourier transform (FFT)
  - Standardized for ADSL and VDSL

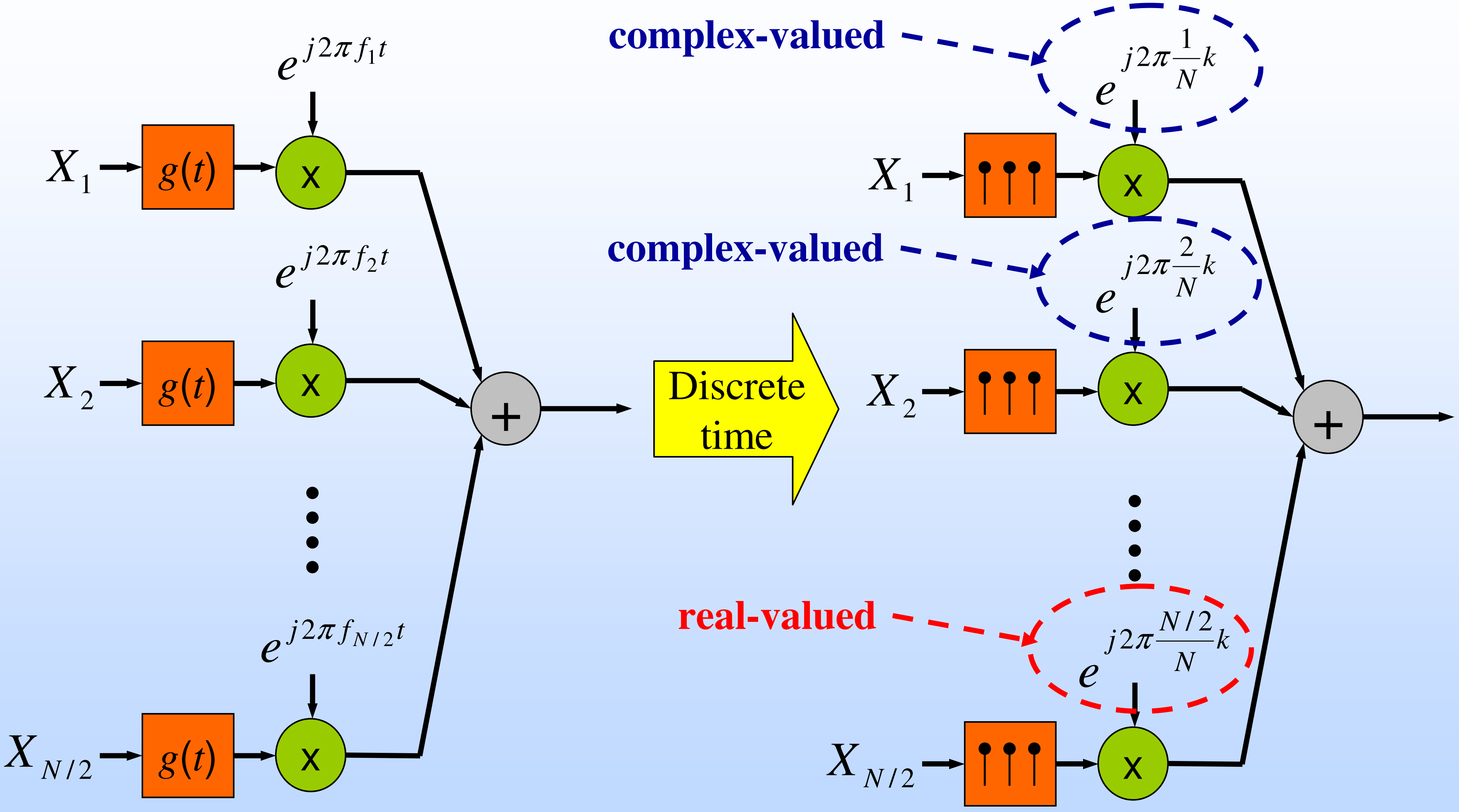


$$\frac{\sin(\omega_c k)}{\pi\, k}$$



Subchannels are 4.3 kHz wide in ADSL and VDSL

Multicarrier Modulation by Inverse FFT Filter Bank

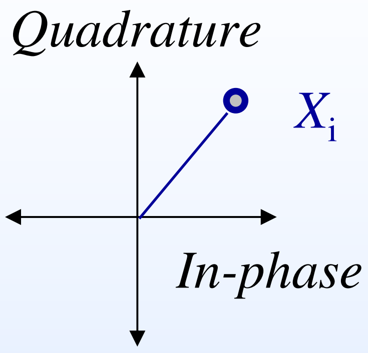


$g(t)$: pulse shaping filter

X_i : i^{th} subsymbol from encoder

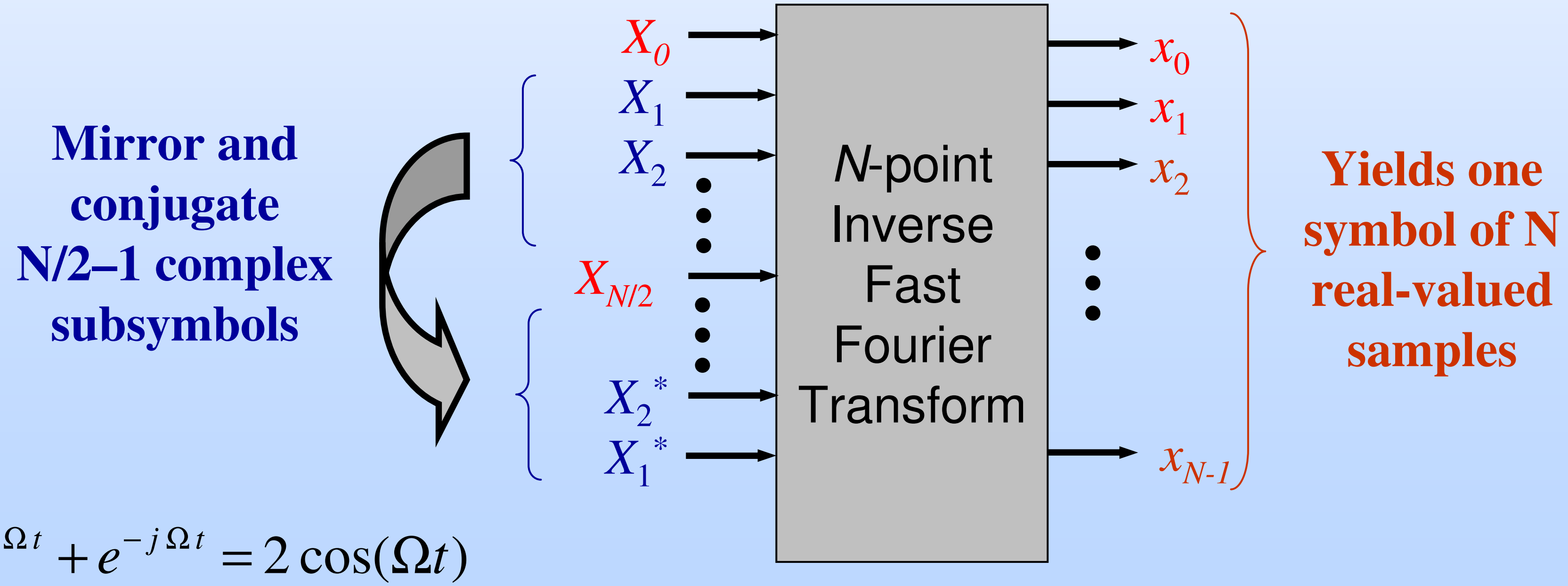
Discrete Multitone Modulation Symbol

- ***N*/2 subsymbols are in general complex-valued**
  - ADSL uses 4-level Quadrature Amplitude Modulation (QAM) during training
  - ADSL uses QAM of 2^2, 2^3, 2^4, …, 2^{15} levels during data transmission



QAM

- **Multicarrier modulation using inverse FFT**



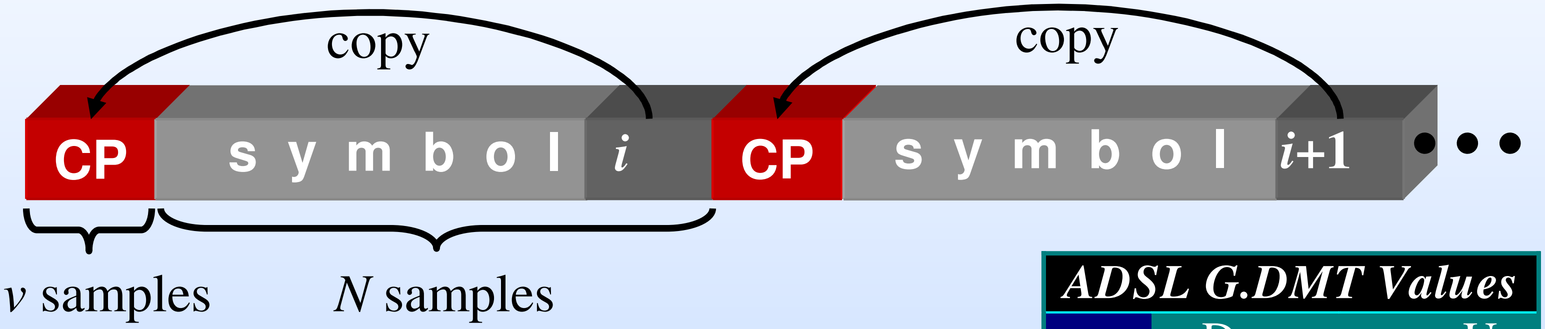
Mirror and conjugate N/2–1 complex subsymbols

Yields one symbol of N real-valued samples

$$e^{j\Omega t} + e^{-j\Omega t} = 2\cos(\Omega t)$$

Discrete Multitone Modulation Frame

- **Frame is sent through D/A converter and transmitted**
  - Frame is the symbol with cyclic prefix prepended
  - Cyclic prefix (CP) consists of last ν samples of the symbol



  - CP reduces throughput by factor of $\dfrac{N}{N+\nu} = \dfrac{16}{17}$

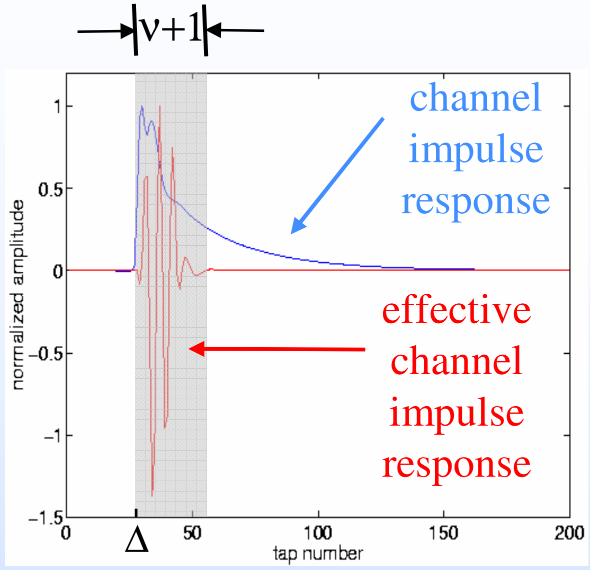
ADSL G.DMT Values		
	Down stream	Up stream
ν	32	4
N	512	64

- **Linear convolution of frame with channel impulse response**
  - Is circular convolution if channel length is CP length plus one or shorter
  - Circular convolution ⇨ frequency-domain equalization in FFT domain
  - Time-domain equalization to reduce effective channel length and ISI

Eliminating ISI in Discrete Multitone Modulation

- **Time domain equalizer (TEQ)**
  - Finite impulse response (FIR) filter
  - *Effective channel impulse response*: convolution of TEQ impulse response with channel impulse response
- **Frequency domain equalizer (FEQ)**
  - Compensates magnitude/phase distortion of equalized channel by dividing each FFT coefficient by complex number
  - Generally updated during data transmission
- **ADSL G.DMT equalizer training**
  - *Reverb*: same symbol sent 1,024 to 1,536 times
  - *Medley*: aperiodic pseudo-noise sequence of 16,384 symbols
  - Receiver returns number of bits (0-15) to transmit each subchannel *i*

$$b_i \le \log\left(1+\frac{\text{SNR}_i}{\Gamma_i}\right)$$



Δ: *transmission delay*

ν: *cyclic prefix length*

ADSL G.DMT Values

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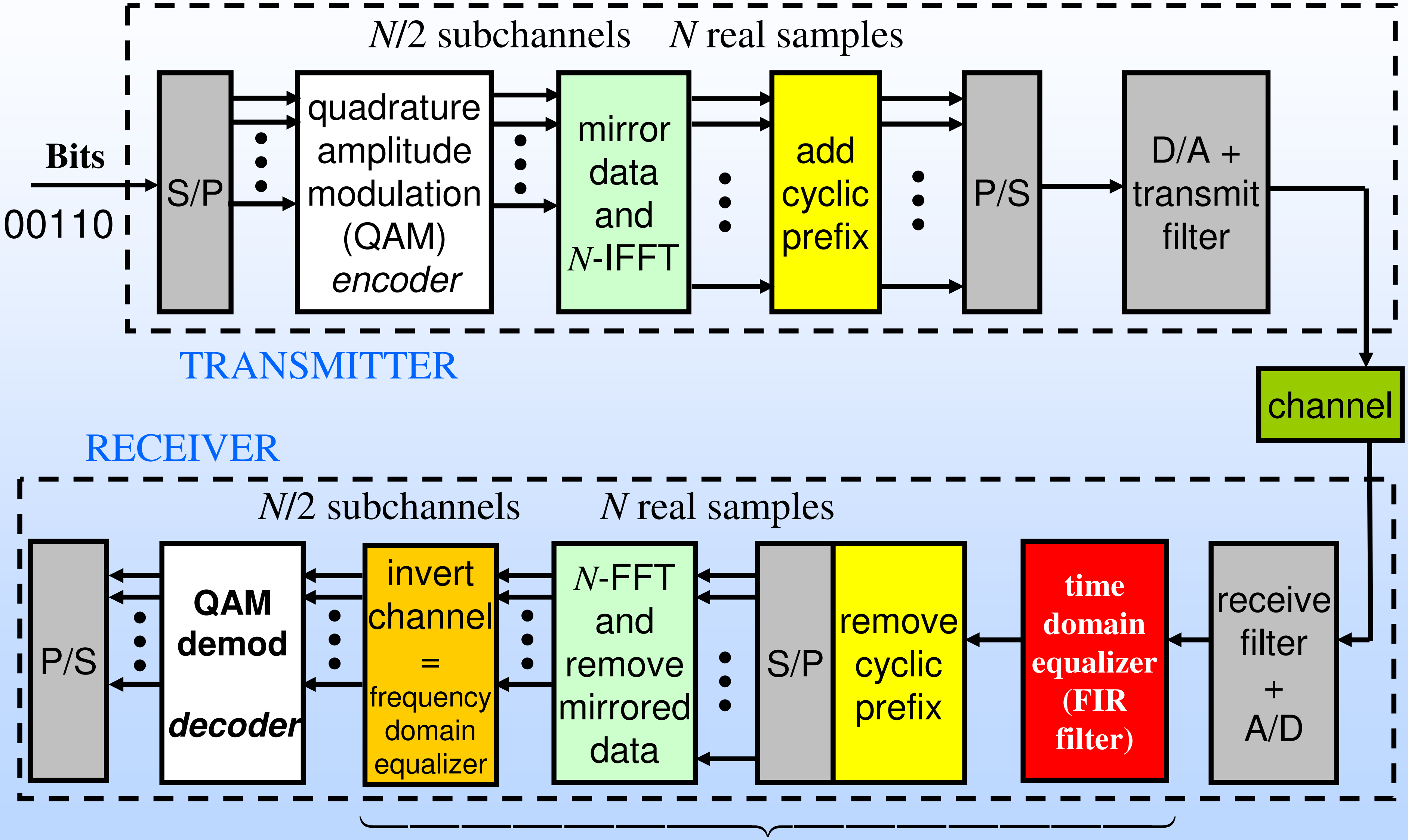
ADSL Transceiver: Data Transmission



N/2 subchannels *N* real samples

Bits

00110

S/P

quadrature amplitude modulation (QAM) *encoder*

mirror data and *N*-IFFT

add cyclic prefix

P/S

D/A + transmit filter

TRANSMITTER

channel

RECEIVER

N/2 subchannels *N* real samples

receive filter + A/D

time domain equalizer (FIR filter)

remove cyclic prefix

S/P

N-FFT and remove mirrored data

invert channel = frequency domain equalizer

QAM demod ***decoder***

P/S

conventional ADSL equalizer structure

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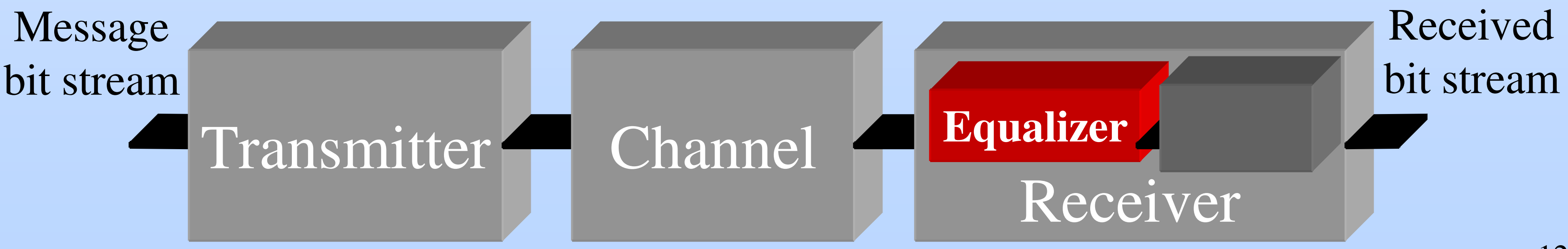
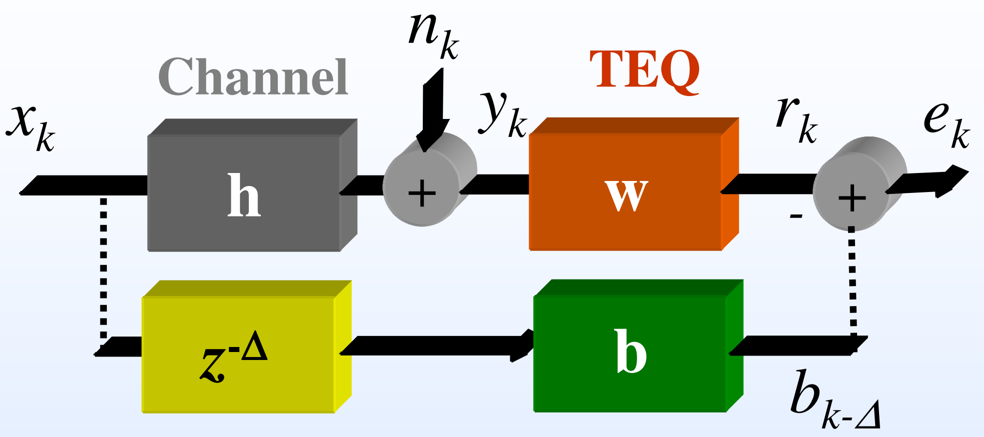
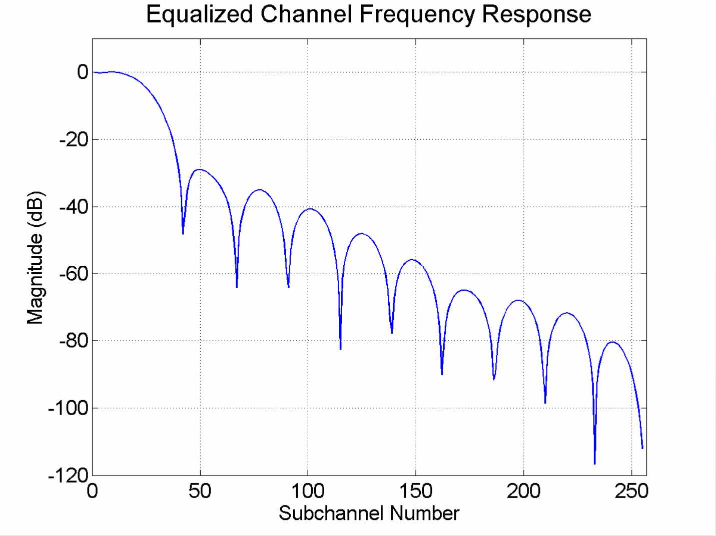


Minimum Mean Squared Error TEQ Design





- **Minimize** $E\{e_k^2\}$ [Chow & Cioffi, 1992]
  - Chose length of $\mathbf{b}$ (e.g. $\nu+1$) to shorten length of $\mathbf{h} * \mathbf{w}$
  - $\mathbf{b}$ is eigenvector of minimum eigenvalue of symmetric channel-dependent matrix $\mathbf{R}_\Delta = \mathbf{R}_{xx} - \mathbf{R}_{xy}\,\mathbf{R}_{yy}^{-1}\,\mathbf{R}_{yx}$
  - Minimum MSE when $\mathbf{R}_{yy}\,\mathbf{w} = \mathbf{R}_{xy}\,\mathbf{b}$ where $\mathbf{w} \neq \mathbf{0}$

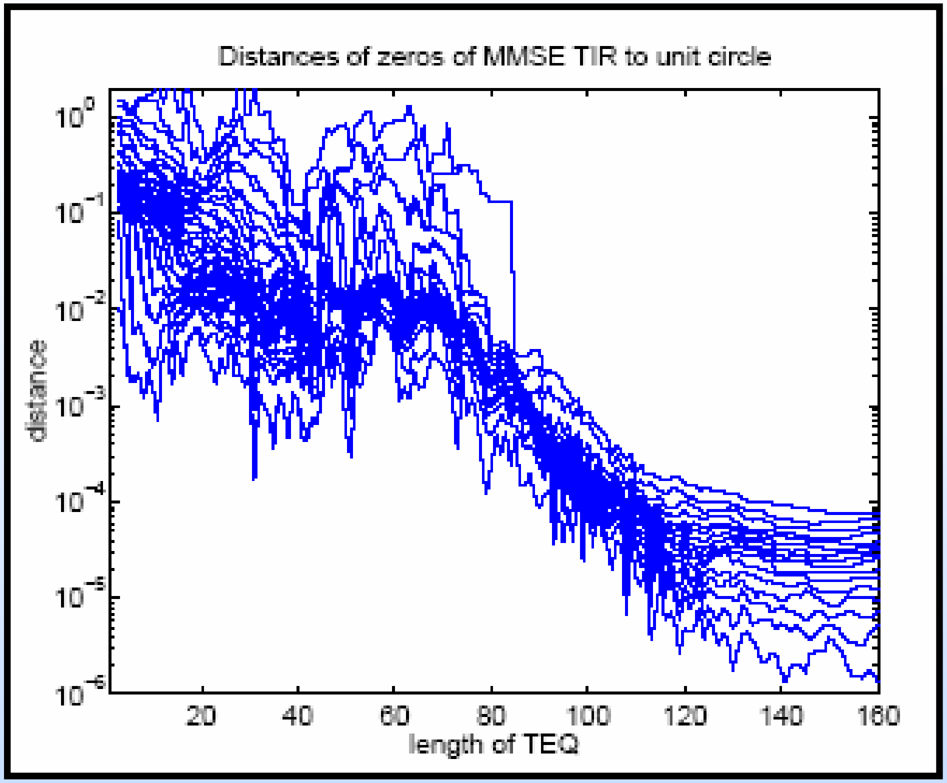
$\mathbf{R}_{xy}$ is cross correlation matrix

- **Disadvantages**
  - Does not consider *bit rate*
  - Deep notches in equalized frequency response

Why?

Infinite Length MMSE TEQ Analysis

- **As TEQ length goes to infinity, $\mathbf{R}_\Delta$ becomes Toeplitz** [Martin *et al.* 2003]
  - Eigenvector of minimum eigenvalue of symmetric Toeplitz matrix has zeros on unit circle [Makhoul 1981]
  - Zeros of target impulse response **b** on unit circle kills ν subchannels



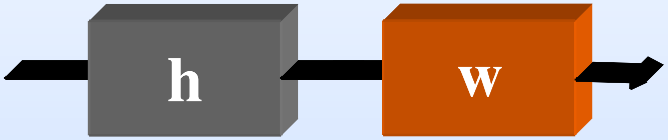
- **Finite length TEQ plot**
  - Each trace is a different zero of **b**
  - Distance of 32 zeros of **b** to unit circle averaged over 8 ADSL test channels for each TEQ length
  - Zeros cluster at 0.01 and 10^{-4} from UC for TEQ lengths 32 and 100

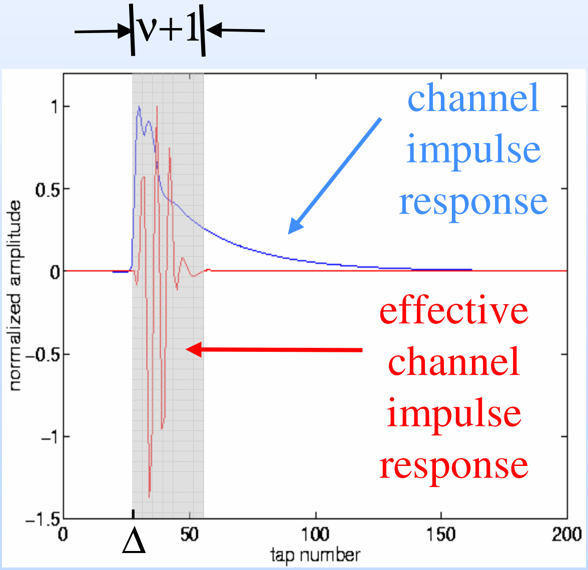
Longer MMSE TEQ may be worse

Maximum Shortening SNR TEQ Design

- **Minimize energy leakage outside shortened channel length**
- **For each possible position of window** [Melsa, Younce & Rohrs, 1996]

$$\max_{\mathbf{w}}\left(\text{SSNR in dB}\right) = \max_{\mathbf{w}} 10\log_{10}\left(\frac{\text{energy inside window after TEQ}}{\text{energy outside window after TEQ}}\right)$$



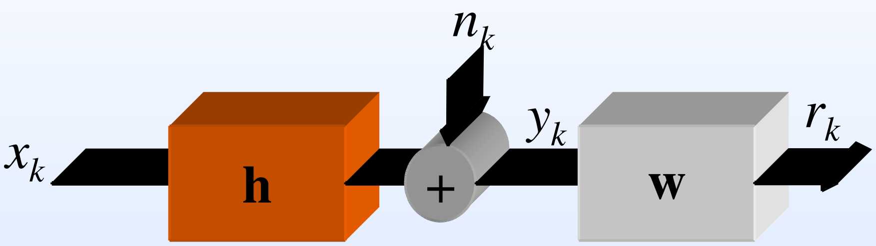


- **Equivalent to noise-free MMSE TEQ**
- **Disadvantages**
  - Does not consider channel noise
  - Does not consider *bit rate*
  - Deep notches in equalized frequency response (zeros of target impulse response near unit circle kill subchannels)
  - Requires Cholesky decomposition, which is computationally-intensive and does not allow TEQ lengths longer than cyclic prefix

Maximum Shortening SNR TEQ Design

- **Choose w to minimize energy outside window of desired length**

  Locate window to capture maximum channel impulse response energy



$$\mathbf{h}_{wall}^T \mathbf{h}_{wall} = \mathbf{w}^T \mathbf{H}_{wall}^T \mathbf{H}_{wall} \mathbf{w} = \mathbf{w}^T \mathbf{A} \mathbf{w}$$

$$\mathbf{h}_{win}^T \mathbf{h}_{win} = \mathbf{w}^T \mathbf{H}_{win}^T \mathbf{H}_{win} \mathbf{w} = \mathbf{w}^T \mathbf{B} \mathbf{w}$$

$h_{win,}\ h_{wall}$: equalized channel within and outside the window

- **Objective function is shortening SNR (SSNR)**

$$\max_{\mathbf{w}}\left(\mathrm{SSNR}\right) = \max_{\mathbf{w}} 10\log_{10}\left(\frac{\mathbf{w}^T\mathbf{B}\mathbf{w}}{\mathbf{w}^T\mathbf{A}\mathbf{w}}\right) \text{ subject to } \mathbf{w}^T\mathbf{B}\mathbf{w} = 1$$

Cholesky decomposition of **B** to find eigenvector for minimum generalized eigenvalue of **A** and **B**

$$\mathbf{C} = \left(\sqrt{\mathbf{B}}\right)^{-1} \mathbf{A} \left(\sqrt{\mathbf{B}^T}\right)^{-1}$$

$$\mathbf{w}_{opt} = \left(\sqrt{\mathbf{B}^T}\right)^{-1} \mathbf{q}_{\min} \qquad \mathbf{q}_{\min} : \text{eigenvector of min eigenvalue of } \mathbf{C}$$

Modeling Achievable Bit Rate

- **Bit allocation bounded by subchannel SNRs:** $\log(1 + \text{SNR}_i / \Gamma_i)$
- **Model i^{th} subchannel SNR** [Arslan, Evans & Kiaei, 2001]

$$\text{SNR}_i = \frac{\text{signal power}}{\text{noise power} + \text{ISI power}}$$

Used in Maximum Bit Rate Method

$$\text{SNR}_i = \frac{S_{x,i} \times \text{signal transfer function}}{S_{n,i} \times \text{noise transfer function} + S_{x,i} \times \text{ISI transfer function}}$$

$S_{x,i}$: transmitted signal power in subchannel i

$S_{n,i}$: channel noise power in subchannel i

- **Divide numerator and denominator of SNR_i by noise power spectral density $S_{n,i}$**

$$\text{SNR}_i = \frac{\dfrac{S_{x,i}}{S_{n,i}} \left| H_i^{signal} \right|^2}{\left| H_i^{noise} \right|^2 + \dfrac{S_{x,i}}{S_{n,i}} \left| H_i^{ISI} \right|^2}$$

Conventional subchannel SNR_i

Used in Minimum ISI Method

Maximum Bit Rate (MBR) TEQ Design

- Subchannel SNR as nonlinear function of equalizer taps $\mathbf{w}$

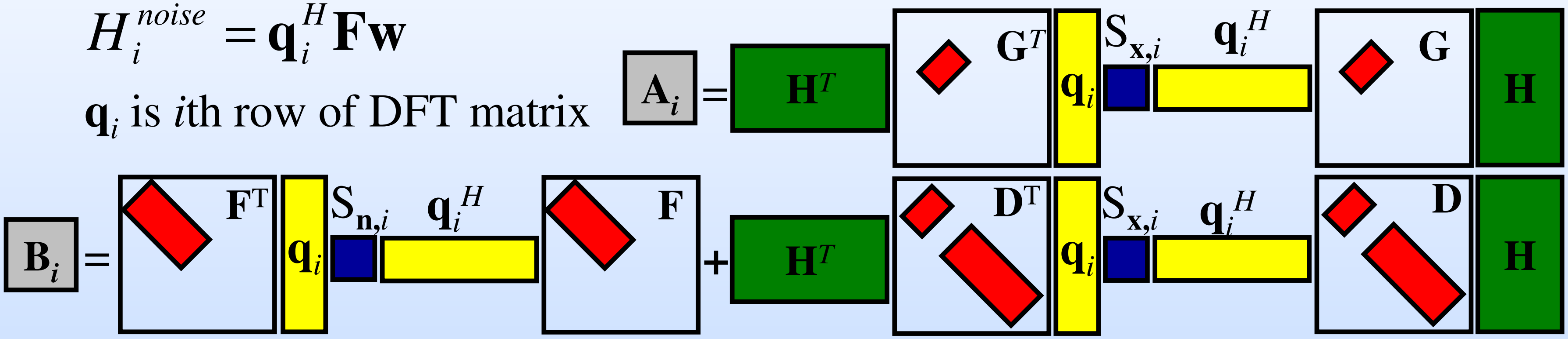
$$H_i^{signal} = \mathbf{q}_i^H \mathbf{GHw}$$

$$H_i^{ISI} = \mathbf{q}_i^H \mathbf{DHw}$$

$$H_i^{noise} = \mathbf{q}_i^H \mathbf{Fw}$$

$\mathbf{q}_i$ is ith row of DFT matrix

$$\mathrm{SNR}_i = \frac{S_{x,i}\left|\mathbf{q}_i^H \mathbf{GHw}\right|^2}{S_{n,i}\left|\mathbf{q}_i^H \mathbf{Fw}\right|^2 + S_{x,i}\left|\mathbf{q}_i^H \mathbf{DHw}\right|^2} = \frac{\mathbf{w}^T \mathbf{A}_i \mathbf{w}}{\mathbf{w}^T \mathbf{B}_i \mathbf{w}}$$



- Maximize nonlinear function of bits/symbol with respect to $\mathbf{w}$

$$b_{DMT} = \sum_{i=1}^{N/2} \log_2\left(1 + \frac{1}{\Gamma}\frac{\mathbf{w}^T \mathbf{A}_i \mathbf{w}}{\mathbf{w}^T \mathbf{B}_i \mathbf{w}}\right)$$

Fractional bits for optimization

  - Good performance measure for comparison of TEQ design methods
  - Not an efficient TEQ design method in computational sense

Minimum-ISI (Min-ISI) TEQ Design

- **Rewrite subchannel SNR**
  [Arslan, Evans & Kiaei, 2001]

ISI power weighted in frequency domain by inverse of noise spectrum →

$$\mathrm{SNR}_i = \frac{\frac{S_{x,i}}{S_{n,i}}\left|H_i^{signal}\right|^2}{\left|H_i^{noise}\right|^2 \boxed{+ \frac{S_{x,i}}{S_{n,i}}\left|H_i^{ISI}\right|^2}}$$

- **Generalize MSSNR method by weighting ISI in frequency**
  - Minimize frequency weighted sum of subchannel ISI power $$\sum_i \mathrm{ISI}_i = \sum_i K_i \left|\mathbf{q}_i^H \mathbf{DHw}\right|^2 = \mathbf{w}^T \mathbf{Xw}$$
  - Penalize ISI power in high conventional SNR subchannels: $K_i = S_{x,i} / S_{n,i}$
  - Constrain signal path gain to one to prevent all-zero solution for **w** $$| h^{signal} |^2 = | \mathbf{GHw} |^2 = \mathbf{w}^T \mathbf{Yw} = 1$$
  - Solution is eigenvector of minimum generalized eigenvalue of **X** and **Y**
- **Iterative Min-ISI method** [Ding *et al.* 2003]
  - Avoids Cholesky decomposition by using adaptive filter theory
  - Designs arbitrary length TEQs without loss in bit rate
  - Overcomes disadvantages of Maximum SSNR method

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- Multicarrier modulation
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- **Per-tone equalizer** [Catholic University, Leuven, Belgium]
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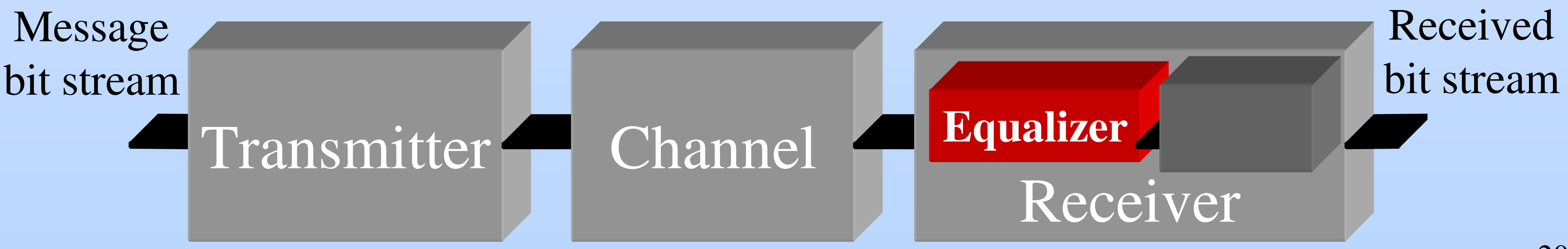
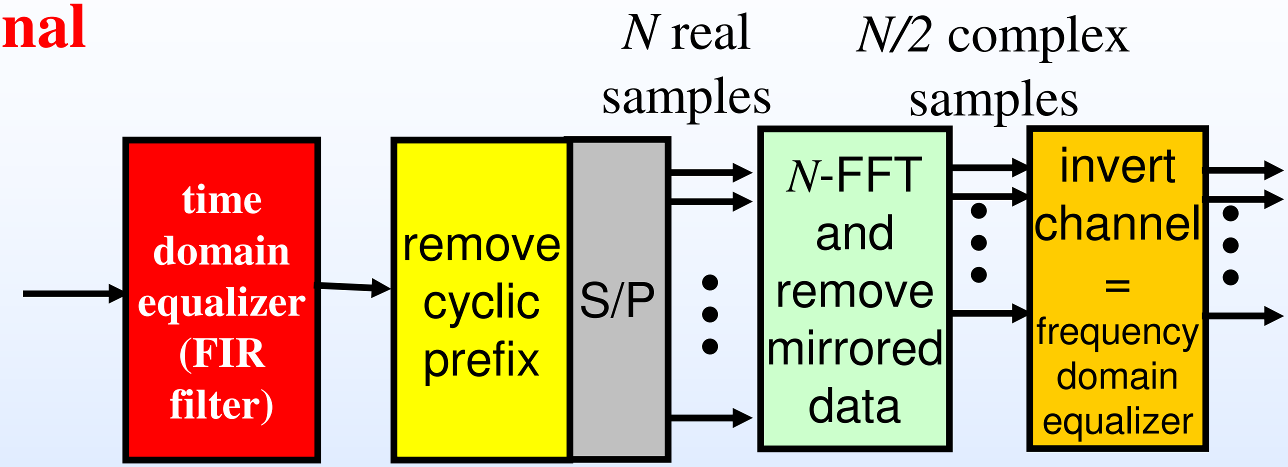


Drawbacks to Using Single FIR Filter for TEQ

- **Conventional equalizer**



- **Equalizes all tones in combined fashion: may limit bit rate**
- **Output of conventional equalizer for tone *i* computed using sequence of linear operations**

$$Z_i = D_i \,\mathrm{row}_i(\mathbf{Q}_N)\,\mathbf{Y}\,\mathbf{w}$$

D_i is the complex scalar value of one-tap FEQ for tone i
$\mathbf{Q}_N$ is the $N \times N$ complex-valued FFT matrix
$\mathbf{Y}$ is an $N \times L_w$ real-valued Toeplitz matrix of received samples
$\mathbf{w}$ is a $L_w \times 1$ column vector of real-valued TEQ taps

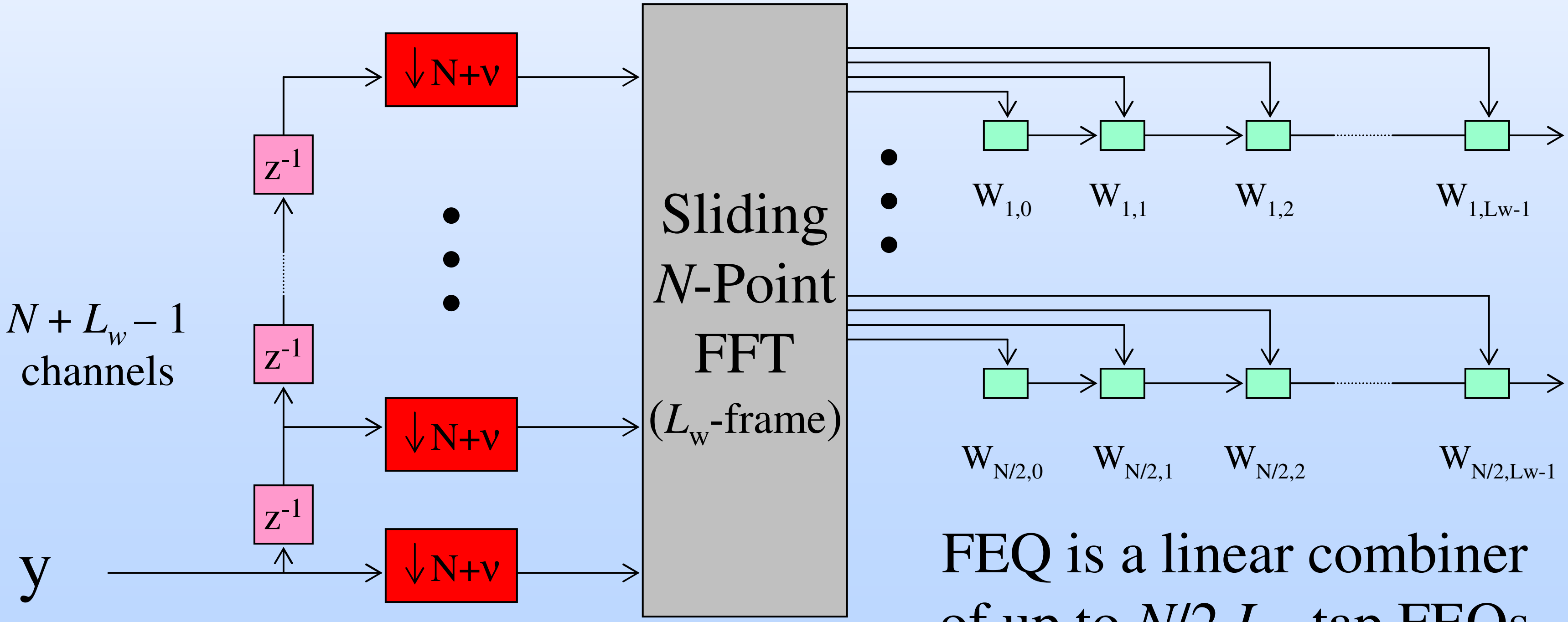
Y w
represents convolution

Frequency-Domain Per Tone Equalizer

- **Rewrite equalized FFT coefficient for each of $N/2$ tones**
  [Van Acker, Leus, Moonen, van de Wiel, Pollet, 2001]

$$Z_i = D_i \,\mathrm{row}_i(\mathbf{Q}_N)\,\mathbf{Y}\,\mathbf{w} = \mathrm{row}_i(\mathbf{Q}_N\,\mathbf{Y})\,(\,\mathbf{w}\,D_i\,) = \mathrm{row}_i(\mathbf{Q}_N\,\mathbf{Y})\,\mathbf{w}_i$$

  - Take sliding FFT to produce $N \times L_w$ matrix product Q_N Y
  - Design $\mathbf{w}_i$ for each tone



FEQ is a linear combiner of up to $N/2$ L_w-tap FEQs

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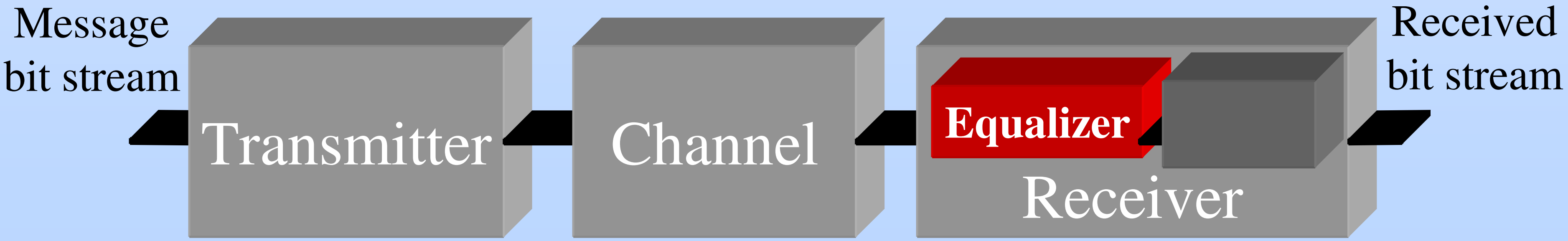
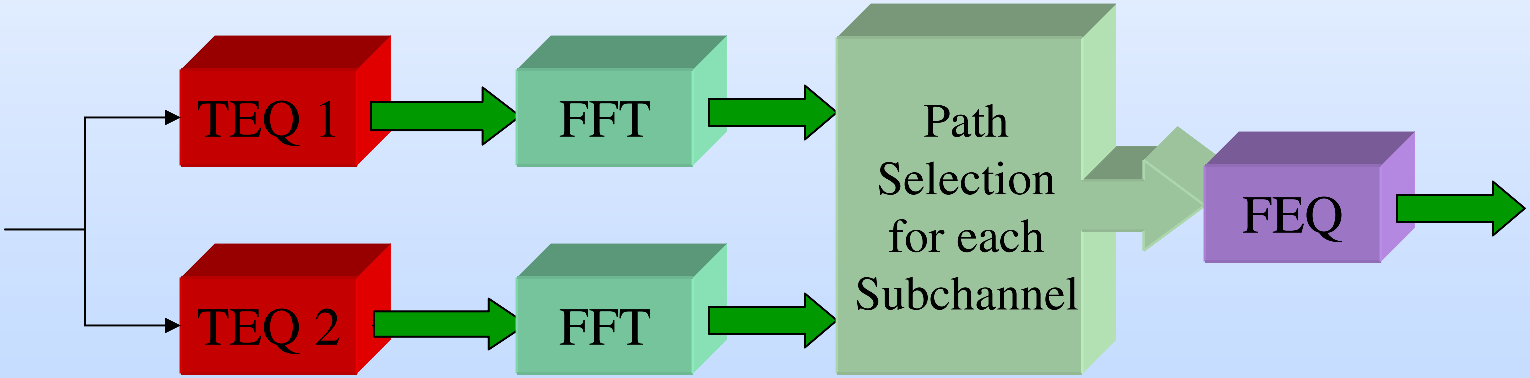


Dual-Path Time Domain Equalizer (DP-TEQ)

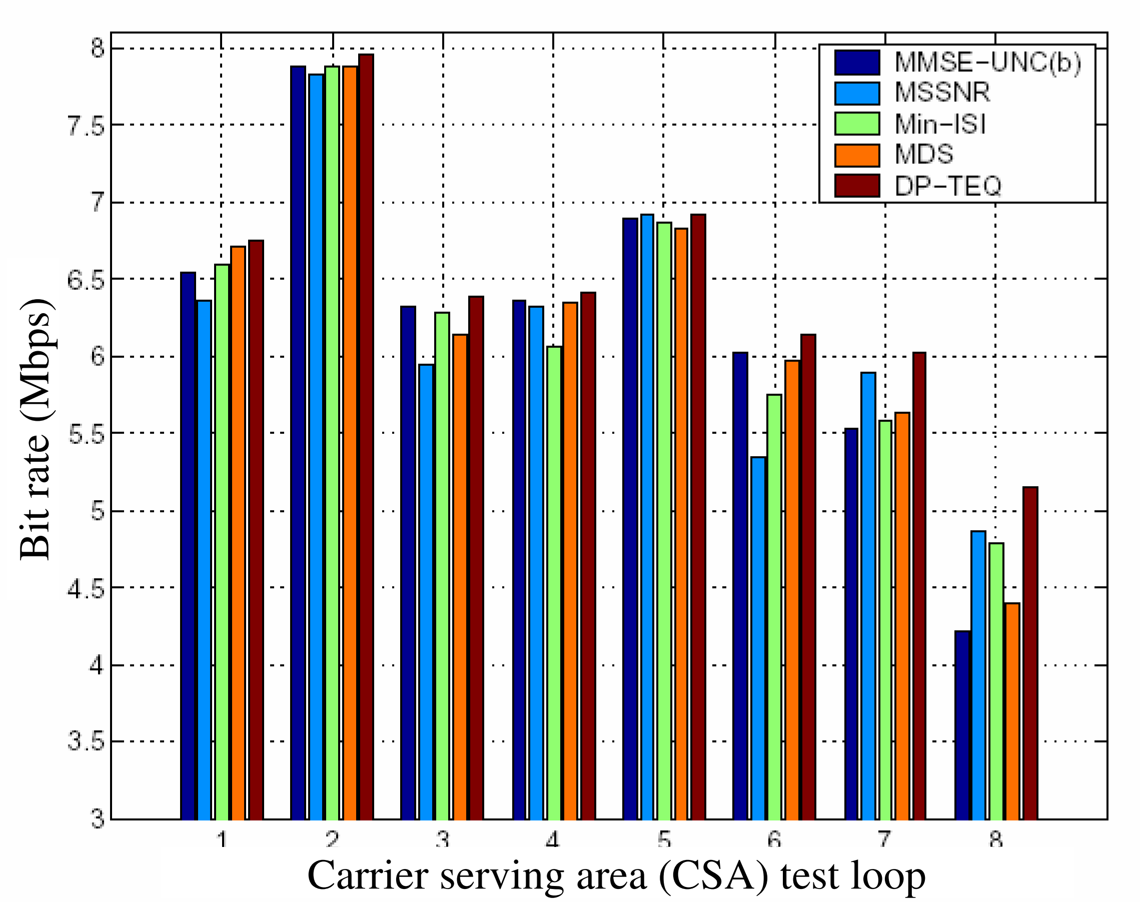
[Ding, Redfern & Evans, 2002]

- **First FIR TEQ equalizes entire available bandwidth**
- **Second FIR TEQ tailored for subset of subchannels**
  - Subchannels with higher SNR
  - Subchannels difficult to equalize, e.g. at boundary of upstream and downstream channels in frequency-division multiplexed ADSL
- **Minimum ISI method is good match for second FIR TEQ**



- **Path selection for each subchannel is fixed during training**
- **Up to 20% improvement in bit rate over MMSE TEQs**
- **Enables reuse of VLSI designs of conventional equalizers**

Simulation Results for 17-Tap Equalizers



Parameters

Cyclic prefix length	32
FFT size (*N*)	512
Coding gain (dB)	4.2
Margin (dB)	6
Input power (dBm)	23
Noise power (dBm/Hz)	-140
Crosstalk noise	24 ISDN disturbers

Downstream transmission

Figure 1 in [Martin, Vanbleu, Ding, Ysebaert, Milosevic, Evans, Moonen & Johnson, Oct. 2005]

UNC(b) means unit norm constraint on target impulse response **b**, i.e. || **b** || = 1

MDS is Maximum Delay Spread design method [Schur & Speidel, 2001]

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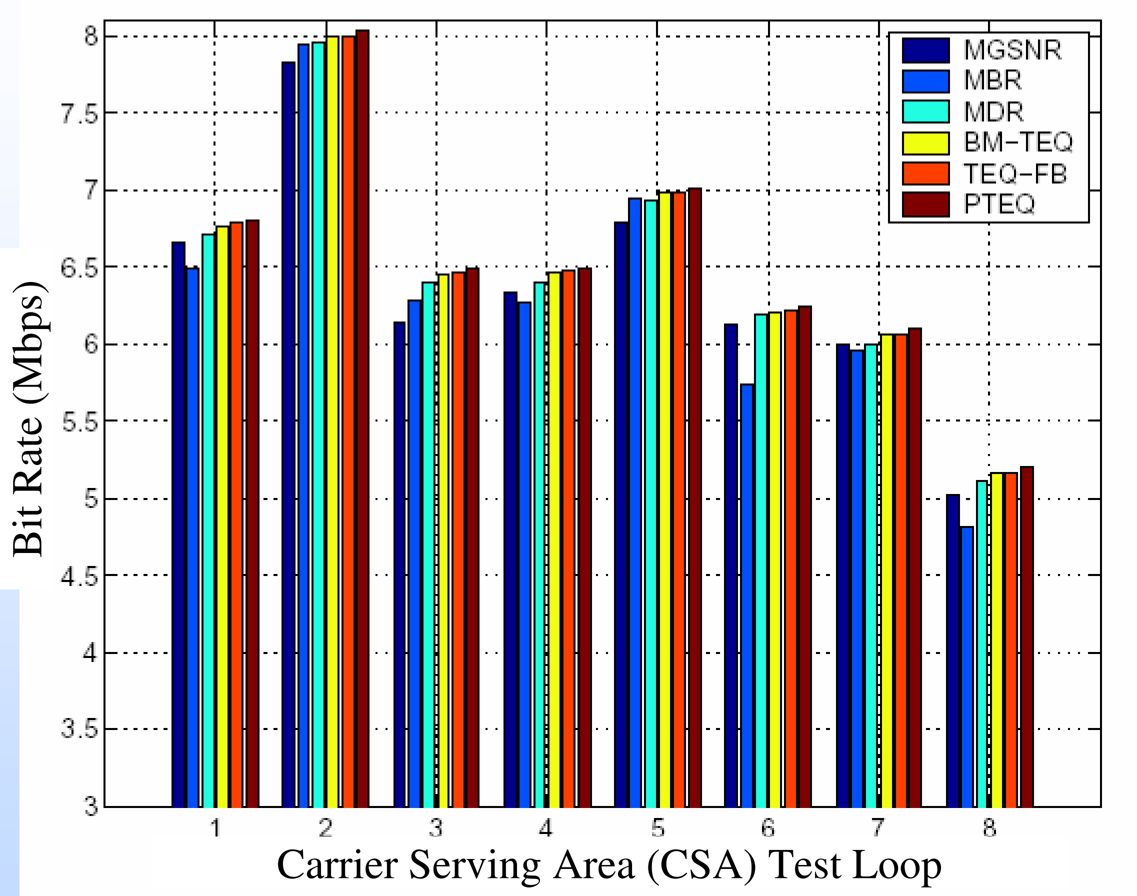

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Downstream transmission

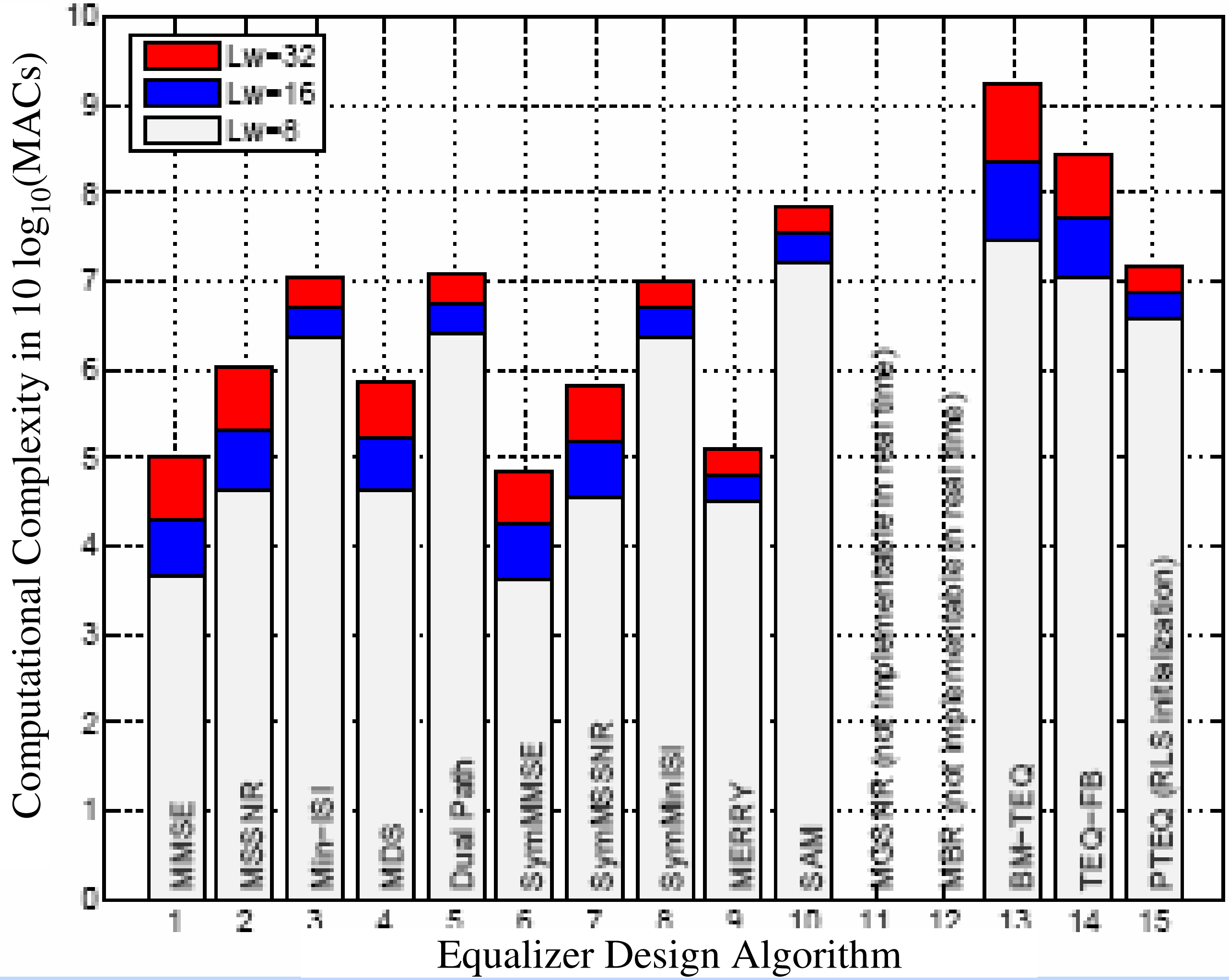
Figure 3 in [Martin, Vanbleu, Ding, Ysebaert, Milosevic, Evans, Moonen & Johnson, Oct. 2005]

MDR is Maximum Data Rate design method [Milosevic *et al.*, 2002]

BM-TEQ is Bit Rate Maximizing design method [Vanbleu *et al.*, 2003]

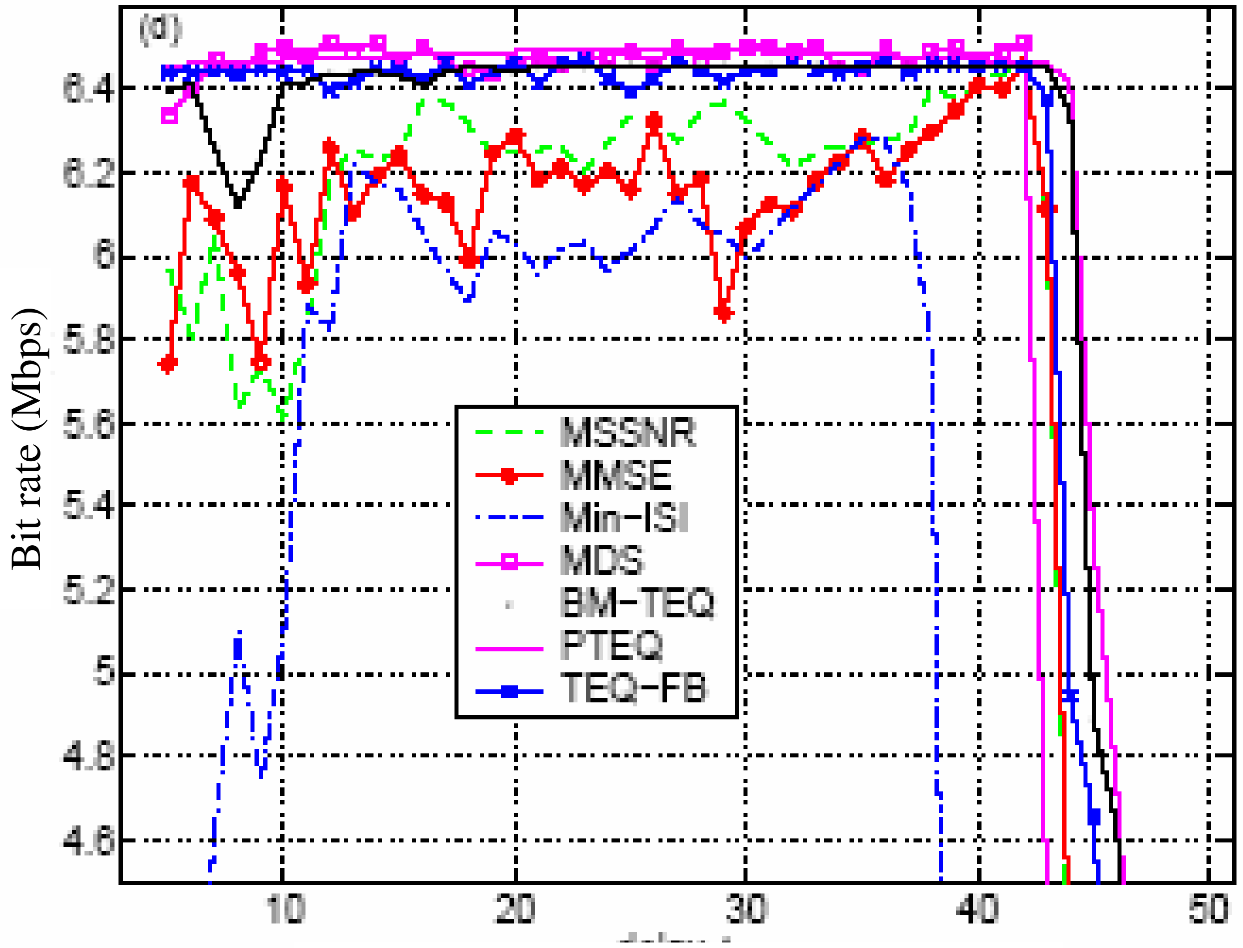
PTEQ is Per Tone Equalizer structure and design method [Acker *et al.*, 2001]

Estimated Computational Complexity



MAC means a multiplication-accumulation operation

Achievable Bit Rate vs. Delay Parameter



Delay Parameter Δ for CSA Test Loop 4

Large plateau of near-optimal delays (optimal choice requires search)
One choice is to set the delay parameter equal to cyclic prefix length

Contributions by Research Group

- **New methods for single-path time-domain equalizer design**
  - Maximum Bit Rate method maximizes bit rate (*upper bound*)
  - Minimum Inter-Symbol Interference method (*real-time, fixed-point*)
- **Minimum Inter-Symbol Interference TEQ design method**
  - Generalizes Maximum Shortening SNR by frequency weighting ISI
  - Improve bit rate in an ADSL transceiver by change of software only
  - Implemented in real-time on three fixed-point digital signal processors: Motorola 56000, TI TMS320C6200 and TI TMS320C5000

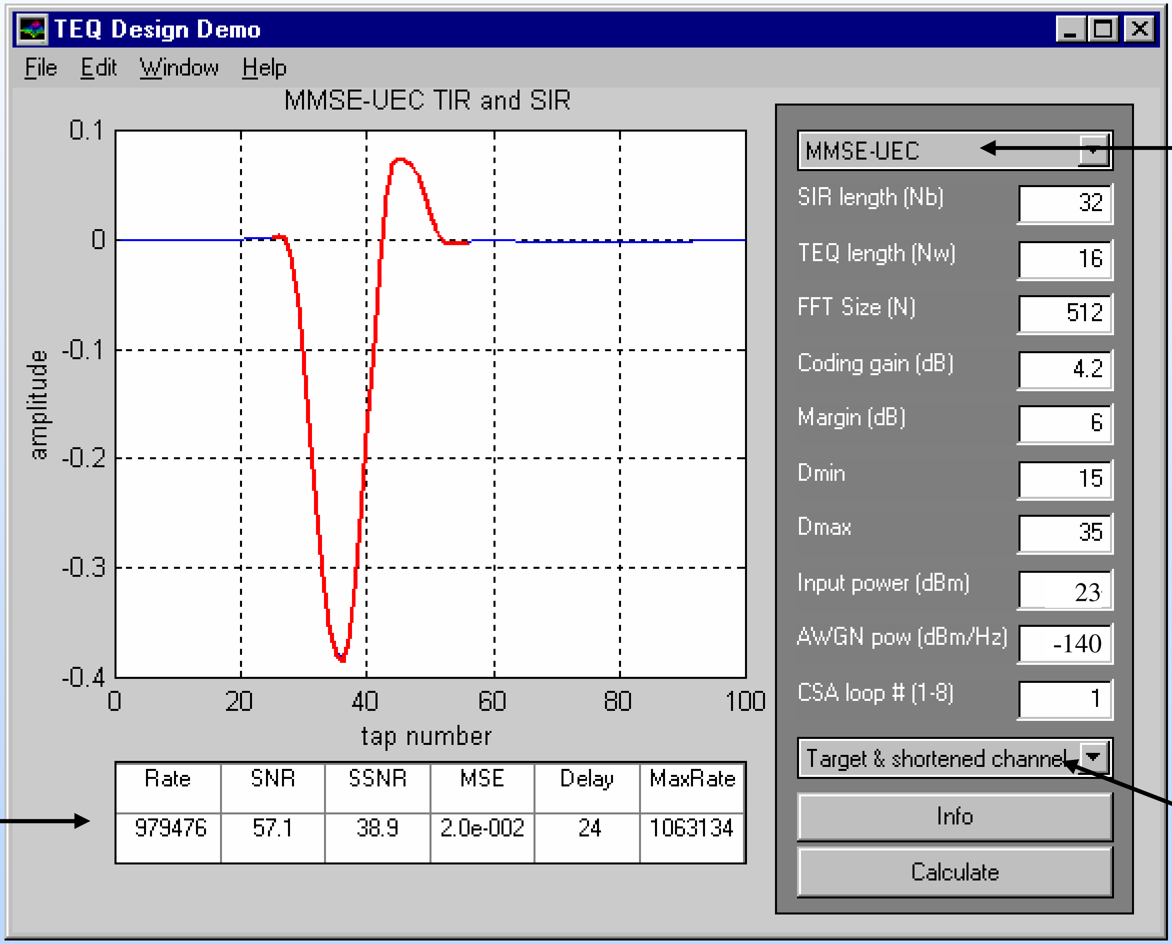
http://www.ece.utexas.edu/~bevans/projects/adsl

- **New dual-path time-domain equalizer**
  - Achieves bit rates between conventional and per tone equalizers
  - Lower implementation complexity in training than per tone equalizers
  - Enables reuse of ASIC designs

Matlab DMTTEQ Toolbox 3.1

- **Single-path, dual-path, per-tone & TEQ filter bank equalizers**

Available at http://www.ece.utexas.edu/~bevans/projects/adsl/dmtteq/



Rate	SNR	SSNR	MSE	Delay	MaxRate
979476	57.1	38.9	2.0e-002	24	1063134

MMSE-UEC

SIR length (Nb)	32
TEQ length (Nw)	16
FFT Size (N)	512
Coding gain (dB)	4.2
Margin (dB)	6
Dmin	15
Dmax	35
Input power (dBm)	23
AWGN pow (dBm/Hz)	-140
CSA loop # (1-8)	1

Target & shortened channel

18 design methods

default parameters from G.DMT ADSL standard

various performance measures

different graphical views

Backup Slides

Applications of Broadband Access

Residential

Application	*Downstream rate (kb/s)*	*Upstream rate (kb/s)*	*Willing to pay*	*Demand Potential*
Database Access	384	9	High	Medium
On-line directory; yellow pages	384	9	Low	High
Video Phone	1,500	1,500	High	Medium
Home Shopping	1,500	64	Low	Medium
Video Games	1,500	1,500	Medium	Medium
Internet	3,000	384	High	Medium
Broadcast Video	6,000	0	Low	High
High definition TV	24,000	0	High	Medium

Business

Application	*Downstream rate (kb/s)*	*Upstream rate (kb/s)*	*Willing to pay*	*Demand Potential*
On-line directory; yellow pages	384	9	Medium	High
Financial news	1,500	9	Medium	Low
Video phone	1,500	1,500	High	Low
Internet	3,000	384	High	High
Video conference	3,000	3,000	High	Low
Remote office	6,000	1,500	High	Medium
LAN interconnection	10,000	10,000	Medium	Medium
Supercomputing, CAD	45,000	45,000	High	Low

Selected DSL Standards

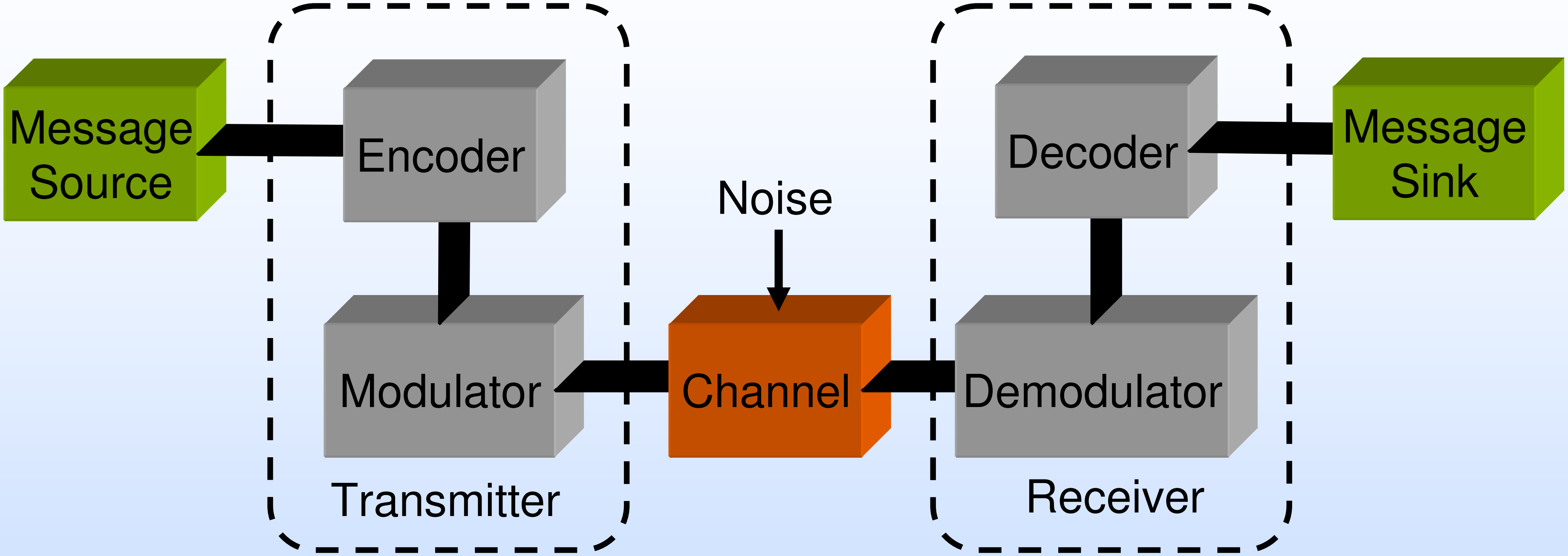
Standard	*Meaning*	*Data Rate*	*Mode*	*Applications*
ISDN	Integrated Services Digital Network	144 kbps	Symmetric	Internet Access, Voice, Pair Gain (2 channels)
T1	T-Carrier One (requires two pairs)	1.544 Mbps	Symmetric	Enterprise, Expansion, Internet Service
HDSL	High-Speed Digital Subscriber Line (requires two pairs)	1.544 Mbps	Symmetric	Pair Gain (12 channels), Internet Access, T1/E1 replacement
HDSL2	Single Line HDSL	1.544 Mbps	Symmetric	Same as HDSL except pair gain is 24 channels
G.Lite ADSL	Splitterless Asymmetric Digital Subscriber Line	up to 1.5 Mbps up to 512 kbps	Downstream Upstream	Internet Access, Digital Video
G.DMT ADSL	Asymmetric Digital Subscriber Line	up to 10 Mbps up to 1 Mbps	Downstream Upstream	Internet Access, Digital Video
VDSL	Very High-Speed Digital Subscriber Line (proposed)	up to 22 Mbps up to 3 Mbps up to 13 Mbps	Downstream Upstream Symmetric	Internet Access, Digital Video, Broadcast Video

Courtesy of Shawn McCaslin (National Instruments, Austin, TX)

Discrete Multitone DSL Standards

- **Discrete multitone (DMT) modulation uses multiple carriers**
- **ADSL – Asymmetric DSL (G.DMT)**
  - *Asymmetric*: 8 Mbps downstream and 1 Mbps upstream
  - *Data band*: 25 kHz – 1.1 MHz
  - Maximum data rates possible in standard (ideal case)
    - Echo cancelled: 14.94 Mbps downstream, 1.56 Mbps upstream
    - Frequency division multiplexing: 13.38 Mbps downstream, 1.56 Mbps up
  - Widespread deployment in US, Canada, Western Europe, Hong Kong
    - Central office providers only installing frequency-division ADSL
    - ADSL modems have about 1/3 of market, and cable modems have 2/3
- **VDSL – Very High Rate DSL**
  - *Asymmetric*: either 22/3 or 13/3 Mbps downstream/upstream
  - *Symmetric*: 13, 9, or 6 Mbps each direction
  - *Data band*: 1 – 12 MHz
  - DMT and single carrier modulation supported
  - DMT VDSL essentially higher speed version of G.DMT ADSL

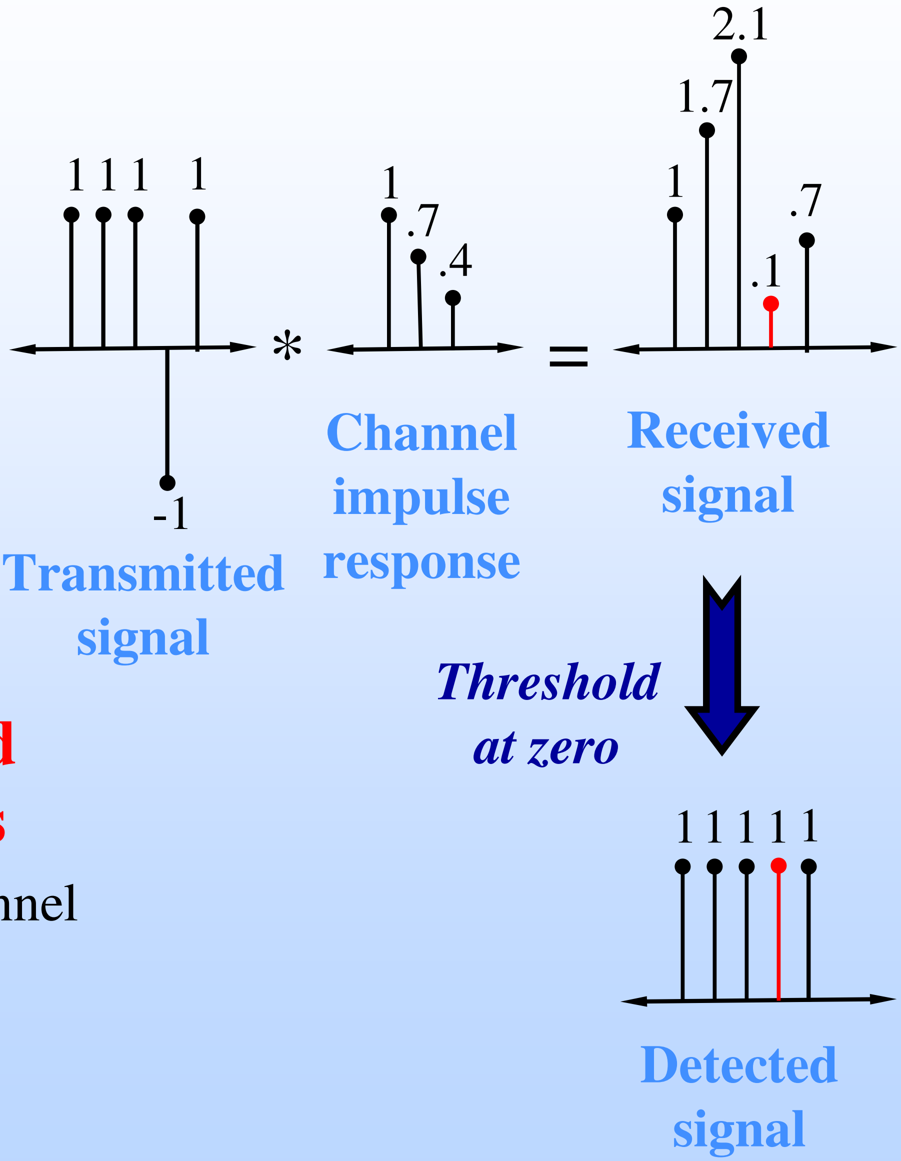
A Digital Communications System



- **Encoder maps a group of message bits to data symbols**
- **Modulator maps these symbols to analog waveforms**
- **Demodulator maps received waveforms back to symbols**
- **Decoder maps the symbols back to binary message bits**

Intersymbol Interference (ISI)

- **Ideal channel**
  - Impulse response is impulse
  - Flat frequency response
- **Non-ideal channel**
  - Causes ISI
  - Channel memory
  - Magnitude and phase variation
- **Received symbol is weighted sum of neighboring symbols**
  - Weights are determined by channel impulse response



Transmitted signal

Channel impulse response

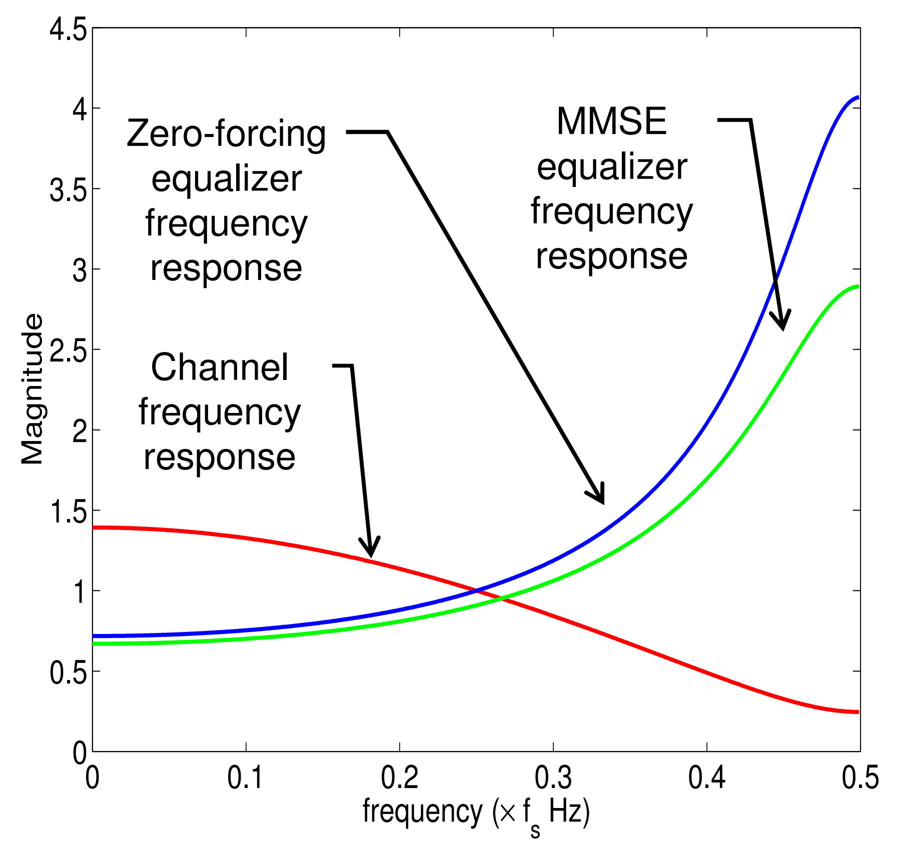
Received signal

Threshold at zero

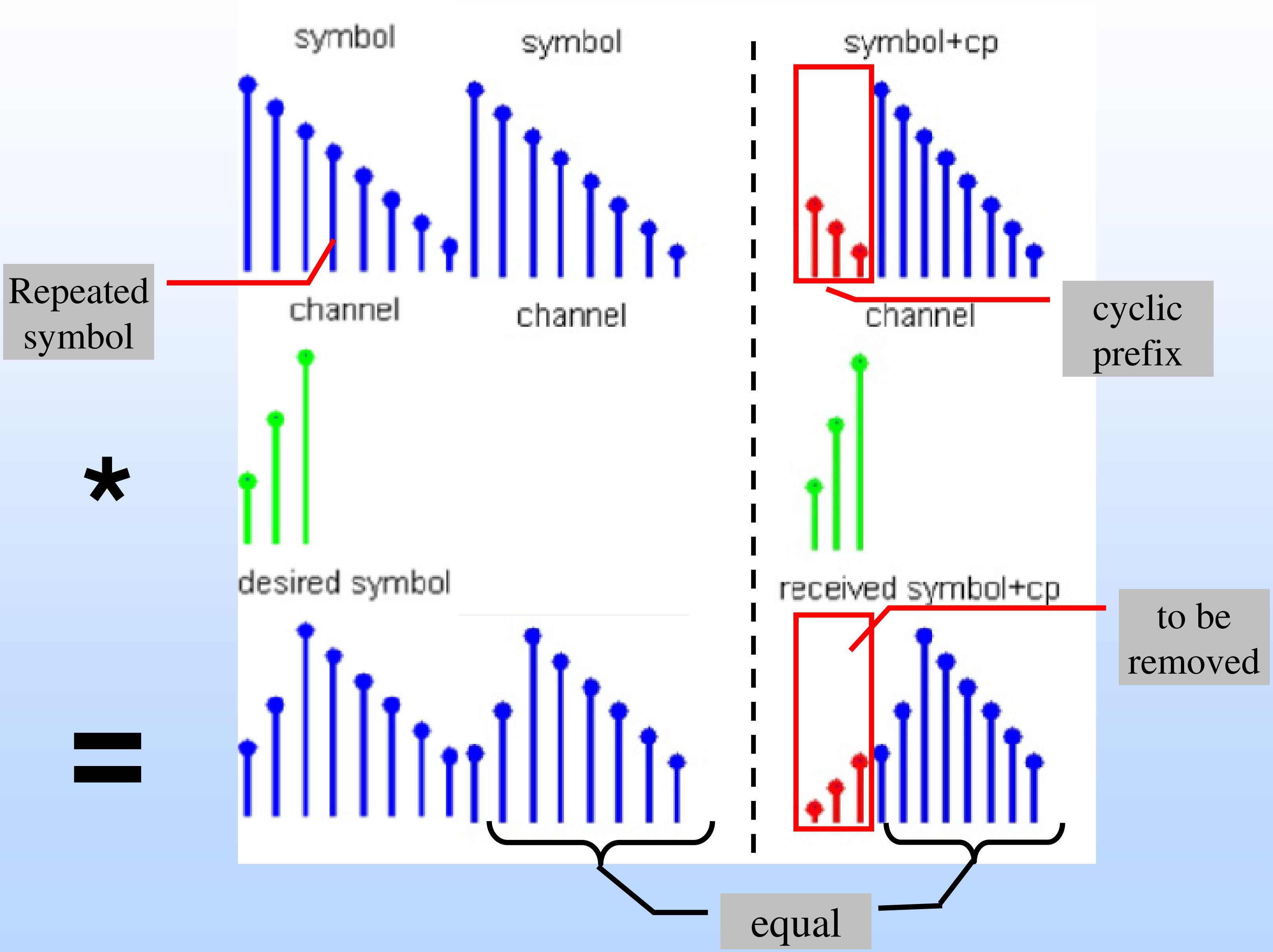
Detected signal

Combat ISI with Equalization

- **Equalization because channel response is not flat**
- **Zero-forcing equalizer**
  - Inverts channel
  - Flattens freq. response
  - Amplifies noise
- **MMSE equalizer**
  - Optimizes trade-off between noise amplification and ISI
- **Decision-feedback equalizer**
  - Increases complexity
  - Propagates error



Cyclic Prefix



Repeated symbol

cyclic prefix

to be removed

equal

Open Issues for Multicarrier Modulation

- **Advantages**
  - Efficient use of bandwidth without full channel equalization
  - Robust against impulsive noise and narrowband interference
  - Dynamic rate adaptation
- **Disadvantages**
  - ***Transmitter:*** High signal peak-to-average power ratio
  - ***Receiver:*** Sensitive to frequency and phase offset in carriers
- **Open issues**
  - Pulse shapes of subchannels (*orthogonal, efficient realization*)
  - Channel equalizer design (*increase bit rate, reduce complexity*)
  - Synchronization (*timing recovery, symbol synchronization*)
  - Bit loading (*allocation of bits in each subchannel*)
  - Echo cancellation

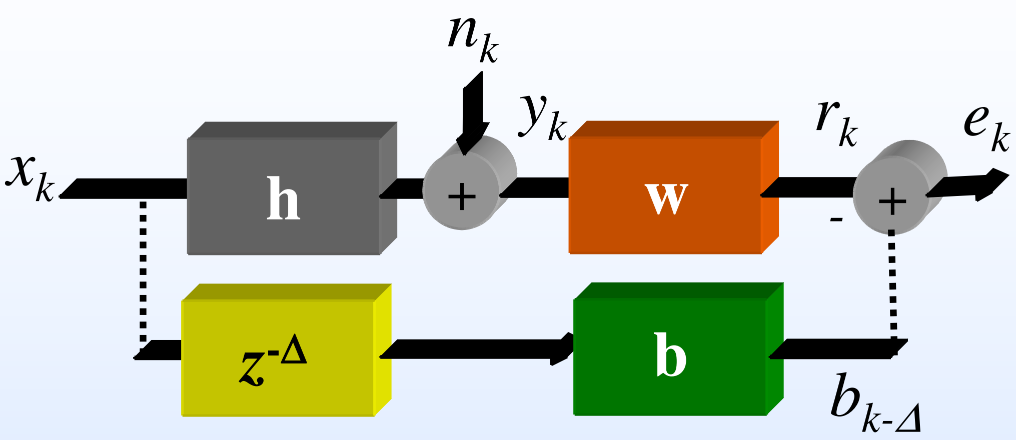
TEQ Algorithm

- **ADSL standards**
  - Set aside 1024 frames (~.25s) for TEQ estimation
  - Reserved ~16,000 frames for channel and noise estimation for the purpose of SNR calculation
- **TEQ is estimated before the SNR calculations**
- **Noise power and channel impulse response can be estimated before time slot reserved for TEQ if the TEQ algorithm needs that information**

Single-FIR Time-Domain Equalizer Design Methods

- ***All methods below perform optimization at TEQ output***
- **Minimizing the mean squared error**
  - Minimize mean squared error (MMSE) method [Chow & Cioffi, 1992]
  - Geometric SNR method [Al-Dhahir & Cioffi, 1996]
- **Minimizing energy outside of shortened (equalized) channel impulse response**
  - Maximum Shortening SNR method [Melsa, Younce & Rohrs, 1996]
  - Divide-and-conquer methods [Lu, Evans, Clark, 2000]
  - Minimum ISI method [Arslan, Evans & Kiaei, 2000]
- **Maximizing bit rate** [Arslan, Evans & Kiaei, 2000]
- **Implementation**
  - Geometric SNR is difficult to automate (requires human intervention)
  - Maximum bit rate method needs nonlinear optimization solver
  - Other methods implemented on fixed-point digital signal processors

Minimum Mean Squared Error (MMSE) TEQ



$$\mathbf{w} = \left[w_0 \;\; w_1 \;\; \cdots \;\; w_{L_w-1}\right]^T$$

$$\mathbf{b} = \left[b_0 \;\; b_1 \;\; \cdots \;\; b_\nu\right]^T$$

$$\hat{\mathbf{b}} = \left[\mathbf{0}_\Delta \mid \mathbf{b}^T \mid \mathbf{0}_{L_h-\Delta-\nu-1}\right]^T$$

$$\mathrm{MSE} = \varepsilon\{e_k^2\} = \hat{\mathbf{b}}^T \mathbf{R}_{\mathbf{xx}} \hat{\mathbf{b}} - 2\hat{\mathbf{b}}^T \mathbf{R}_{\mathbf{xy}} \mathbf{w} + \mathbf{w}^T \mathbf{R}_{\mathbf{yy}} \mathbf{w}$$

minimum MSE is achieved only if $\mathbf{b}^T \mathbf{R}_{\mathbf{xy}} = \mathbf{w}^T \mathbf{R}_{\mathbf{yy}}$

$$\mathrm{MSE} = \hat{\mathbf{b}}^T \left[\mathbf{R}_{\mathbf{xx}} - \mathbf{R}_{\mathbf{xy}} \mathbf{R}_{\mathbf{yy}}^{-1} \mathbf{R}_{\mathbf{yx}}\right] \hat{\mathbf{b}} = \hat{\mathbf{b}}^T \mathbf{R}_{\mathbf{x|y}} \hat{\mathbf{b}}$$

Define $\mathbf{R}_\Delta = \mathbf{O}^T \mathbf{R}_{\mathbf{x|y}} \mathbf{O}$ then $\mathrm{MSE} = \mathbf{b}^T \mathbf{R}_\Delta \mathbf{b}$

O **selects the proper part out of** $\mathrm{R}_{\mathrm{x|y}}$ **corresponding to the delay Δ**

Near-optimal Minimum-ISI (Min-ISI) TEQ Design

- **Generalizes MSSNR method by frequency weighting ISI**
  - ISI power in *i*th subchannel is $\mathrm{ISI}_i = S_{x,i} \left| \mathbf{q}_i^H \mathbf{DHw} \right|^2$
  - Minimize ISI power as a frequency weighted sum of subchannel ISI

$$\sum_i \mathrm{ISI}_i = \sum_i K_i \left| \mathbf{q}_i^H \mathbf{DHw} \right|^2 = \mathbf{w}^T \mathbf{X} \mathbf{w}$$

  - Constrain signal path gain to one to prevent all-zero solution

$$| h^{signal} |^2 = | \mathbf{GHw} |^2 = \mathbf{w}^T \mathbf{Y} \mathbf{w} = 1$$

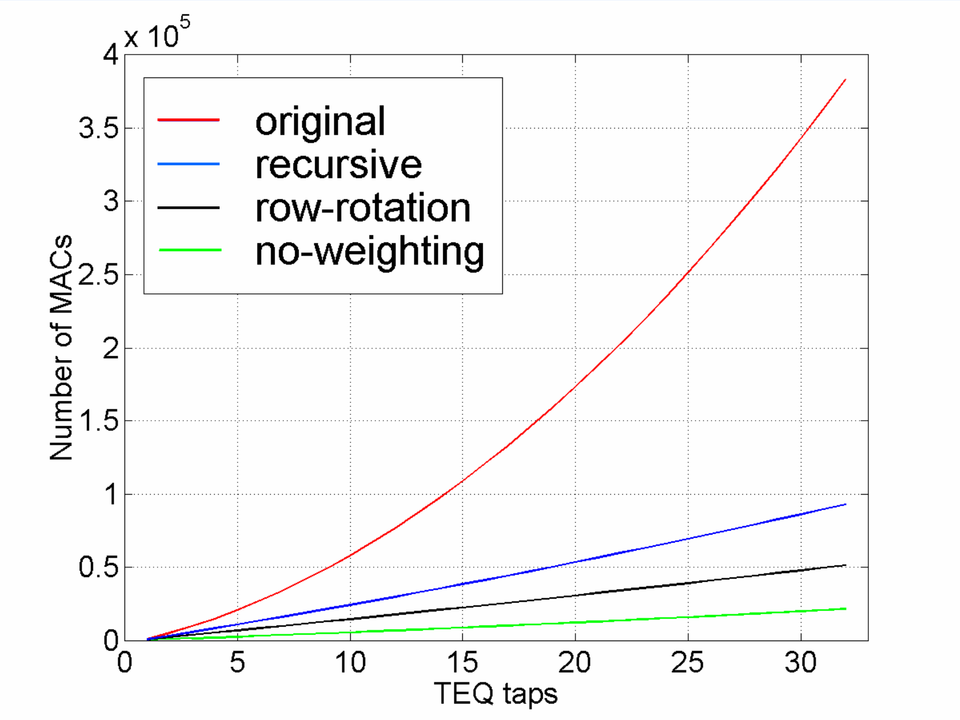
  - Solution is a generalized eigenvector of **X** and **Y**

- **Possible weightings**
  - Amplify ISI objective function in subchannels with low noise power (high SNR) to put ISI in low SNR bins: $K_i = \dfrac{S_{x,i}}{S_{n,i}}$
  - Set weighting equal to input power spectrum: $K_i = S_{x,i}$
  - Set weighting to be constant in all subchannels (MSSNR): $K_i = 1$

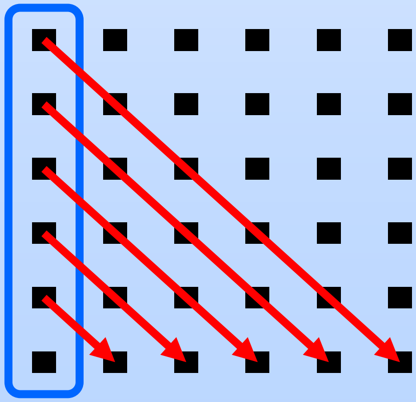
- **Performance virtually equal to MBR (optimal) method**

Efficient Implementations of Min-ISI Method

- **Generalized eigenvalue problem can solved with generalized power iteration:** $\mathbf{X}\mathbf{w}^{k+1} = \mathbf{Y}\mathbf{w}^{k}$
- **Recursively calculate diagonal elements of X and Y from first column** [Wu, Arslan, Evans, 2000]



Method	*Bit Rate*	*MACs*
Original	99.6%	132,896
Recursive	99.5%	44,432
Row-rotation	99.5%	25,872
No-weighting	97.8%	10,064



Motivation for Divide-and-Conquer Methods

- **Fast methods for implementing Maximum SSNR method**
- **Maximum SSNR Method**
  - For each Δ, maximum SSNR method requires
    - Multiplications: $(L_h + \frac{7}{6}) L_w + \frac{5}{2} L_w^2 + \frac{25}{3} L_w^3$
    - Additions: $(L_h - \frac{5}{6}) L_w - \frac{3}{2} L_w^2 + \frac{25}{3} L_w^3$
    - Divisions: L_w^2
  - Exhaustive search for the optimal delay Δ

$$0 \le \Delta \le L_h + L_w - \nu - 2 \Rightarrow 0 \le \Delta \le 499$$

- **Divide L_w TEQ taps into (L_w - *1*) two-tap filters in cascade**
  - Design first two-tap filter then second and so forth (greedy approach)
- **Develop heuristic to estimate the optimal delay**

Divide-and-Conquer Approach

- **The i^{th} two-tap filter is initialized as either**
  - Unit tap constraint (UTC) $\mathbf{w}_i = \begin{bmatrix} 1 \\ g_i \end{bmatrix}$
  - Unit norm constraint (UNC) $\mathbf{w}_i = \begin{bmatrix} \sin\theta_i \\ \cos\theta_i \end{bmatrix}$
- **Calculate best g_i or θ_i by using a greedy approach either by**
  - Minimizing $\frac{1}{\text{SSNR}}$ (Divide-and-conquer TEQ minimization)
  - Minimizing energy in $\mathbf{h}_{\text{wall}}$ (Divide-and conquer TEQ cancellation)
- **Convolve two-tap filters to obtain TEQ**

Divide-and-Conquer TEQ Minimization (UTC)

- **At i^{th} iteration, minimize J_i over g_i**

$$J_i = \frac{\mathbf{w}_i^T \mathbf{A} \mathbf{w}_i}{\mathbf{w}_i^T \mathbf{B} \mathbf{w}_i} = \frac{\begin{bmatrix} 1 & g_i \end{bmatrix} \begin{bmatrix} a_{1,i} & a_{2,i} \\ a_{2,i} & a_{3,i} \end{bmatrix} \begin{bmatrix} 1 \\ g_i \end{bmatrix}}{\begin{bmatrix} 1 & g_i \end{bmatrix} \begin{bmatrix} b_{1,i} & b_{2,i} \\ b_{2,i} & b_{3,i} \end{bmatrix} \begin{bmatrix} 1 \\ g_i \end{bmatrix}} = \frac{a_{1,i} + 2a_{2,i} g_i + a_{3,i} g_i^2}{b_{1,i} + 2b_{2,i} g_i + b_{3,i} g_i^2}$$

- **Closed-form solution**

$$g_{i(1,2)} = \frac{-\left(a_{3,i} b_{1,i} - a_{1,i} b_{3,i}\right)}{2\left(a_{3,i} b_{2,i} - a_{2,i} b_{3,i}\right)} \pm \frac{\sqrt{D}}{2\left(a_{3,i} b_{2,i} - a_{2,i} b_{3,i}\right)}$$

$$D = \left(a_{3,i} b_{1,i} - a_{1,i} b_{3,i}\right)^2 - 4\left(a_{3,i} b_{2,i} - a_{2,i} b_{3,i}\right)\left(a_{2,i} b_{1,i} - a_{1,i} b_{2,i}\right)$$

Divide-and-Conquer TEQ Minimization (UNC)

- **At i^{th} iteration, minimize J_i over η_i**

$$J_i = \frac{\mathbf{w}_i^T \mathbf{A} \mathbf{w}_i}{\mathbf{w}_i^T \mathbf{B} \mathbf{w}_i} = \frac{\left(\sin\theta_i \begin{bmatrix} 1 & \eta_i \end{bmatrix}\right) \begin{bmatrix} a_{1,i} & a_{2,i} \\ a_{2,i} & a_{3,i} \end{bmatrix} \left(\sin\theta_i \begin{bmatrix} 1 \\ \eta_i \end{bmatrix}\right)}{\left(\sin\theta_i \begin{bmatrix} 1 & \eta_i \end{bmatrix}\right) \begin{bmatrix} b_{1,i} & b_{2,i} \\ b_{2,i} & b_{3,i} \end{bmatrix} \left(\sin\theta_i \begin{bmatrix} 1 \\ \eta_i \end{bmatrix}\right)}$$

$$= \frac{\begin{bmatrix} 1 & \eta_i \end{bmatrix} \begin{bmatrix} a_{1,i} & a_{2,i} \\ a_{2,i} & a_{3,i} \end{bmatrix} \begin{bmatrix} 1 \\ \eta_i \end{bmatrix}}{\begin{bmatrix} 1 & \eta_i \end{bmatrix} \begin{bmatrix} b_{1,i} & b_{2,i} \\ b_{2,i} & b_{3,i} \end{bmatrix} \begin{bmatrix} 1 \\ \eta_i \end{bmatrix}}$$

Calculate η_i in the same way as g_i for UTC version of this method

- **where** $\mathbf{w}_i = \begin{bmatrix} \sin\theta_i \\ \cos\theta_i \end{bmatrix} = \sin\theta_i \begin{bmatrix} 1 \\ \cos\theta_i / \sin\theta_i \end{bmatrix} = \sin\theta_i \begin{bmatrix} 1 \\ \eta_i \end{bmatrix}$

Divide-and-Conquer TEQ Cancellation (UTC)

- **At i^{th} iteration, minimize J_i over g_i**

$$J_i = \tilde{\mathbf{h}}_{\text{wall}}^T \tilde{\mathbf{h}}_{\text{wall}} = \sum_{k \in S} \left(\tilde{h}_{i-1}(k) + g_i \tilde{h}_{i-1}(k-1) \right)^2,$$

$$S = \left\{ 1, 2, \cdots, \Delta, \Delta + \nu + 2, \cdots, L_{\tilde{h}_{i-1}} \right\}$$

- **Closed-form solution for the i^{th} two-tap FIR filter**

$$g_i = -\frac{\sum_{k \in S} \tilde{h}_{i-1}(k-1) \tilde{h}_{i-1}(k)}{\sum_{k \in S} \tilde{h}_{i-1}^2(k-1)}$$

Divide-and-Conquer TEQ Cancellation (UNC)

- **At i^{th} iteration, minimize J_i over θ_I**

$$J_i = \tilde{\mathbf{h}}_{\text{wall}}^T \tilde{\mathbf{h}}_{\text{wall}} = \sum_{k \in S} \left(\tilde{h}_{i-1}(k) \sin\theta_i + \tilde{h}_{i-1}(k-1) \cos\theta_i \right)^2,$$

$$S = \left\{ 1, 2, \cdots, \Delta, \Delta + \nu + 2, \cdots, L_{\tilde{h}_{i-1}} \right\}$$

- **Closed-form solution**

$$\sin\theta_i = \pm\sqrt{0.5\left(1 \pm \sqrt{\frac{a^2}{a^2+4b^2}}\right)}, \cos\theta_i = \pm\sqrt{0.5\left(1 \mp \sqrt{\frac{a^2}{a^2+4b^2}}\right)}$$

$$a = \sum_{k \in S} \left(\tilde{h}_{i-1}^2(k) - \tilde{h}_{i-1}^2(k-1) \right), \; b = \sum_{k \in S} \tilde{h}_{i-1}(k-1)\, \tilde{h}_{i-1}(k)$$

Computational Complexity

- **Computational complexity for each candidate Δ**

Method	×	+	÷	*Memory (words)*
Maximum SSNR	120379	118552	441	1899
DC-TEQ-minimization (UTC)	53240	52980	60	563
DC-TEQ-cancellation (UNC)	42280	42160	20	555
DC-TEQ-cancellation (UTC)	41000	40880	20	554

G.DMT
ADSL
$L_h = 512$
$\nu = 32$
$L_w = 21$

- **Divide-and-conquer methods vs. maximum SSNR method**
  - Reduces multiplications, additions, divisions, and memory
  - No matrix calculations (saves on memory accesses)
  - Avoids matrix inversion, and eigenvalue and Cholesky decompositions

Heuristic Search for the Optimal Delay

- **Estimate optimal delay Δ before computing TEQ taps**

$$\Delta_{\text{ratio}} = \arg\max_{\Delta} \frac{\text{energy inside a window of original } \mathbf{h}}{\text{energy outside a window of original } \mathbf{h}}$$

- **Total computational cost**
  - Multiplications: L_h
  - Additions: $3L_h - 3$
  - Divisions: L_h
- **Performance of heuristic vs. exhaustive search**
  - Reduce computational complexity by factor of 500
  - 2% loss in SSNR for TEQ with four taps or more
  - 8% loss in SSNR for two-tap TEQ

Comparison of Earlier Methods

Method	*MMSE*	*MSSNR*	*Geometric*
Advantages			
Maximize bit rate			✓
Minimize ISI		✓	
Bit Rate	Low-medium	High	Low-medium
Disadvantages			
Nonlinear optimization			✓
Computational complexity	Low	Medium	High
Artificial constraints	✓		✓
Ad-hoc parameters			✓
Lowpass frequency response	✓		✓
Unrealistic assumptions			✓

MBR TEQ vs. Geometric TEQ

Method	***MBR***	***Geometric***
Advantages		
Maximize channel capacity	✓	✓
Minimize ISI	✓	
Bit rate	optimal	Low-medium
Disadvantages		
Low-pass frequency response		✓
Computationally complex	✓	✓
Artificial constraints		✓
Ad-hoc parameters		✓
Nonlinear optimization	✓	✓
Unrealistic assumptions		✓

Min-ISI TEQ vs. MSSNR TEQ

Method	***Min-ISI***	***MSSNR***
Advantages		
Maximize channel capacity		
Minimize ISI	✓	✓
Frequency domain weighting	✓	
Bit rate	high	high
Disadvantages		
Computationally complex	very high	high

- **Min-ISI weights ISI power with the SNR**
  - Residual ISI power should be placed in high noise frequency bands

$$\mathrm{SNR}_i = \frac{\text{signal power}}{\text{noise power} + \text{ISI power}}$$

$$\mathrm{SNR}_{50} = \frac{1}{10} = 0.1$$

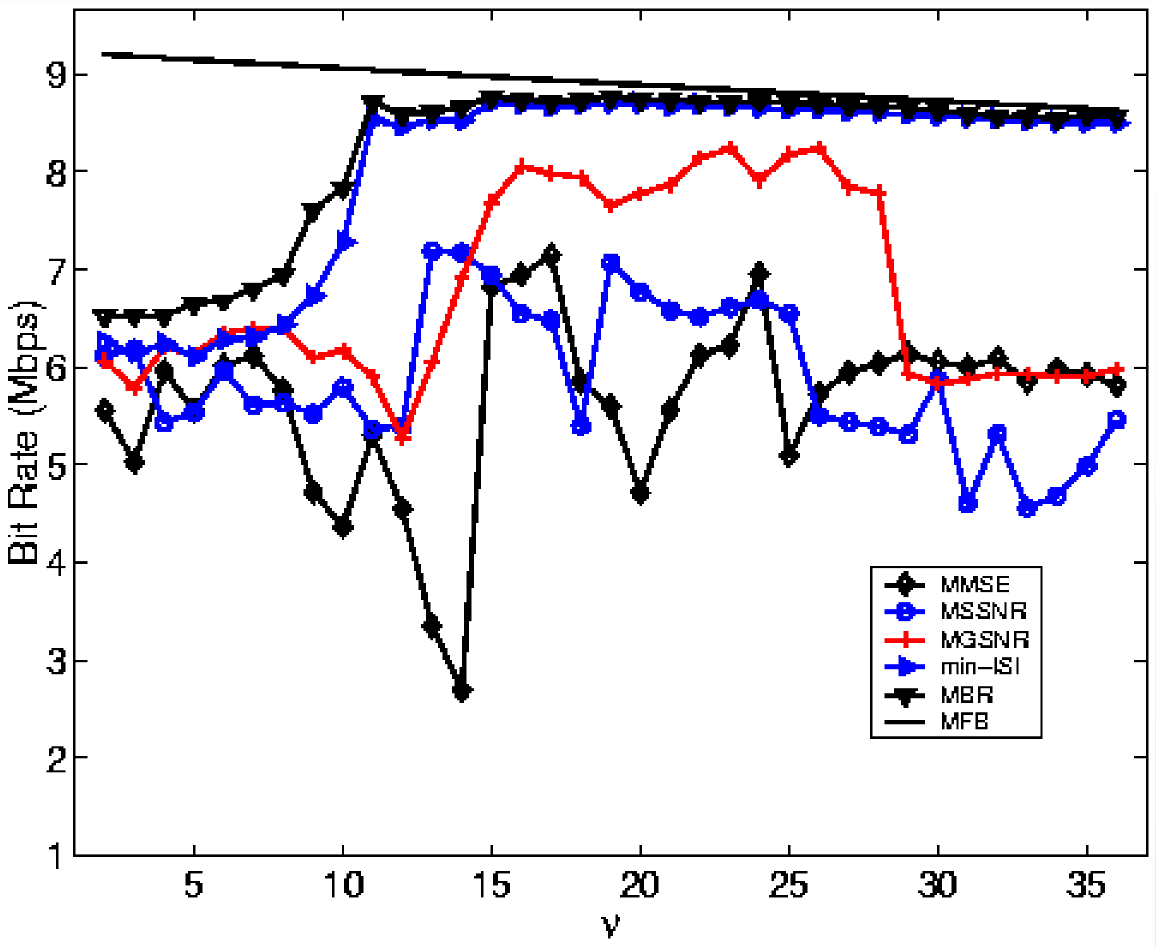
$$\mathrm{SNR}_{2} = \frac{1}{0.1} = 10$$

$$\mathrm{SNR}_{50} = \frac{1}{10+1} = 0.09$$

$$\mathrm{SNR}_{2} = \frac{1}{0.1+1} = 0.9$$

Bit Rate vs. Cyclic Prefix (CP) Size

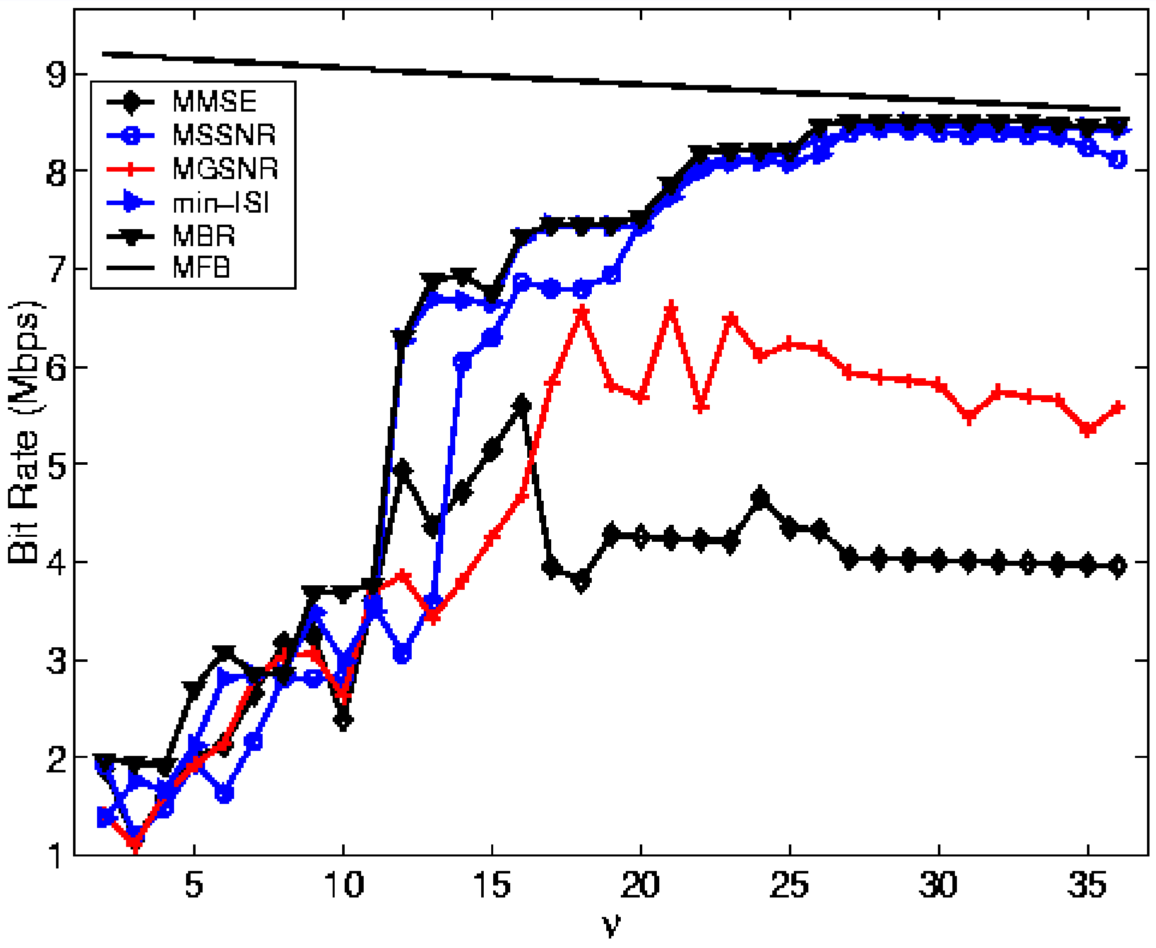
- Matched filter bound decreases because CP has no new information
- Min-ISI and MBR achieve bound with 16-sample CP
- Other design methods are erratic
- MGSNR better for 15-28 sample CPs



TEQ taps (L_w)	17		
FFT size (N)	512	**input power**	23 dBm
coding gain	4.2 dB	**noise power**	-140 dBm/Hz
margin	6 dB	**crosstalk noise**	8 ADSL disturbers

Simulation Results

- **Min-ISI, MBR, and MSSNR achieve matched filter bound owith CP of 27 samples**
- **Min-ISI with 13-sample CP beats MMSE with 32-sample CP**
- **MMSE is worst**

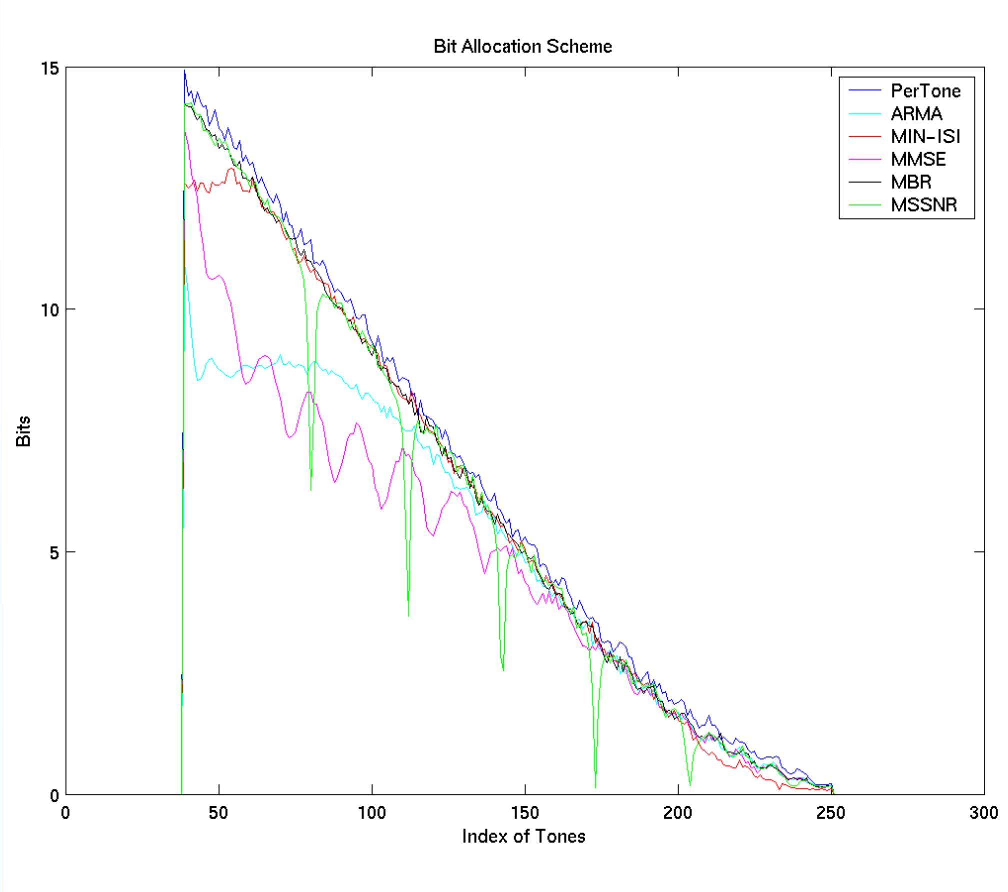


TEQ taps (L_w)	3		
FFT size (N)	512	**input power**	23 dBm
coding gain	4.2 dB	**noise power**	-140 dBm/Hz
margin	6 dB	**crosstalk noise**	8 ADSL disturbers

Bit Allocation Comparison

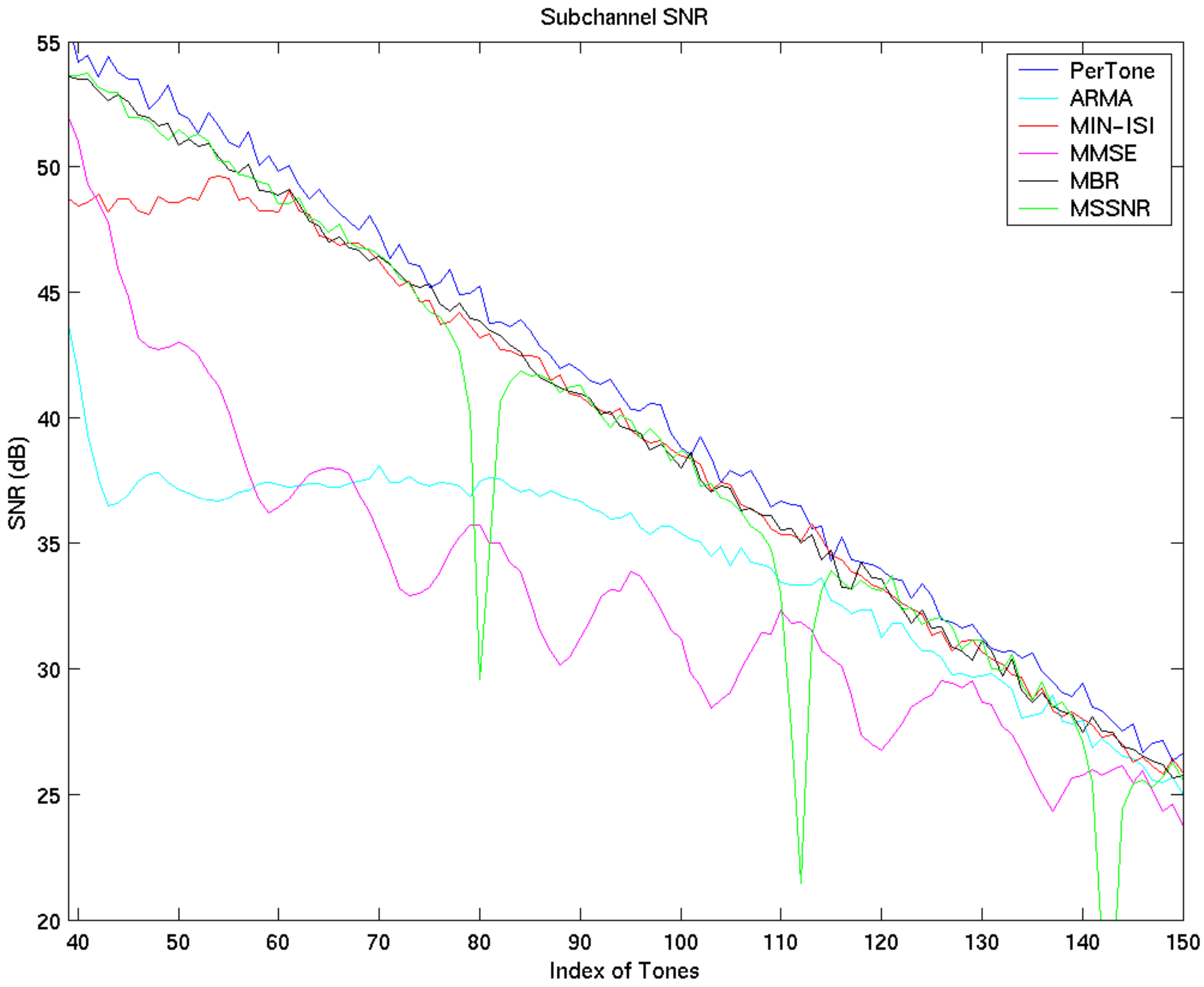
- **AWG 26 Loop: 12000 ft + AWGN**

Equalizer	*Bit Rate*
Per Tone	5.7134 Mbps
MBR	5.4666 Mbps
MSSNR	5.2903 Mbps
Min ISI	5.2586 Mbps
ARMA	4.5479 Mbps
MMSE	4.4052 Mbps



- **Simulation**
  - NEXT from 24 DSL disturbers
  - 32-tap equalizers: least squares training used for per-tone equalizer

Subchannel SNR



Frequency-Domain Per-Tone Equalizer

- **Rearrange computation of FFT coefficient for tone *i***

  [Van Acker, Leus, Moonen, van de Wiel, Pollet, 2001]

  $$Z_i = D_i \, \mathrm{row}_i(\mathbf{Q}_N) \, \mathbf{Y} \, \mathbf{w} = \mathrm{row}_i(\mathbf{Q}_N \, \mathbf{Y}) \, (\, \mathbf{w} \, D_i \,)$$

  $\mathbf{Q}_N \, \mathbf{Y}$ produces $N \times L_w$ complex-valued matrix produced by sliding FFT

  Z_i is inner product of ith row of $\mathbf{Q}_N \, \mathbf{Y}$ (complex) and $\mathbf{w} \, D_i$ (complex)

  TEQ has been moved into FEQ to create multi-tap FEQ as linear combiner

- **After FFT demodulation, each tone equalized separately**

  Equalize each carrier independently of other carriers (N/2 carriers)

  Maximize bit rate at *output of FEQ* by maximizing subchannel SNR

- **Sliding FFT to produce $N \times L_w$ matrix product $\mathbf{Q}_N \, \mathbf{Y}$**

  Receive one ADSL frame (symbol + cyclic prefix) of $N + \nu$ samples

  Take FFT of first N samples to form the first column

  Advance one sample

  Take FFT of N samples to form the second column, etc.

Per-Tone Equalizer: Implementation Complexity

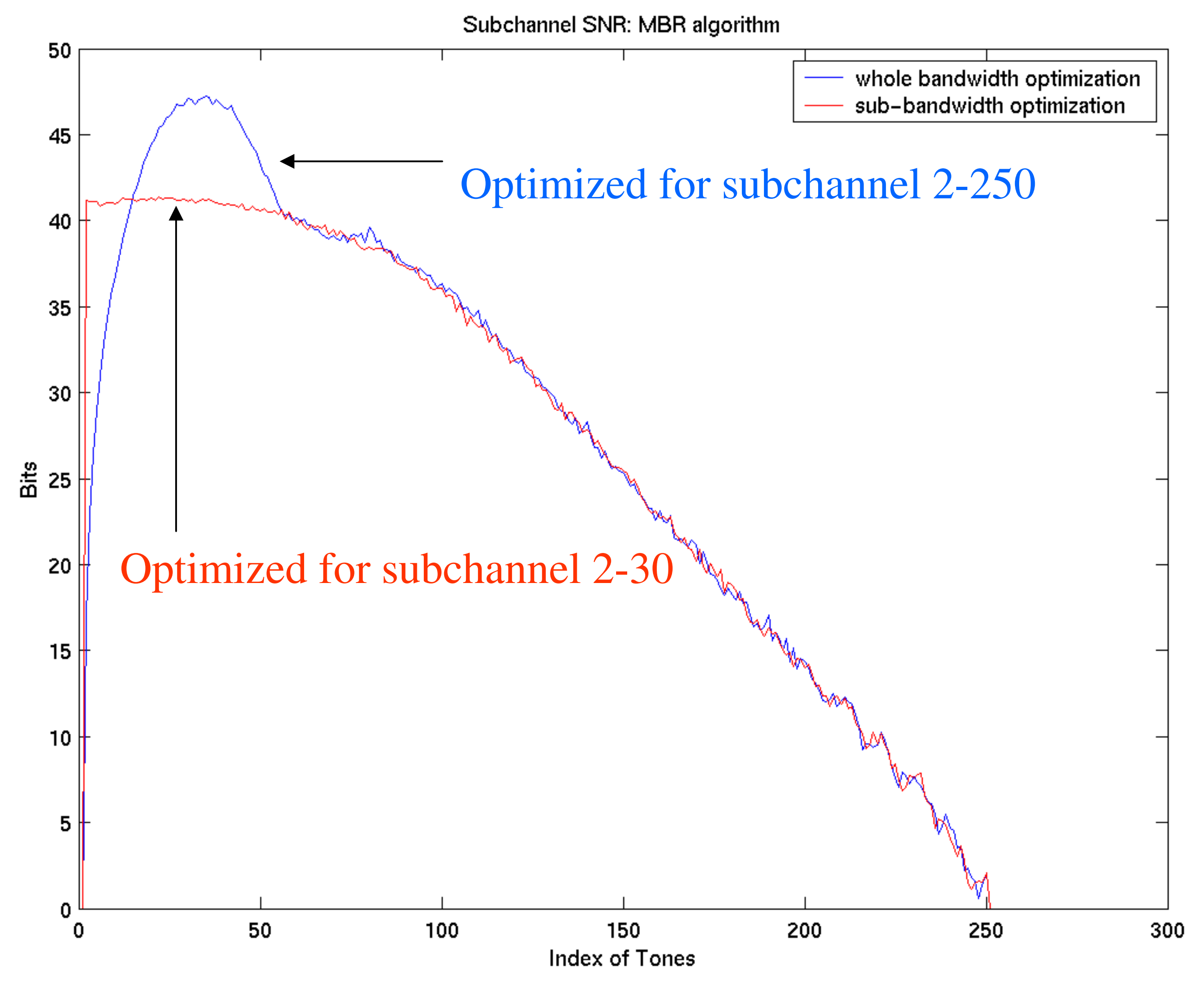
Conventional	Real MACs	Words
TEQ	$L_w f_s$	$2 L_w$
FFT	$2 N \log_2(N) f_{sym}$	$4 N$
FEQ	$4 N_u f_{sym}$	$4 N_u$

Per Tone	Real MACs	Words
FFT	$2 N \log_2(N) f_{sym}$	$4 N + 2 \nu$
Sliding FFT	$2 (L_w - 1) N f_{sym}$	N
Combiner	$4 L_w N_u f_{sym}$	$2 (L_w + 1) N_u$

Parameter	Symbol	Value
Sampling rate	f_s	2.208 MHz
Symbol rate	f_{sym}	4 kHz
TEQ length	L_w	3-32
Symbol length	N	512
Subchannels used	N_u	256
Cyclic prefix length	ν	32

Modified. Per Tone	Real MACs	Adds	Words
FFT	$2 N \log_2(N) f_{sym}$		$4 N$
Differencing		$(L_w - 1) f_{sym}$	$L_w - 1$
Combiner	$2 (L_w + 1) N_u f_{sym}$		$2 L_w N_u$

Dual-Path TEQ (Simulated Channel)



Optimized for subchannel 2-250

Optimized for subchannel 2-30

Motorola CopperGold ADSL Chip

- **Announced in March 1998**
- **5 million transistors, 144 pins, clocked at 55 MHz**
- **1.5 W power consumption**
- **DMT processor consists**
  - Motorola MC56300 DSP core
  - Several application specific ICs
    - 512-point FFT
    - 17-tap FIR filter for time-domain channel equalization based on MMSE method (20 bits precision per tap)
- **DSP core and memory occupies about 1/3 of chip area**