Equalizer Design to Maximize Bit Rate in ADSL Transceivers

Prof. Brian L. Evans
Dept. of Electrical and Comp. Eng.
The University of Texas at Austin
http://signal.ece.utexas.edu

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UT graduate students: Mr. Zukang Shen, Mr. Daifeng Wang, Mr. Ian Wong

UT Ph.D. graduates: Dr. Güner Arslan (Silicon Labs), Dr. Biao Lu (Schlumberger), Dr. Ming Ding (Bandspeed), Dr. Milos Milosevic (Schlumberger)

UT senior design students: Wade Berglund, Jerel Canales, David J. Love, Ketan Mandke, Scott Margo, Esther Resendiz, Jeff Wu

Other collaborators: Dr. Lloyd D. Clark (Schlumberger), Prof. C. Richard Johnson, Jr. (Cornell), Prof. Sayfe Kiaei (ASU), Prof. Rick Martin (AFIT), Prof. Marc Moonen (KU Leuven), Dr. Lucio F. C. Pessoa (Motorola), Dr. Arthur J. Redfern (Texas Instruments)
Introduction

Digital Subscriber Line (DSL) Broadband Access

DSLAM - Digital Subscriber Line Access Multiplexer

LPF – Lowpass Filter (passes voiceband frequencies)
## Discrete Multitone (DMT) DSL Standards

### ADSL – Asymmetric DSL

- **1997**
- Maximum data rates supported in G.DMT standard (ideal case)
  - **Echo cancelled**: 14.94 Mbps downstream, 1.56 Mbps upstream
  - **Frequency division multiplexing** (FDM): 13.38 Mbps downstream, 1.56 Mbps upstream
- Widespread deployment in US, Canada, Western Europe, and Hong Kong
- **Central office providers** only installing frequency-division multiplexed (FDM)
- ADSL: **cable modem** market
  - 1:2 in US & 2:1 worldwide
- ADSL+ 8 Mbps downstream min.
- ADSL2 doubles analog bandwidth

### VDSL – Very High Rate DSL

- **2003**
- Asymmetric
  - Faster G.DMT FDM ADSL
  - $2^m$ subcarriers $m \in [8, 12]$
- Symmetric: 13, 9, or 6 Mbps
- **Optional** 12-17 MHz band

### Table: G.DMT ADSL vs Asymmetric DMT VDSL

<table>
<thead>
<tr>
<th></th>
<th>G.DMT ADSL</th>
<th>Asymmetric DMT VDSL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data band</strong></td>
<td>0.025 – 1.1 MHz</td>
<td>0.138 – 12 MHz</td>
</tr>
<tr>
<td><strong>Upstream subcarriers</strong></td>
<td>32</td>
<td>256</td>
</tr>
<tr>
<td><strong>Downstream subcarriers</strong></td>
<td>256</td>
<td>2048/4096</td>
</tr>
<tr>
<td><strong>Target upstream rate</strong></td>
<td>1 Mbps</td>
<td>3 Mbps</td>
</tr>
<tr>
<td><strong>Target downstream rate</strong></td>
<td>8 Mbps</td>
<td>13/22 Mbps</td>
</tr>
</tbody>
</table>
Outline

• Multicarrier modulation

• Conventional equalizer training methods
  – Minimum Mean Squared Error design [Stanford]
  – Maximum Shortening Signal-to-Noise Ratio design [Tellabs]
  – Maximum Bit Rate design (optimal) [UT Austin]
  – Minimum Inter-symbol Interference design (near-optimal) [UT Austin]

• Per-tone equalizer [Catholic University, Leuven, Belgium]
• Dual-path equalizer [UT Austin]

• Conclusion
Single Carrier Modulation

- **Ideal (non-distorting) channel over transmission band**
  - Flat magnitude response
  - Linear phase response: delay is constant for all spectral components
  - No intersymbol interference

- **Impulse response for ideal channel over all frequencies**
  - Continuous time: $g \delta(t-T)$
  - Discrete time: $g \delta[k-\Delta]$

- **Equalizer**
  - Shortens channel impulse response (time domain)
  - Compensates for frequency distortion (frequency domain)
Multicarrier Modulation

- **Divide channel into narrowband subchannels**
  - No inter-symbol interference (ISI) in subchannels if constant gain within every subchannel and if ideal sampling

- **Discrete multitone modulation**
  - Baseband transmission
  - Based on fast Fourier transform (FFT)
  - Standardized for ADSL and VDSL

Subchannels are 4.3 kHz wide in ADSL and VDSL
Multicarrier Modulation by Inverse FFT Filter Bank

$X_1 \mapsto g(t) \mapsto x \mapsto e^{j2\pi f_1 t}$

$X_2 \mapsto g(t) \mapsto x \mapsto e^{j2\pi f_2 t}$

$X_{N/2} \mapsto g(t) \mapsto x \mapsto e^{j2\pi f_{N/2} t}$

Discrete time

$X_i$ : $i^{th}$ subsymbol from encoder

$g(t)$ : pulse shaping filter

$X_1$ : complex-valued

$X_2$ : complex-valued

$X_{N/2}$ : real-valued

$e^{j2\pi \frac{1}{N} k}$

$e^{j2\pi \frac{2}{N} k}$

$e^{j2\pi \frac{N/2}{N} k}$
Discrete Multitone Modulation Symbol

- **N/2 subsymbols are in general complex-valued**
  - ADSL uses 4-level Quadrature Amplitude Modulation (QAM) during training
  - ADSL uses QAM of $2^2$, $2^3$, $2^4$, ..., $2^{15}$ levels during data transmission

- **Multicarrier modulation using inverse FFT**

  $$e^{j\Omega t} + e^{-j\Omega t} = 2\cos(\Omega t)$$

  ![Diagram of Multicarrier Modulation](image.png)
Multicarrier Modulation

Discrete Multitone Modulation Frame

• Frame is sent through D/A converter and transmitted
  – Frame is the symbol with cyclic prefix prepended
  – Cyclic prefix (CP) consists of last \( v \) samples of the symbol

\[
\begin{align*}
\text{CP} & \quad \text{symbol} \quad i \quad \text{CP} \\
v \text{ samples} & \quad N \text{ samples} \\
\end{align*}
\]

– CP reduces throughput by factor of \( \frac{N}{N + v} = \frac{16}{17} \)

• Linear convolution of frame with channel impulse response
  – Is circular convolution if channel length is CP length plus one or shorter
  – Circular convolution \( \rightarrow \) frequency-domain equalization in FFT domain
  – Time-domain equalization to reduce effective channel length and ISI

\[
\begin{align*}
& \text{ADSL G.DMT Values} \\
& \begin{array}{|c|c|c|}
\hline
& \text{Down} & \text{Up} \\
\text{stream} & \text{stream} \\
\hline
v & 32 & 4 \\
N & 512 & 64 \\
\hline
\end{array}
\end{align*}
\]
Eliminating ISI in Discrete Multitone Modulation

- **Time domain equalizer (TEQ)**
  - Finite impulse response (FIR) filter
  - *Effective channel impulse response*: convolution of TEQ impulse response with channel impulse response
- **Frequency domain equalizer (FEQ)**
  - Compensates magnitude/phase distortion of equalized channel by dividing each FFT coefficient by complex number
  - Generally updated during data transmission
- **ADSL G.DMT equalizer training**
  - *Reverb*: same symbol sent 1,024 to 1,536 times
  - *Medley*: aperiodic pseudo-noise sequence of 16,384 symbols
  - Receiver returns number of bits (0-15) to transmit each subchannel $i$

\[
b_i \leq \log \left(1 + \frac{\text{SNR}_i}{I_i}\right)
\]

$\Delta$: transmission delay
$v$: cyclic prefix length

<table>
<thead>
<tr>
<th>ADSL G.DMT Values</th>
<th>Downstream</th>
<th>Upstream</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>32</td>
<td>4</td>
</tr>
<tr>
<td>$N$</td>
<td>512</td>
<td>64</td>
</tr>
</tbody>
</table>
ADSL Transceiver: Data Transmission

**TRANSMITTER**

- Bits
- S/P
- quadrature amplitude modulation (QAM) encoder
- mirror data and \( N \)-IFFT
- add cyclic prefix
- P/S
- D/A + transmit filter

**RECEIVER**

- N/2 subchannels
- N real samples
- P/S
- QAM demod decoder
- invert channel = frequency domain equalizer
- \( N \)-FFT and remove mirrored data
- remove cyclic prefix
- S/P
- time domain equalizer (FIR filter)
- receive filter + A/D

**conventional ADSL equalizer structure**

**Multicarrier Modulation**
Outline

- Multicarrier modulation
- **Conventional equalizer training methods**
  - Minimum Mean Squared Error design [Stanford]
  - Maximum Shortening Signal-to-Noise Ratio design [Tellabs]
  - Maximum Bit Rate design (*optimal*) [UT Austin]
  - Minimum Inter-symbol Interference design (*near-optimal*) [UT Austin]
- Per-tone equalizer
- Dual-path equalizer
- Conclusion
Minimum Mean Squared Error TEQ Design

- **Minimize** $E\{e_k^2\}$ [Chow & Cioffi, 1992]
  - Chose length of $b$ (e.g. $n+1$) to shorten length of $h^*w$
  - $b$ is eigenvector of minimum eigenvalue of symmetric channel-dependent matrix $R_\Delta = R_{xx} - R_{xy} R_{yy}^{-1} R_{yx}$
  - Minimum MSE when $R_{yy} w = R_{xy} b$ where $w \neq 0$

- **Disadvantages**
  - Does not consider *bit rate*
  - Deep notches in equalized frequency response

- $R_{xy}$ is cross correlation matrix

Why?
Infinite Length MMSE TEQ Analysis

- As TEQ length goes to infinity, $R_A$ becomes Toeplitz [Martin et al. 2003]
  - Eigenvector of minimum eigenvalue of symmetric Toeplitz matrix has zeros on unit circle [Makhoul 1981]
  - Zeros of target impulse response $b$ on unit circle kills $v$ subchannels

- Finite length TEQ plot
  - Each trace is a different zero of $b$
  - Distance of 32 zeros of $b$ to unit circle averaged over 8 ADSL test channels for each TEQ length
  - Zeros cluster at 0.01 and $10^{-4}$ from UC for TEQ lengths 32 and 100

**Longer MMSE TEQ may be worse**
**Maximum Shortening SNR TEQ Design**

- Minimize energy leakage outside shortened channel length
- For each possible position of window [Melsa, Younce & Rohrs, 1996]

\[
\text{max}_w (\text{SSNR in dB}) = \text{max}_w 10 \log_{10} \left( \frac{\text{energy inside window after TEQ}}{\text{energy outside window after TEQ}} \right)
\]

- Equivalent to noise-free MMSE TEQ
- Disadvantages
  - Does not consider channel noise
  - Does not consider *bit rate*
  - Deep notches in equalized frequency response (zeros of target impulse response near unit circle kill subchannels)
  - Requires Cholesky decomposition, which is computationally-intensive and does not allow TEQ lengths longer than cyclic prefix

**Conventional Equalizer**
**Maximum Shortening SNR TEQ Design**

- **Choose w to minimize energy outside window of desired length**
  Locate window to capture maximum channel impulse response energy

- **Objective function is shortening SNR (SSNR)**

  \[
  \max_{w} \text{(SSNR)} = \max_{w} 10 \log_{10} \left( \frac{w^{T} Bw}{w^{T} Aw} \right) \quad \text{subject to} \quad w^{T} Bw = 1
  \]

  Cholesky decomposition of \( B \) to find eigenvector for minimum generalized eigenvalue of \( A \) and \( B \)

  \[
  C = \left( \sqrt{B} \right)^{-1} A \left( \sqrt{B^{T}} \right)^{-1}
  \]

  \[
  w_{opt} = \left( \sqrt{B^{T}} \right)^{-1} q_{min} \quad q_{min} : \text{eigenvector of min eigenvalue of } C
  \]
Modeling Achievable Bit Rate

- **Bit allocation bounded by subchannel SNRs:** \( \log(1 + \frac{\text{SNR}_i}{\Gamma_i}) \)
- **Model** \( i^{th} \) **subchannel SNR** [Arslan, Evans & Kiaei, 2001]

\[
\text{SNR}_i = \frac{\text{signal power}}{\text{noise power + ISI power}}
\]

\[
\text{SNR}_i = \frac{S_{x,i} \times \text{signal transfer function}}{S_{n,i} \times \text{noise transfer function} + S_{x,i} \times \text{ISI transfer function}}
\]

\( S_{x,i} \) : transmitted signal power in subchannel \( i \)
\( S_{n,i} \) : channel noise power in subchannel \( i \)

- **Divide numerator and denominator of** \( \text{SNR}_i \) **by noise power spectral density** \( S_{n,i} \)

\[
\text{SNR}_i = \frac{\frac{S_{x,i}}{S_{n,i}} |H_i^{\text{signal}}|^2}{|H_i^{\text{noise}}|^2 + \frac{S_{x,i}}{S_{n,i}} |H_i^{\text{ISI}}|^2}
\]

**Used in Maximum Bit Rate Method**

**Used in Minimum ISI Method**
Maximum Bit Rate (MBR) TEQ Design

- Subchannel SNR as nonlinear function of equalizer taps $w$

$$
H_{i}^{\text{signal}} = q_i^{H} G w
$$

$$
H_{i}^{\text{ISI}} = q_i^{H} D H w
$$

$$
H_{i}^{\text{noise}} = q_i^{H} F w
$$

$q_i$ is $i$th row of DFT matrix

$$
\text{SNR}_i = \frac{S_{x,i} |q_i^{H} G w|^2}{S_{n,i} |q_i^{H} F w|^2 + S_{x,i} |q_i^{H} D H w|^2} = \frac{w^T A_i w}{w^T B_i w}
$$

- Maximize nonlinear function of bits/symbol with respect to $w$

$$
\log_2 (1 + \frac{1}{\Gamma} \frac{w^T A_i w}{w^T B_i w})
$$

$\Gamma$ Fractional bits for optimization

- Good performance measure for comparison of TEQ design methods
- Not an efficient TEQ design method in computational sense
Minimum-ISI (Min-ISI) TEQ Design

- **Rewrite subchannel SNR**
  [Arslan, Evans & Kiaei, 2001]

  \[ SNR_i = \frac{S_{x,i} |H_i^{\text{signal}}|^2}{S_{n,i} \left( |H_i^{\text{noise}}|^2 + \frac{S_{x,i}}{S_{n,i}} |H_i^{\text{ISI}}|^2 \right)} \]

  ISI power weighted in frequency domain by inverse of noise spectrum

- **Generalize MSSNR method by weighting ISI in frequency**
  - Minimize frequency weighted sum of subchannel ISI power
  \[ \sum_i \text{ISI}_i = \sum_i K_i |q_i^H DHw|^2 = w^T X w \]
  - Penalize ISI power in high conventional SNR subchannels: \( K_i = \frac{S_{x,i}}{S_{n,i}} \)
  - Constrain signal path gain to one to prevent all-zero solution for \( w \)
  \[ |h_i^{\text{signal}}|^2 = |GHw|^2 = w^T Y w = 1 \]
  - Solution is eigenvector of minimum generalized eigenvalue of \( X \) and \( Y \)

- **Iterative Min-ISI method** [Ding et al. 2003]
  - Avoids Cholesky decomposition by using adaptive filter theory
  - Designs arbitrary length TEQs without loss in bit rate
  - Overcomes disadvantages of Maximum SSNR method
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- **Per-tone equalizer**
- Dual-path equalizer
- Conclusion

Message bit stream

Transmitter

Channel

Equalizer

Receiver

Received bit stream
Drawbacks to Using Single FIR Filter for TEQ

- **Conventional equalizer**

  \[ Z_i = D_i \text{ row}_i(Q_N) \ Y \ w \]

  - \( D_i \) is the complex scalar value of one-tap FEQ for tone \( i \)
  - \( Q_N \) is the \( N \times N \) complex-valued FFT matrix
  - \( Y \) is an \( N \times L_w \) real-valued Toeplitz matrix of received samples
  - \( w \) is a \( L_w \times 1 \) column vector of real-valued TEQ taps

- Equalizes all tones in combined fashion: may limit bit rate

- Output of conventional equalizer for tone \( i \) computed using sequence of linear operations

Per-Tone Equalizer
Per-Tone Equalizer

**Frequency-Domain Per Tone Equalizer**

- **Rewrite equalized FFT coefficient for each of \( N/2 \) tones**
  
  \[ Z_i = D_i \, \text{row}_i(Q_N) \, Y \quad w = \text{row}_i(Q_N \, Y) \, (w \, D_i) = \text{row}_i(Q_N \, Y) \, w_i \]

  - Take sliding FFT to produce \( N \times L_w \) matrix product \( Q_N \, Y \)
  - Design \( w_i \) for each tone

**Diagram:**

- **FEQ is a linear combiner of up to \( N/2 \) \( L_w \)-tap FEQs**

- **N + \( L_w \) – 1 channels**

\[ y \]

\[ \downarrow \text{N+v} \]

\[ \downarrow \text{N+v} \]

\[ \downarrow \text{N+v} \]

\[ \downarrow \text{N+v} \]

\[ \text{Sliding N-Point FFT (} L_w \text{-frame)} \]

\[ Z^{-1} \]

\[ Z^{-1} \]

\[ Z^{-1} \]
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[UT Austin]
Dual-Path Equalizer

Dual-Path Time Domain Equalizer (DP-TEQ)
[Ding, Redfern & Evans, 2002]

• First FIR TEQ equalizes entire available bandwidth
• Second FIR TEQ tailored for subset of subchannels
  – Subchannels with higher SNR
  – Subchannels difficult to equalize, e.g. at boundary of upstream and downstream channels in frequency-division multiplexed ADSL
• Minimum ISI method is good match for second FIR TEQ

Path selection for each subchannel is fixed during training
• Up to 20% improvement in bit rate over MMSE TEQs
• Enables reuse of VLSI designs of conventional equalizers
Simulation Results for 17-Tap Equalizers

**Parameters**
- Cyclic prefix length: 32
- FFT size (N): 512
- Coding gain (dB): 4.2
- Margin (dB): 6
- Input power (dBm): 23
- Noise power (dBm/Hz): -140
- Crosstalk noise: 24 ISDN disturbers

**Downstream transmission**

Figure 1 in [Martin, Vanbleu, Ding, Ysebaert, Milosevic, Evans, Moonen & Johnson, Oct. 2005]

UNC(b) means unit norm constraint on target impulse response $b$, i.e. $\| b \| = 1$

MDS is Maximum Delay Spread design method [Schur & Speidel, 2001]
Simulation Results

Simulation Results for 17-Tap Equalizers

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- Crosstalk noise: 24 ISDN disturbers

Figure 3 in [Martin, Vanbleu, Ding, Ysebaert, Milosevic, Evans, Moonen & Johnson, Oct. 2005]
MDR is Maximum Data Rate design method [Milosevic et al., 2002]
BM-TEQ is Bit Rate Maximizing design method [Vanbleu et al., 2003]
PTEQ is Per Tone Equalizer structure and design method [Acker et al., 2001]
Simulation Results

Estimated Computational Complexity

Computational Complexity in $10 \log_{10}(\text{MACs})$

Equalizer Design Algorithm

MAC means a multiplication-accumulation operation
Simulation Results

Achievable Bit Rate vs. Delay Parameter

Large plateau of near-optimal delays (optimal choice requires search)
One choice is to set the delay parameter equal to cyclic prefix length
Contribution by Research Group

• New methods for single-path time-domain equalizer design
  – Maximum Bit Rate method maximizes bit rate (upper bound)
  – Minimum Inter-Symbol Interference method (real-time, fixed-point)

• Minimum Inter-Symbol Interference TEQ design method
  – Generalizes Maximum Shortening SNR by frequency weighting ISI
  – Improve bit rate in an ADSL transceiver by change of software only
  – Implemented in real-time on three fixed-point digital signal processors: Motorola 56000, TI TMS320C6200 and TI TMS320C5000

http://www.ece.utexas.edu/~bevans/projects/adsl

• New dual-path time-domain equalizer
  – Achieves bit rates between conventional and per tone equalizers
  – Lower implementation complexity in training than per tone equalizers
  – Enables reuse of ASIC designs
Conclusion

Matlab DMTTEQ Toolbox 3.1

- Single-path, dual-path, per-tone & TEQ filter bank equalizers
  Available at http://www.ece.utexas.edu/~bevans/projects/adsl/dmtteq/

various performance measures

18 design methods
default parameters from G.DMT ADSL standard
different graphical views
Backup Slides
### Applications of Broadband Access

#### Residential

<table>
<thead>
<tr>
<th>Application</th>
<th>Downstream rate (kb/s)</th>
<th>Upstream rate (kb/s)</th>
<th>Willing to pay</th>
<th>Demand Potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Database Access</td>
<td>384</td>
<td>9</td>
<td>High</td>
<td>Medium</td>
</tr>
<tr>
<td>On-line directory; yellow pages</td>
<td>384</td>
<td>9</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Video Phone</td>
<td>1,500</td>
<td>1,500</td>
<td>High</td>
<td>Medium</td>
</tr>
<tr>
<td>Home Shopping</td>
<td>1,500</td>
<td>64</td>
<td>Low</td>
<td>Medium</td>
</tr>
<tr>
<td>Video Games</td>
<td>1,500</td>
<td>1,500</td>
<td>Medium</td>
<td>Medium</td>
</tr>
<tr>
<td>Internet</td>
<td>3,000</td>
<td>384</td>
<td>High</td>
<td>Medium</td>
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<tr>
<td>Broadcast Video</td>
<td>6,000</td>
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<td>Low</td>
<td>High</td>
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<tr>
<td>High definition TV</td>
<td>24,000</td>
<td>0</td>
<td>High</td>
<td>Medium</td>
</tr>
</tbody>
</table>

#### Business

<table>
<thead>
<tr>
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<th>Demand Potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>On-line directory; yellow pages</td>
<td>384</td>
<td>9</td>
<td>Medium</td>
<td>High</td>
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<td>Financial news</td>
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<td>Video phone</td>
<td>1,500</td>
<td>1,500</td>
<td>High</td>
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<td>Internet</td>
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<td>Video conference</td>
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<td>Medium</td>
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<td>LAN interconnection</td>
<td>10,000</td>
<td>10,000</td>
<td>Medium</td>
<td>Medium</td>
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<td>Supercomputing, CAD</td>
<td>45,000</td>
<td>45,000</td>
<td>High</td>
<td>Low</td>
</tr>
</tbody>
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## Selected DSL Standards

<table>
<thead>
<tr>
<th>Standard</th>
<th>Meaning</th>
<th>Data Rate</th>
<th>Mode</th>
<th>Applications</th>
</tr>
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<tbody>
<tr>
<td><strong>ISDN</strong></td>
<td>Integrated Services Digital Network</td>
<td>144 kbps</td>
<td>Symmetric</td>
<td>Internet Access, Voice, Pair Gain (2 channels)</td>
</tr>
<tr>
<td><strong>T1</strong></td>
<td>T-Carrier One (requires two pairs)</td>
<td>1.544 Mbps</td>
<td>Symmetric</td>
<td>Enterprise, Expansion, Internet Service</td>
</tr>
<tr>
<td><strong>HDSL</strong></td>
<td>High-Speed Digital Subscriber Line (requires two pairs)</td>
<td>1.544 Mbps</td>
<td>Symmetric</td>
<td>Pair Gain (12 channels), Internet Access, T1/E1 replacement</td>
</tr>
<tr>
<td><strong>HDSL2</strong></td>
<td>Single Line HDSL</td>
<td>1.544 Mbps</td>
<td>Symmetric</td>
<td>Same as HDSL except pair gain is 24 channels</td>
</tr>
<tr>
<td><strong>G.Lite ADSL</strong></td>
<td>Splitterless Asymmetric Digital Subscriber Line</td>
<td>up to 1.5 Mbps, up to 512 kbps</td>
<td>Downstream, Upstream</td>
<td>Internet Access, Digital Video</td>
</tr>
<tr>
<td><strong>G.DMT ADSL</strong></td>
<td>Asymmetric Digital Subscriber Line</td>
<td>up to 10 Mbps, up to 1 Mbps</td>
<td>Downstream, Upstream</td>
<td>Internet Access, Digital Video</td>
</tr>
<tr>
<td><strong>VDSL</strong></td>
<td>Very High-Speed Digital Subscriber Line (proposed)</td>
<td>up to 22 Mbps, up to 3 Mbps</td>
<td>Downstream, Symmetric</td>
<td>Internet Access, Digital Video, Broadcast Video</td>
</tr>
</tbody>
</table>

Courtesy of Shawn McCaslin (National Instruments, Austin, TX)
Discrete Multitone DSL Standards

- Discrete multitone (DMT) modulation uses multiple carriers

- ADSL – Asymmetric DSL (G.DMT)
  - *Asymmetric*: 8 Mbps downstream and 1 Mbps upstream
  - *Data band*: 25 kHz – 1.1 MHz
  - Maximum data rates possible in standard (ideal case)
    - Echo cancelled: 14.94 Mbps downstream, 1.56 Mbps upstream
    - Frequency division multiplexing: 13.38 Mbps downstream, 1.56 Mbps up
  - Widespread deployment in US, Canada, Western Europe, Hong Kong
    - Central office providers only installing frequency-division ADSL
    - ADSL modems have about 1/3 of market, and cable modems have 2/3

- VDSL – Very High Rate DSL
  - *Asymmetric*: either 22/3 or 13/3 Mbps downstream/upstream
  - *Symmetric*: 13, 9, or 6 Mbps each direction
  - *Data band*: 1 – 12 MHz
  - DMT and single carrier modulation supported
  - DMT VDSL essentially higher speed version of G.DMT ADSL
A Digital Communications System

- Encoder maps a group of message bits to data symbols
- Modulator maps these symbols to analog waveforms
- Demodulator maps received waveforms back to symbols
- Decoder maps the symbols back to binary message bits
Intersymbol Interference (ISI)

- **Ideal channel**
  - Impulse response is impulse
  - Flat frequency response

- **Non-ideal channel**
  - Causes ISI
  - Channel memory
  - Magnitude and phase variation

- **Received symbol is weighted sum of neighboring symbols**
  - Weights are determined by channel impulse response

Introduction
Combat ISI with Equalization

• **Equalization because channel response is not flat**
  - Inverts channel
  - Flattens freq. response
  - Amplifies noise

• **Zero-forcing equalizer**
  - Inverts channel
  - Flattens freq. response
  - Amplifies noise

• **MMSE equalizer**
  - Optimizes trade-off between noise amplification and ISI

• **Decision-feedback equalizer**
  - Increases complexity
  - Propagates error
Introduction

Cyclic Prefix

Repeated symbol

\[ * \]

\[ = \]

equal

cyclic prefix

to be removed
Open Issues for Multicarrier Modulation

• **Advantages**
  – Efficient use of bandwidth without full channel equalization
  – Robust against impulsive noise and narrowband interference
  – Dynamic rate adaptation

• **Disadvantages**
  – *Transmitter*: High signal peak-to-average power ratio
  – *Receiver*: Sensitive to frequency and phase offset in carriers

• **Open issues**
  – Pulse shapes of subchannels (*orthogonal, efficient realization*)
  – Channel equalizer design (*increase bit rate, reduce complexity*)
  – Synchronization (*timing recovery, symbol synchronization*)
  – Bit loading (*allocation of bits in each subchannel*)
  – Echo cancellation
**TEQ Algorithm**

- **ADSL standards**
  - Set aside 1024 frames (~0.25s) for TEQ estimation
  - Reserved ~16,000 frames for channel and noise estimation for the purpose of SNR calculation

- **TEQ is estimated before the SNR calculations**

- **Noise power and channel impulse response can be estimated before time slot reserved for TEQ if the TEQ algorithm needs that information**
Single-FIR Time-Domain Equalizer Design Methods

- **All methods below perform optimization at TEQ output**
- **Minimizing the mean squared error**
  - Minimize mean squared error (MMSE) method [Chow & Cioffi, 1992]
  - Geometric SNR method [Al-Dhahir & Cioffi, 1996]
- **Minimizing energy outside of shortened (equalized) channel impulse response**
  - Maximum Shortening SNR method [Melsa, Younce & Rohrs, 1996]
  - Divide-and-conquer methods [Lu, Evans, Clark, 2000]
  - Minimum ISI method [Arslan, Evans & Kiaei, 2000]
- **Maximizing bit rate** [Arslan, Evans & Kiaei, 2000]
- **Implementation**
  - Geometric SNR is difficult to automate (requires human intervention)
  - Maximum bit rate method needs nonlinear optimization solver
  - Other methods implemented on fixed-point digital signal processors
Minimum Mean Squared Error (MMSE) TEQ

Conventional Equalizer

\[ \text{MSE} = \mathcal{E}\{e_k^2\} = \hat{b}^T R_{xx} \hat{b} - 2 \hat{b}^T R_{xy} w + w^T R_{yy} w \]

minimum MSE is achieved only if \( b^T R_{xy} = w^T R_{yy} \)

\[ \text{MSE} = \hat{b}^T \left[ R_{xx} - R_{xy} R_{yy}^{-1} R_{yx} \right] \hat{b} = \hat{b}^T R_{xly} \hat{b} \]

Define \( R_{\Delta} = O^T R_{xly} O \) then \( \text{MSE} = b^T R_{\Delta} b \)

\( O \) selects the proper part out of \( R_{xly} \) corresponding to the delay \( \Delta \)
Near-optimal Minimum-ISI (Min-ISI) TEQ Design

- Generalizes MSSNR method by frequency weighting ISI
  - ISI power in $i$th subchannel is $ISI_i = S_{x,i} |q_i^H DHw|^2$
  - Minimize ISI power as a frequency weighted sum of subchannel ISI
    $$\sum_i ISI_i = \sum_i K_i |q_i^H DHw|^2 = w^T Xw$$
  - Constrain signal path gain to one to prevent all-zero solution
    $$|h_{signal}|^2 = |GHw|^2 = w^T Yw = 1$$
  - Solution is a generalized eigenvector of $X$ and $Y$

- Possible weightings
  - Amplify ISI objective function in subchannels with low noise power (high SNR) to put ISI in low SNR bins:
    $$K_i = \frac{S_{x,i}}{S_{n,i}}$$
  - Set weighting equal to input power spectrum:
    $$K_i = S_{x,i}$$
  - Set weighting to be constant in all subchannels (MSSNR):
    $$K_i = 1$$

- Performance virtually equal to MBR (optimal) method
Efficient Implementations of Min-ISI Method

- Generalized eigenvalue problem can be solved with generalized power iteration: \( Xw^{k+1} = Yw^k \)
- Recursively calculate diagonal elements of \( X \) and \( Y \) from first column [Wu, Arslan, Evans, 2000]

<table>
<thead>
<tr>
<th>Method</th>
<th>Bit Rate</th>
<th>MACs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>99.6%</td>
<td>132,896</td>
</tr>
<tr>
<td>Recursive</td>
<td>99.5%</td>
<td>44,432</td>
</tr>
<tr>
<td>Row-rotation</td>
<td>99.5%</td>
<td>25,872</td>
</tr>
<tr>
<td>No-weighting</td>
<td>97.8%</td>
<td>10,064</td>
</tr>
</tbody>
</table>

Conventional Equalizer
Motivation for Divide-and-Conquer Methods

- Fast methods for implementing Maximum SSNR method
- Maximum SSNR Method
  - For each $\Delta$, maximum SSNR method requires
    - Multiplications: $(L_n + \frac{7}{6})L_w + \frac{5}{2}L_w^2 + \frac{25}{3}L_w^3$
    - Additions: $(L_n - \frac{5}{6})L_w - \frac{3}{2}L_w^2 + \frac{25}{3}L_w^3$
    - Divisions: $L_w^2$
  - Exhaustive search for the optimal delay $\Delta$
    \[0 \leq \Delta \leq L_n + L_w - v - 2 \Rightarrow 0 \leq \Delta \leq 499\]
- Divide $L_w$ TEQ taps into $(L_w - 1)$ two-tap filters in cascade
  - Design first two-tap filter then second and so forth (greedy approach)
- Develop heuristic to estimate the optimal delay

Conventional Equalizer
**Divide-and-Conquer Approach**

- The $i^{th}$ two-tap filter is initialized as either
  - Unit tap constraint (UTC) $w_i = \begin{bmatrix} 1 \\ g_i \end{bmatrix}$
  - Unit norm constraint (UNC) $w_i = \begin{bmatrix} \sin \theta_i \\ \cos \theta_i \end{bmatrix}$

- Calculate best $g_i$ or $\theta_i$ by using a greedy approach either by
  - Minimizing $\frac{1}{SSNR}$ (Divide-and-conquer TEQ minimization)
  - Minimizing energy in $h_{\text{wall}}$ (Divide-and-conquer TEQ cancellation)

- Convolve two-tap filters to obtain TEQ
Conventional Equalizer

Divide-and-Conquer TEQ Minimization (UTC)

- At $i^{th}$ iteration, minimize $J_i$ over $g_i$

$$J_i = \frac{w_i^T A w_i}{w_i^T B w_i} = \frac{1}{\left[ \begin{array}{c} a_{1,i} \\ a_{2,i} \\ a_{3,i} \end{array} \right]} \left[ \begin{array}{c} b_{1,i} \\ b_{2,i} \end{array} \right]$$

- Closed-form solution

$$g_{i(1,2)} = \frac{-\left( a_{3,i} b_{1,i} - a_{1,i} b_{3,i} \right)}{2 \left( a_{3,i} b_{2,i} - a_{2,i} b_{3,i} \right)} \pm \frac{\sqrt{D}}{2 \left( a_{3,i} b_{2,i} - a_{2,i} b_{3,i} \right)}$$

$$D = \left( a_{3,i} b_{1,i} - a_{1,i} b_{3,i} \right)^2 - 4 \left( a_{3,i} b_{2,i} - a_{2,i} b_{3,i} \right) \left( a_{2,i} b_{1,i} - a_{1,i} b_{2,i} \right)$$
Divide-and-Conquer TEQ Minimization (UNC)

- At $i^{th}$ iteration, minimize $J_i$ over $\eta_i$

\[
J_i = \frac{w_i^T Aw_i}{w_i^T B w_i} = \frac{(\sin \theta_i [1 \eta_i]) \begin{bmatrix} a_{1,i} & a_{2,i} \\ a_{2,i} & a_{3,i} \end{bmatrix} (\sin \theta_i [1 \eta_i])}{(\sin \theta_i [1 \eta_i]) \begin{bmatrix} b_{1,i} & b_{2,i} \\ b_{2,i} & b_{3,i} \end{bmatrix} (\sin \theta_i [1 \eta_i])}
\]

\[
= \frac{\begin{bmatrix} 1 & \eta_i \end{bmatrix} \begin{bmatrix} a_{1,i} & a_{2,i} \\ a_{2,i} & a_{3,i} \end{bmatrix} \begin{bmatrix} 1 \\ \eta_i \end{bmatrix}}{\begin{bmatrix} 1 & \eta_i \end{bmatrix} \begin{bmatrix} b_{1,i} & b_{2,i} \\ b_{2,i} & b_{3,i} \end{bmatrix} \begin{bmatrix} 1 \\ \eta_i \end{bmatrix}}
\]

- where $w_i = \begin{bmatrix} \sin \theta_i \\ \cos \theta_i \end{bmatrix} = \sin \theta_i \begin{bmatrix} 1 \\ \cos \theta_i / \sin \theta_i \end{bmatrix} = \sin \theta_i \begin{bmatrix} 1 \\ \eta_i \end{bmatrix}$

Calculate $\eta_i$ in the same way as $g_i$ for UTC version of this method.
Divide-and-Conquer TEQ Cancellation (UTC)

- At $i^{th}$ iteration, minimize $J_i$ over $g_i$

$$J_i = \tilde{h}_{\text{wall}}^T \tilde{h}_{\text{wall}} = \sum_{k \in S} \left( \tilde{h}_{i-1}(k) + g_i \tilde{h}_{i-1}(k-1) \right)^2,$$

$$S = \left\{ 1, 2, \cdots, \Delta, \Delta + \nu + 2, \cdots, L_{\tilde{h}_{i-1}} \right\}$$

- Closed-form solution for the $i^{th}$ two-tap FIR filter

$$g_i = -\frac{\sum_{k \in S} \tilde{h}_{i-1}(k-1) \tilde{h}_{i-1}(k)}{\sum_{k \in S} \tilde{h}_{i-1}^2(k-1)}$$
Divide-and-Conquer TEQ Cancellation (UNC)

- At $i^{th}$ iteration, minimize $J_i$ over $\theta_i$

\[
J_i = \tilde{h}_{\text{wall}}^T \tilde{h}_{\text{wall}} = \sum_{k \in S} \left( \tilde{h}_{i-1}(k) \sin \theta_i + \tilde{h}_{i-1}(k - 1) \cos \theta_i \right)^2,
\]

\[S = \left\{ 1, 2, \cdots, \Delta, \Delta + \nu + 2, \cdots, L_{\tilde{h}_{i-1}} \right\}\]

- Closed-form solution

\[
\sin \theta_i = \pm \sqrt{0.5 \left( 1 \pm \sqrt{\frac{a^2}{a^2 + 4b^2}} \right)}, \quad \cos \theta_i = \pm \sqrt{0.5 \left( 1 \pm \sqrt{\frac{a^2}{a^2 + 4b^2}} \right)}
\]

\[a = \sum_{k \in S} \left( \tilde{h}_{i-1}^2(k) - \tilde{h}_{i-1}^2(k - 1) \right), \quad b = \sum_{k \in S} \tilde{h}_{i-1}(k - 1) \tilde{h}_{i-1}(k)\]
Computational Complexity

- Computational complexity for each candidate \( \Delta \)

<table>
<thead>
<tr>
<th>Method</th>
<th>( \times )</th>
<th>( \div )</th>
<th>Memory (words)</th>
</tr>
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<tbody>
<tr>
<td>Maximum SSNR</td>
<td>120379</td>
<td>118552</td>
<td>441</td>
</tr>
<tr>
<td>DC-TEQ-minimization (UTC)</td>
<td>53240</td>
<td>52980</td>
<td>60</td>
</tr>
<tr>
<td>DC-TEQ-cancellation (UNC)</td>
<td>42280</td>
<td>42160</td>
<td>20</td>
</tr>
<tr>
<td>DC-TEQ-cancellation (UTC)</td>
<td>41000</td>
<td>40880</td>
<td>20</td>
</tr>
</tbody>
</table>

- Divide-and-conquer methods vs. maximum SSNR method
  - Reduces multiplications, additions, divisions, and memory
  - No matrix calculations (saves on memory accesses)
  - Avoids matrix inversion, and eigenvalue and Cholesky decompositions

\[
\begin{align*}
\text{G.DMT} & \\
\text{ADSL} & \\
L_h & = 512 \\
\nu & = 32 \\
L_w & = 21
\end{align*}
\]
Heuristic Search for the Optimal Delay

- **Estimate optimal delay** $\Delta$ before computing TEQ taps
  \[
  \Delta_{\text{ratio}} = \arg\max_{\Delta} \frac{\text{energy inside a window of original } h}{\text{energy outside a window of original } h}
  \]

- **Total computational cost**
  - Multiplications: $L_h$
  - Additions: $3L_h - 3$
  - Divisions: $L_h$

- **Performance of heuristic vs. exhaustive search**
  - Reduce computational complexity by factor of 500
  - 2% loss in SSNR for TEQ with four taps or more
  - 8% loss in SSNR for two-tap TEQ
### Comparison of Earlier Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>MMSE</th>
<th>MSSNR</th>
<th>Geometric</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Advantages</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximize bit rate</td>
<td></td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>Minimize ISI</td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>Bit Rate</td>
<td>Low-medium</td>
<td>High</td>
<td>Low-medium</td>
</tr>
<tr>
<td><strong>Disadvantages</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlinear optimization</td>
<td></td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td>Computational complexity</td>
<td>Low</td>
<td>Medium</td>
<td>High</td>
</tr>
<tr>
<td>Artificial constraints</td>
<td>✔️</td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td>Ad-hoc parameters</td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>Lowpass frequency response</td>
<td>✔️</td>
<td></td>
<td>✔️</td>
</tr>
<tr>
<td>Unrealistic assumptions</td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
</tbody>
</table>
## MBR TEQ vs. Geometric TEQ

<table>
<thead>
<tr>
<th>Method</th>
<th>MBR</th>
<th>Geometric</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Advantages</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximize channel capacity</td>
<td>✔</td>
<td>✔️</td>
</tr>
<tr>
<td>Minimize ISI</td>
<td>✔</td>
<td></td>
</tr>
<tr>
<td>Bit rate</td>
<td>optimal</td>
<td>Low-medium</td>
</tr>
<tr>
<td><strong>Disadvantages</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-pass frequency response</td>
<td>✔</td>
<td>✔️</td>
</tr>
<tr>
<td>Computationally complex</td>
<td>✔</td>
<td>✔️</td>
</tr>
<tr>
<td>Artificial constraints</td>
<td>✔</td>
<td></td>
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<tr>
<td>Ad-hoc parameters</td>
<td>✔</td>
<td></td>
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<tr>
<td>Nonlinear optimization</td>
<td>✔</td>
<td>✔️</td>
</tr>
<tr>
<td>Unrealistic assumptions</td>
<td>✔</td>
<td>✔️</td>
</tr>
</tbody>
</table>
Min-ISI TEQ vs. MSSNR TEQ

<table>
<thead>
<tr>
<th>Method</th>
<th>Min-ISI</th>
<th>MSSNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advantages</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximize channel capacity</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>Minimize ISI</td>
<td>✔️</td>
<td>✔️</td>
</tr>
<tr>
<td>Frequency domain weighting</td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>Bit rate</td>
<td>high</td>
<td>high</td>
</tr>
<tr>
<td>Disadvantages</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computationally complex</td>
<td>very high</td>
<td>high</td>
</tr>
</tbody>
</table>

- **Min-ISI weights ISI power with the SNR**
  - Residual ISI power should be placed in high noise frequency bands

\[
\text{SNR}_i = \frac{\text{signal power}}{\text{noise power} + \text{ISI power}}
\]

\[
\text{SNR}_{50} = \frac{1}{10} = 0.1
\]

\[
\text{SNR}_{50} = \frac{1}{10+1} = 0.09
\]

\[
\text{SNR}_2 = \frac{1}{0.1} = 10
\]

\[
\text{SNR}_2 = \frac{1}{0.1+1} = 0.9
\]
Bit Rate vs. Cyclic Prefix (CP) Size

- Matched filter bound decreases because CP has no new information
- Min-ISI and MBR achieve bound with 16-sample CP
- Other design methods are erratic
- MGSNR better for 15-28 sample CPs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEQ taps ($L_w$)</td>
<td>17</td>
</tr>
<tr>
<td>FFT size ($N$)</td>
<td>512</td>
</tr>
<tr>
<td>Coding gain</td>
<td>4.2 dB</td>
</tr>
<tr>
<td>Margin</td>
<td>6 dB</td>
</tr>
<tr>
<td>Input power</td>
<td>23 dBm</td>
</tr>
<tr>
<td>Noise power</td>
<td>-140 dBm/Hz</td>
</tr>
<tr>
<td>Crosstalk noise</td>
<td>8 ADSL disturbers</td>
</tr>
</tbody>
</table>
Simulation Results

- Min-ISI, MBR, and MSSNR achieve matched filter bound with CP of 27 samples
- Min-ISI with 13-sample CP beats MMSE with 32-sample CP
- MMSE is worst

Conventional Equalizer

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEQ taps ($L_w$)</td>
<td>3</td>
</tr>
<tr>
<td>FFT size ($N$)</td>
<td>512</td>
</tr>
<tr>
<td>Coding gain</td>
<td>4.2 dB</td>
</tr>
<tr>
<td>Margin</td>
<td>6 dB</td>
</tr>
<tr>
<td>Input power</td>
<td>23 dBm</td>
</tr>
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<td>Noise power</td>
<td>-140 dBm/Hz</td>
</tr>
<tr>
<td>Crosstalk noise</td>
<td>8 ADSL disturbers</td>
</tr>
</tbody>
</table>
**Per-Tone Equalizer**

**Bit Allocation Comparison**

- **AWG 26 Loop:** 12000 ft + AWGN

<table>
<thead>
<tr>
<th>Equalizer</th>
<th>Bit Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per Tone</td>
<td>5.7134 Mbps</td>
</tr>
<tr>
<td>MBR</td>
<td>5.4666 Mbps</td>
</tr>
<tr>
<td>MSSNR</td>
<td>5.2903 Mbps</td>
</tr>
<tr>
<td>Min ISI</td>
<td>5.2586 Mbps</td>
</tr>
<tr>
<td>ARMA</td>
<td>4.5479 Mbps</td>
</tr>
<tr>
<td>MMSE</td>
<td>4.4052 Mbps</td>
</tr>
</tbody>
</table>

- **Simulation**
  - NEXT from 24 DSL disturbers
  - 32-tap equalizers: least squares training used for per-tone equalizer
Per-Tone Equalizer

Subchannel SNR

![Subchannel SNR graph]

- PerTone
- ARMA
- MIN-ISI
- MMSE
- MBR
- MSSNR

Index of Tones
Per-Tone Equalizer

Frequency-Domain Per-Tone Equalizer

- **Rearrange computation of FFT coefficient for tone** $i$
  [Van Acker, Leus, Moonen, van de Wiel, Pollet, 2001]

  \[ Z_i = D_i \text{row}_i(Q_N) \ Y \ w = \text{row}_i(Q_N \ Y) (w \ D_i) \]

  $Q_N Y$ produces $N \times L_w$ complex-valued matrix produced by sliding FFT
  $Z_i$ is inner product of $i$th row of $Q_N Y$ (complex) and $w \ D_i$ (complex)
  TEQ has been moved into FEQ to create multi-tap FEQ as linear combiner

- **After FFT demodulation, each tone equalized separately**
  Equalize each carrier independently of other carriers ($N/2$ carriers)
  Maximize bit rate at *output of FEQ* by maximizing subchannel SNR

- **Sliding FFT to produce** $N \times L_w$ **matrix product** $Q_N Y$
  Receive one ADSL frame (symbol + cyclic prefix) of $N + \nu$ samples
  Take FFT of first $N$ samples to form the first column
  Advance one sample
  Take FFT of $N$ samples to form the second column, etc.
Per-Tone Equalizer: Implementation Complexity

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling rate</td>
<td>$f_s$</td>
<td>2.208 MHz</td>
</tr>
<tr>
<td>Symbol rate</td>
<td>$f_{sym}$</td>
<td>4 kHz</td>
</tr>
<tr>
<td>TEQ length</td>
<td>$L_w$</td>
<td>3-32</td>
</tr>
<tr>
<td>Symbol length</td>
<td>$N$</td>
<td>512</td>
</tr>
<tr>
<td>Subchannels used</td>
<td>$N_u$</td>
<td>256</td>
</tr>
<tr>
<td>Cyclic prefix length</td>
<td>$\nu$</td>
<td>32</td>
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<table>
<thead>
<tr>
<th>Conventional</th>
<th>Real MACs</th>
<th>Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEQ</td>
<td>$L_w f_s$</td>
<td>$2 L_w$</td>
</tr>
<tr>
<td>FFT</td>
<td>$2N \log_2(N) f_{sym}$</td>
<td>$4N$</td>
</tr>
<tr>
<td>FEQ</td>
<td>$4 N_u f_{sym}$</td>
<td>$4 N_u$</td>
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<th>Per Tone</th>
<th>Real MACs</th>
<th>Words</th>
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<tr>
<td>FFT</td>
<td>$2N \log_2(N) f_{sym}$</td>
<td>$4N + 2 \nu$</td>
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<tr>
<td>Sliding FFT</td>
<td>$2 (L_w - 1) N f_{sym}$</td>
<td>$N$</td>
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<tr>
<td>Combiner</td>
<td>$4 L_w N_u f_{sym}$</td>
<td>$2 (L_w + 1) N_u$</td>
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<th>Modified. Per Tone</th>
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<th>Adds</th>
<th>Words</th>
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<tr>
<td>FFT</td>
<td>$2N \log_2(N) f_{sym}$</td>
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<td>$4N$</td>
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<tr>
<td>Differencing</td>
<td>$(L_w - 1) f_{sym}$</td>
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<td>$L_w - 1$</td>
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<tr>
<td>Combiner</td>
<td>$2 (L_w + 1) N_u f_{sym}$</td>
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<td>$2 L_w N_u$</td>
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Dual-Path Equalizer

Dual-Path TEQ (Simulated Channel)

- Optimized for subchannel 2-250
- Optimized for subchannel 2-30
Motorola CopperGold ADSL Chip

- Announced in March 1998
- 5 million transistors, 144 pins, clocked at 55 MHz
- 1.5 W power consumption
- DMT processor consists
  - Motorola MC56300 DSP core
  - Several application specific ICs
    - 512-point FFT
    - 17-tap FIR filter for time-domain channel equalization based on MMSE method (20 bits precision per tap)
- DSP core and memory occupies about 1/3 of chip area