Cascaded Noise-Shaping Modulators for Oversampled Data Conversion

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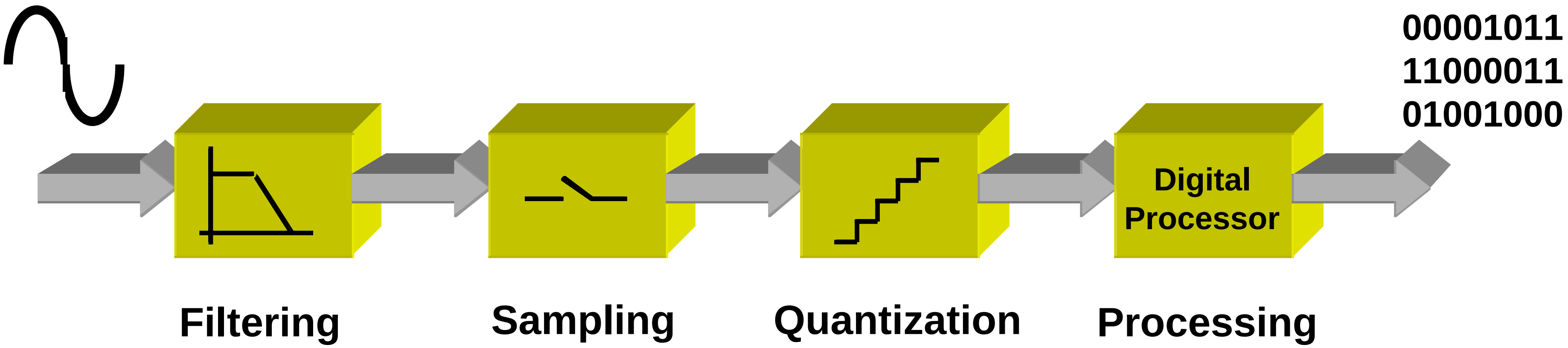
Outline

- Oversampling modulators for A/D conversion
- Cascaded noise-shaping architectures
- Embedded quantization
- Distributed noise shaping zeros
- Bandpass cascaded modulator
- Digital cascaded modulators with semi-digital reconstruction for D/A conversion

Analog-to-Digital Conversion



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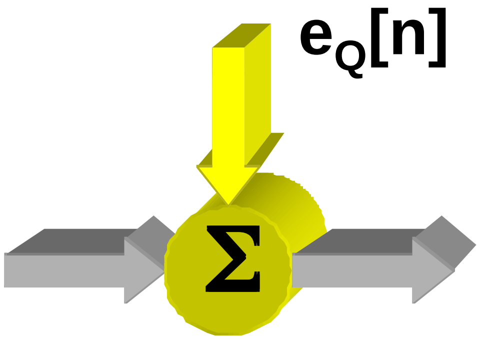
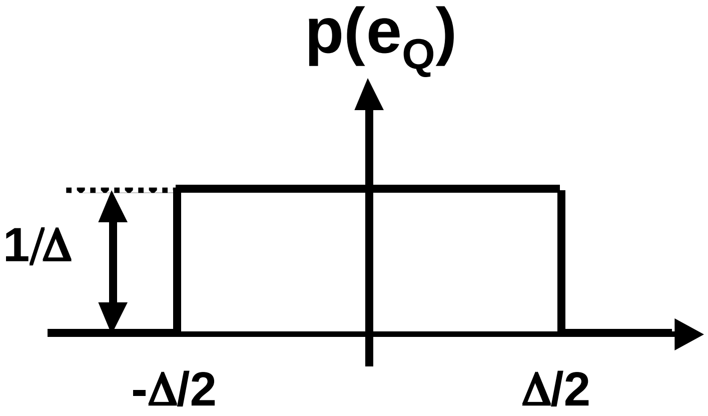
Digital Processor

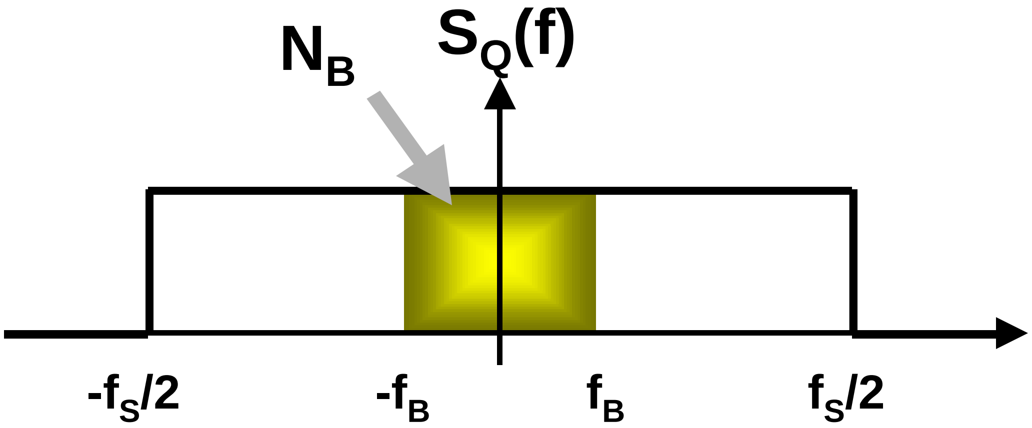
Filtering **Sampling** **Quantization** **Processing**

Quantization Model:

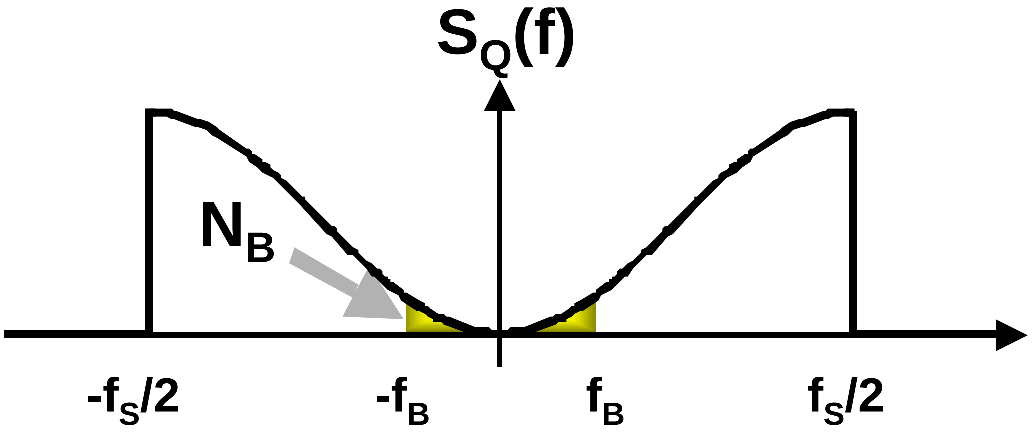




Noise Shaping



Quantizer resolution increased by 3 dB per octave of OVERSAMPLING

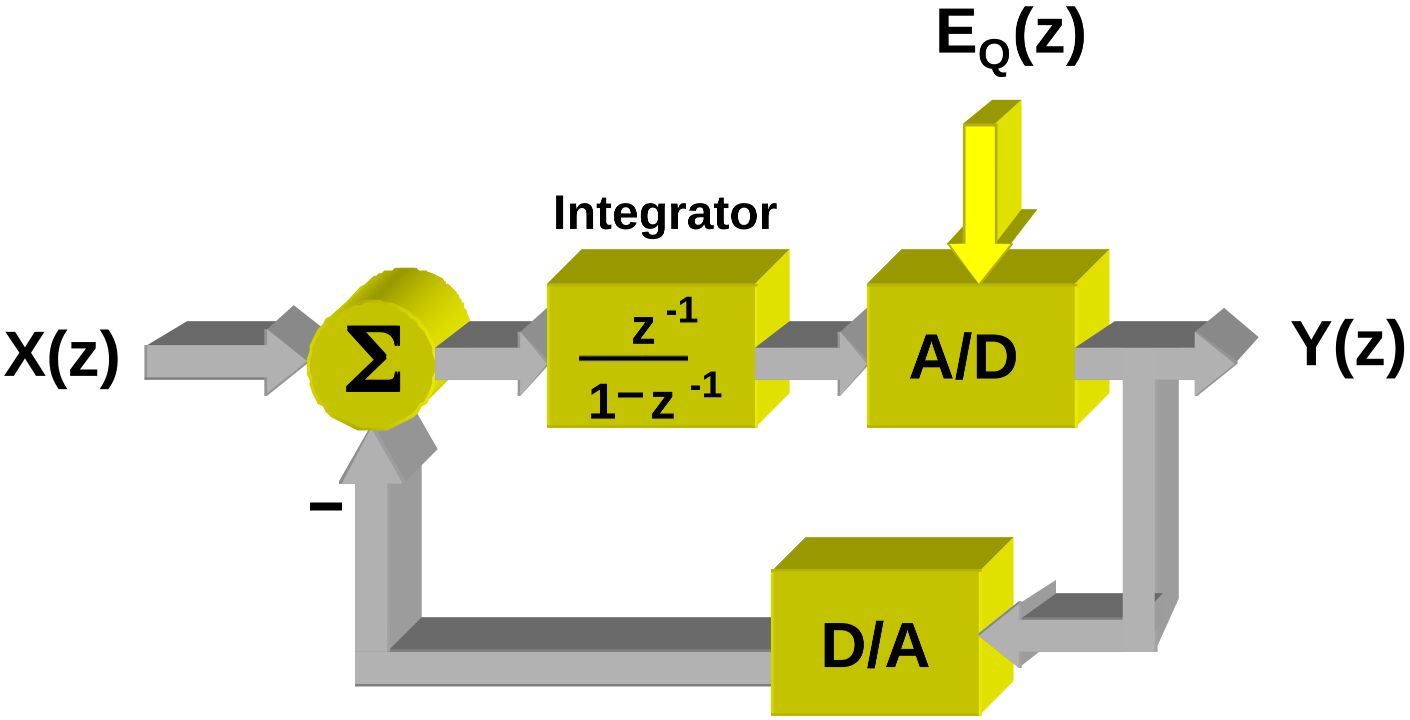


Quantizer resolution increased through NOISE SHAPING

Oversampling Modulators

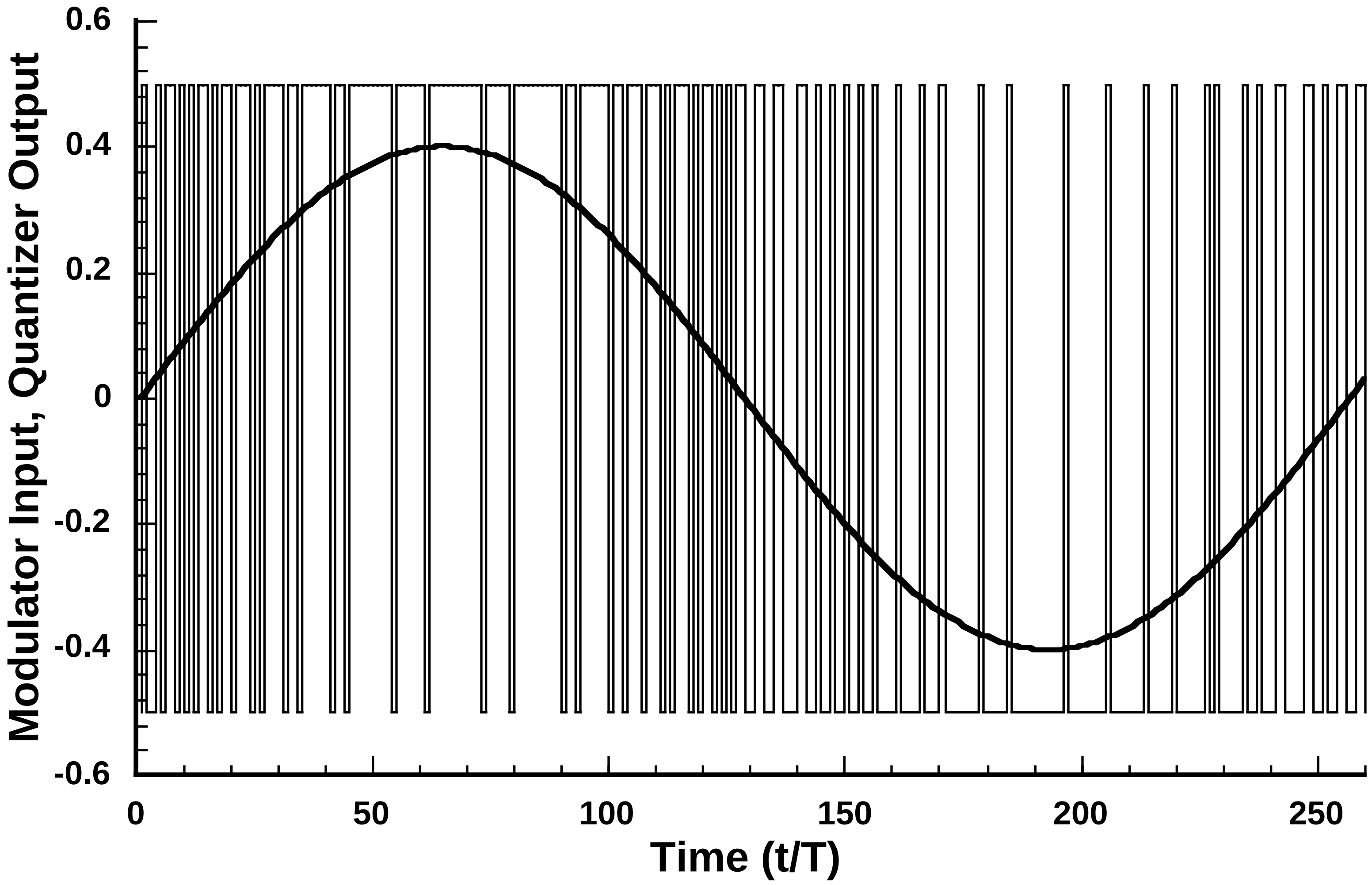
- **Embed quantizer in a feedback loop to achieve larger improvement in resolution with increasing M**
- **Feedback used for PREDICTION (Δ Modulation) or NOISE SHAPING (ΣΔ Modulation)**
- **Noise shaping modulators are more robust and easier to implement than predictive modulators**

Sigma-Delta (Delta-Sigma) Modulation

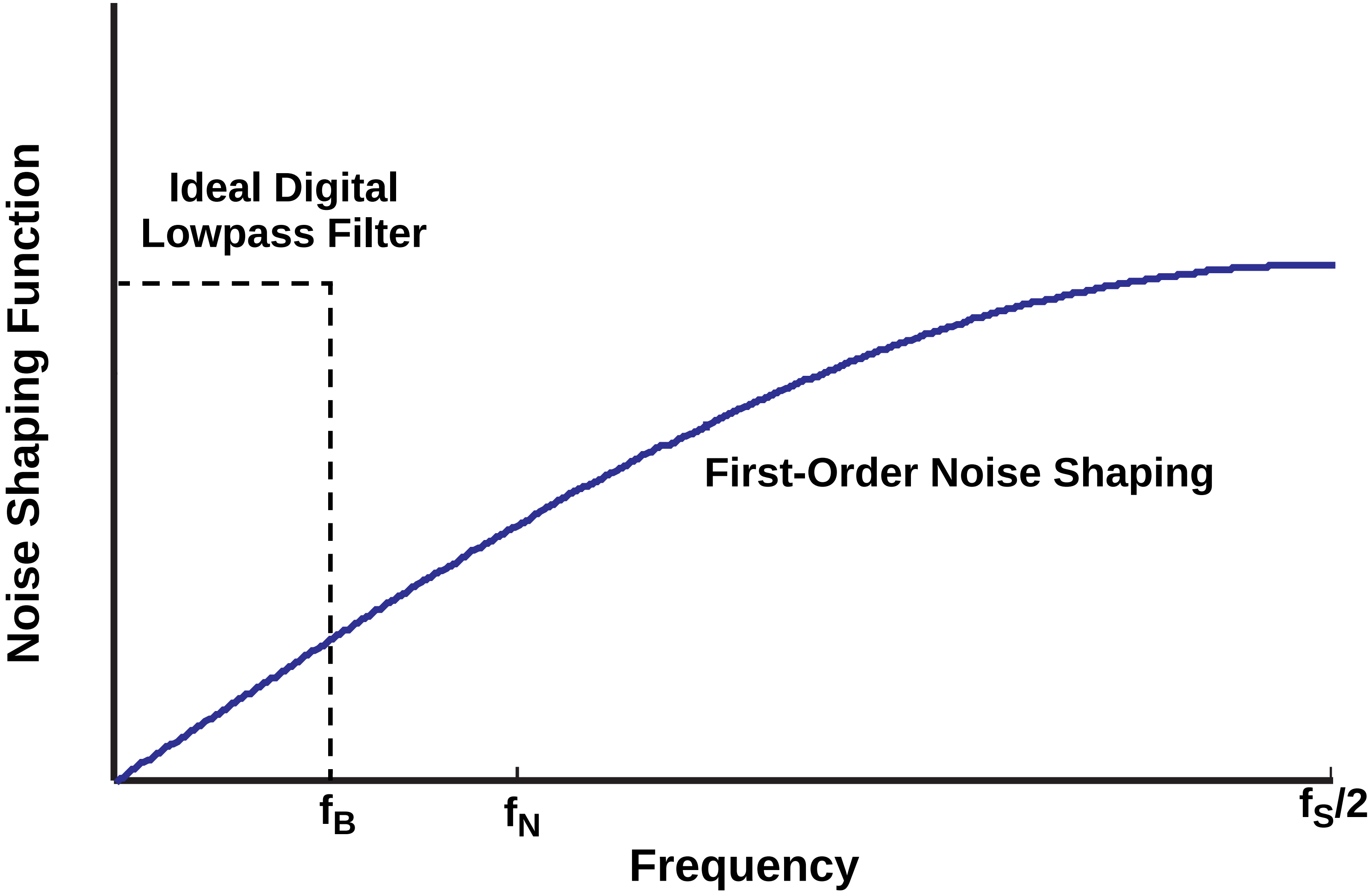


$$Y(z) = z^{-1}X(z) + (1 - z^{-1})E_Q(z)$$

ΣΔ Modulator Response (w/ 1-bit Quantization)

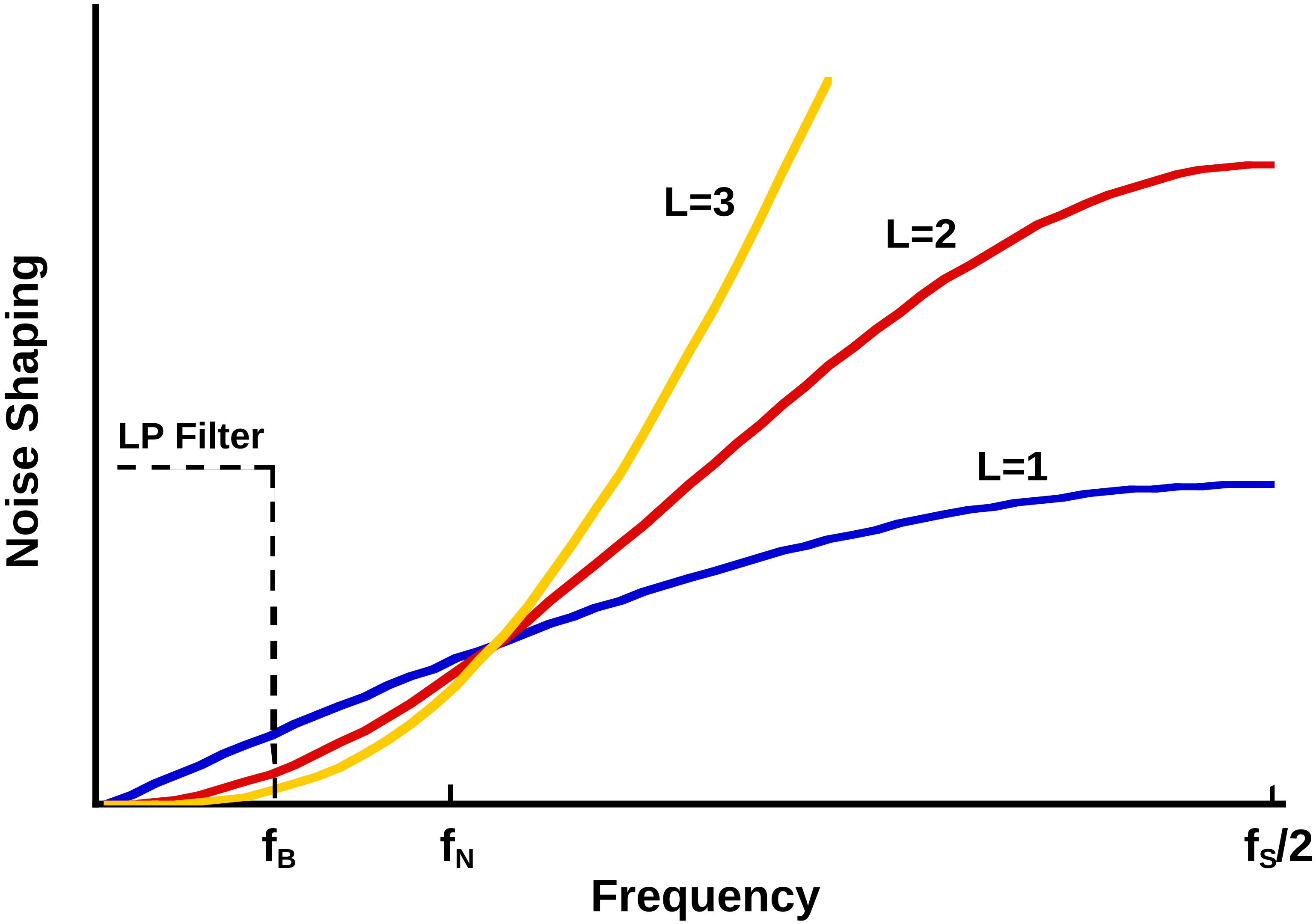


Noise Shaping

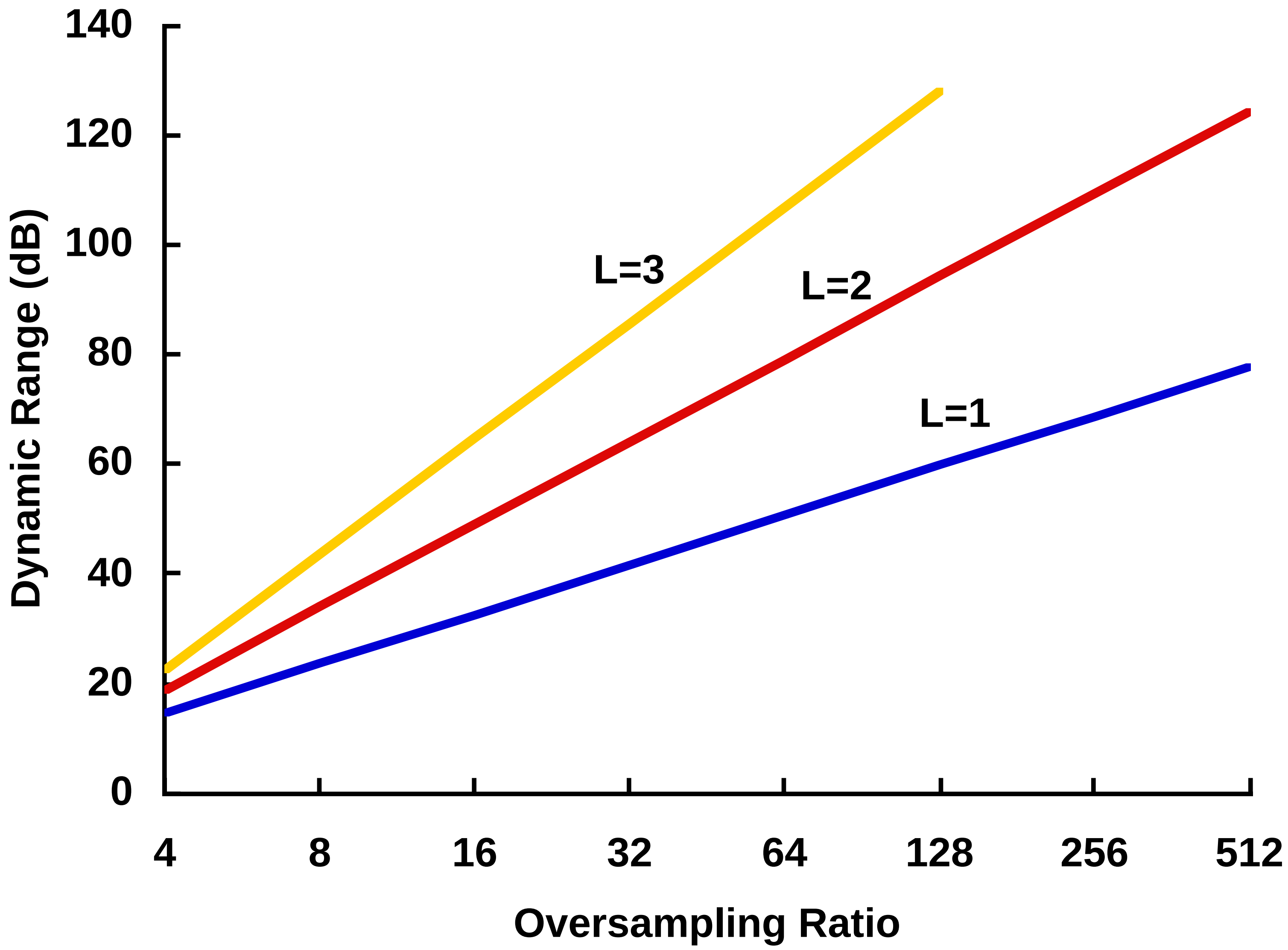


Noise Differencing Modulators (of order L)

$$Y(z) = z^{-1}X(z) + (1 - z^{-1})^{L} E_{Q}(z)$$



Sigma-Delta Modulator Dynamic Range



Higher-Order Noise Shaping Modulators

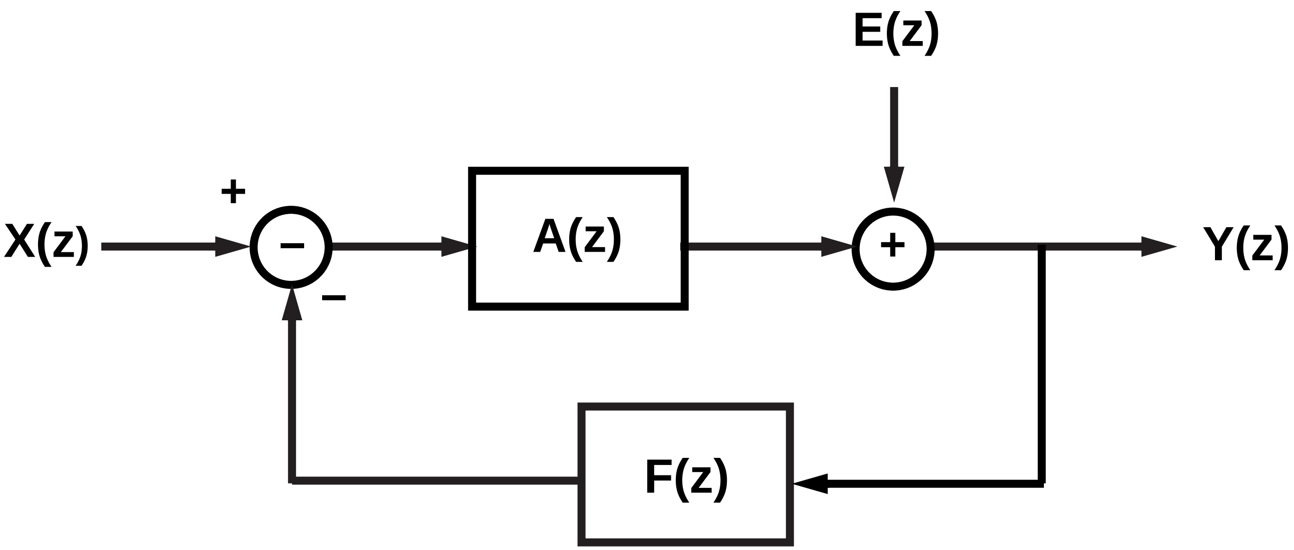
The order of the noise shaping can be increased using either

- Single quantizer modulators
  - Multi-loop noise differencing
  - Single-loop with multi-order filter

or

- Cascaded (multistage) modulators

Single-Quantizer ΣΔ Modulation



$$Y(z) = H_X(z)X(z) + H_E(z)E(z)$$

where

$$H_X(z) = \frac{A(z)}{1 + A(z)F(z)}, \qquad H_E(z) = \frac{1}{1 + A(z)F(z)}$$

and

$$A(z) = \frac{H_X(z)}{H_E(z)}, \qquad F(z) = \frac{1 - H_E(z)}{H_X(z)}$$

Noise Differencing Modulators

Class of modulators where:

$$H_X(z) = z^{-1} \text{ and } H_E(z) = (1 - z^{-1})^L$$

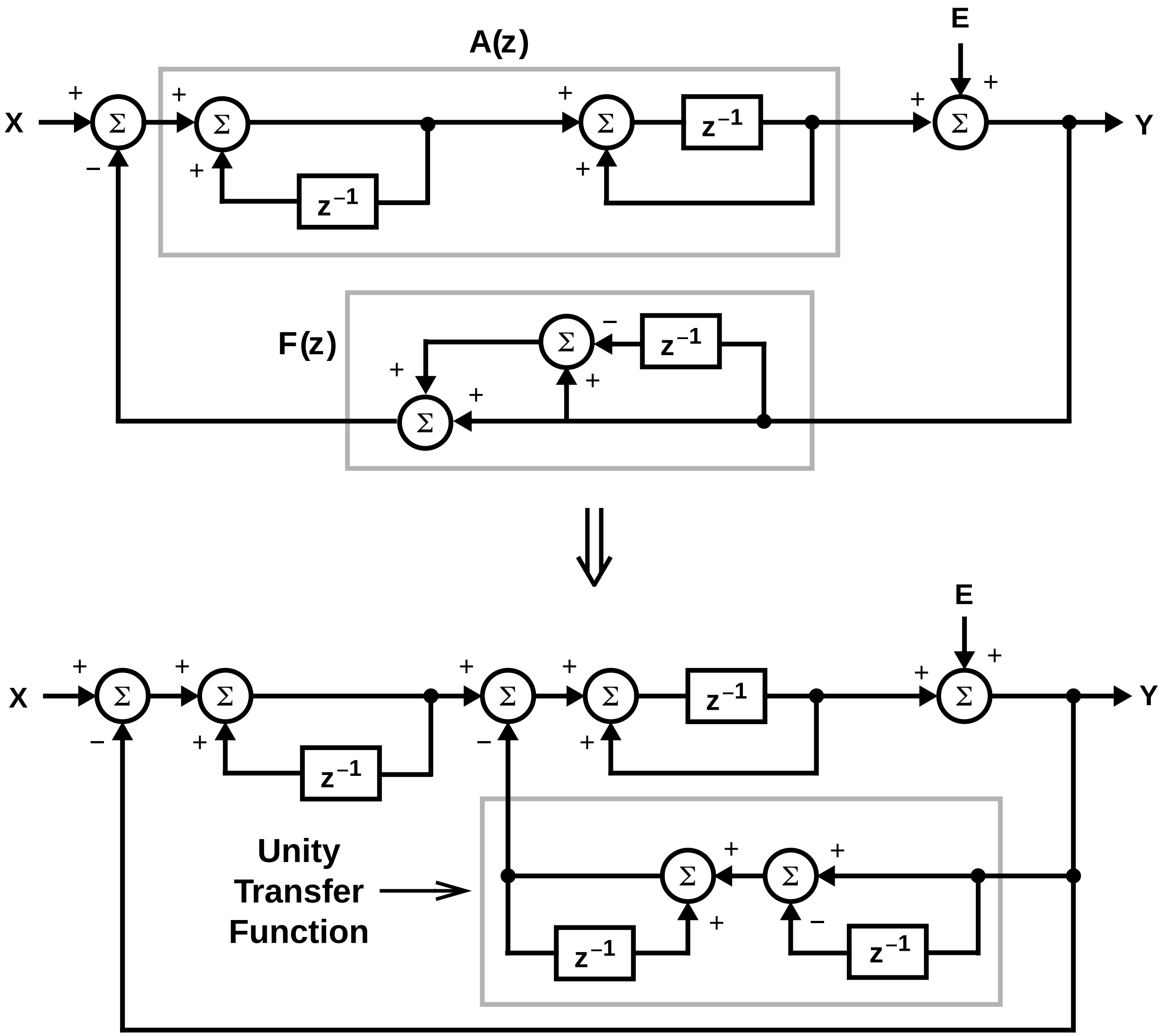
$$L = 1: \quad H_E(z) = (1 - z^{-1}) \quad \Rightarrow \quad A(z) = \frac{z^{-1}}{1 - z^{-1}}$$

$$F(z) = 1$$

$$L = 2: \quad H_E(z) = (1 - z^{-1})^2 \quad \Rightarrow \quad A(z) = \frac{z^{-1}}{(1 - z^{-1})^2}$$

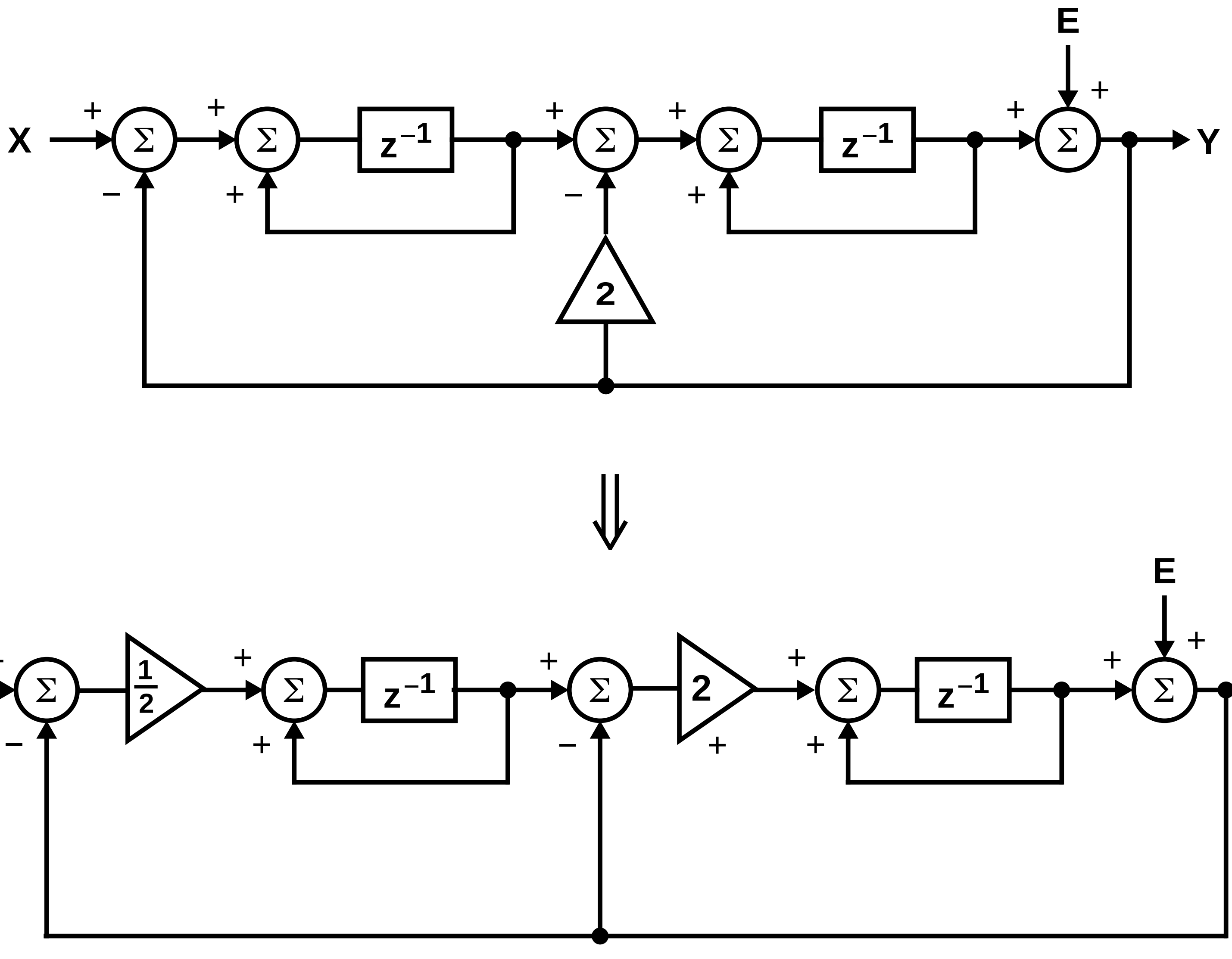
$$F(z) = 2 - z^{-1} \quad (\neq 1)$$

Second-Order Implementation



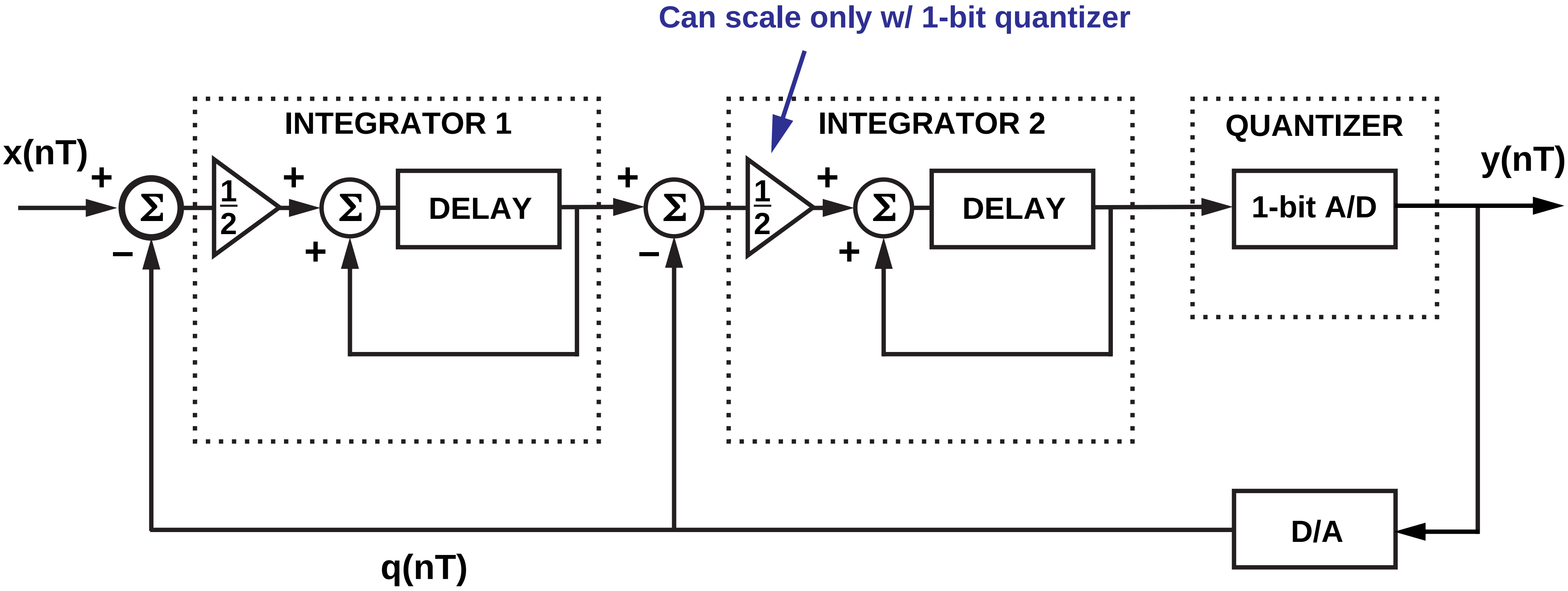
Unity Transfer Function

Second-Order ΣΔ Modulator



Second-Order Noise Differencing ΣΔ Modulator

(with 1-bit quantization)

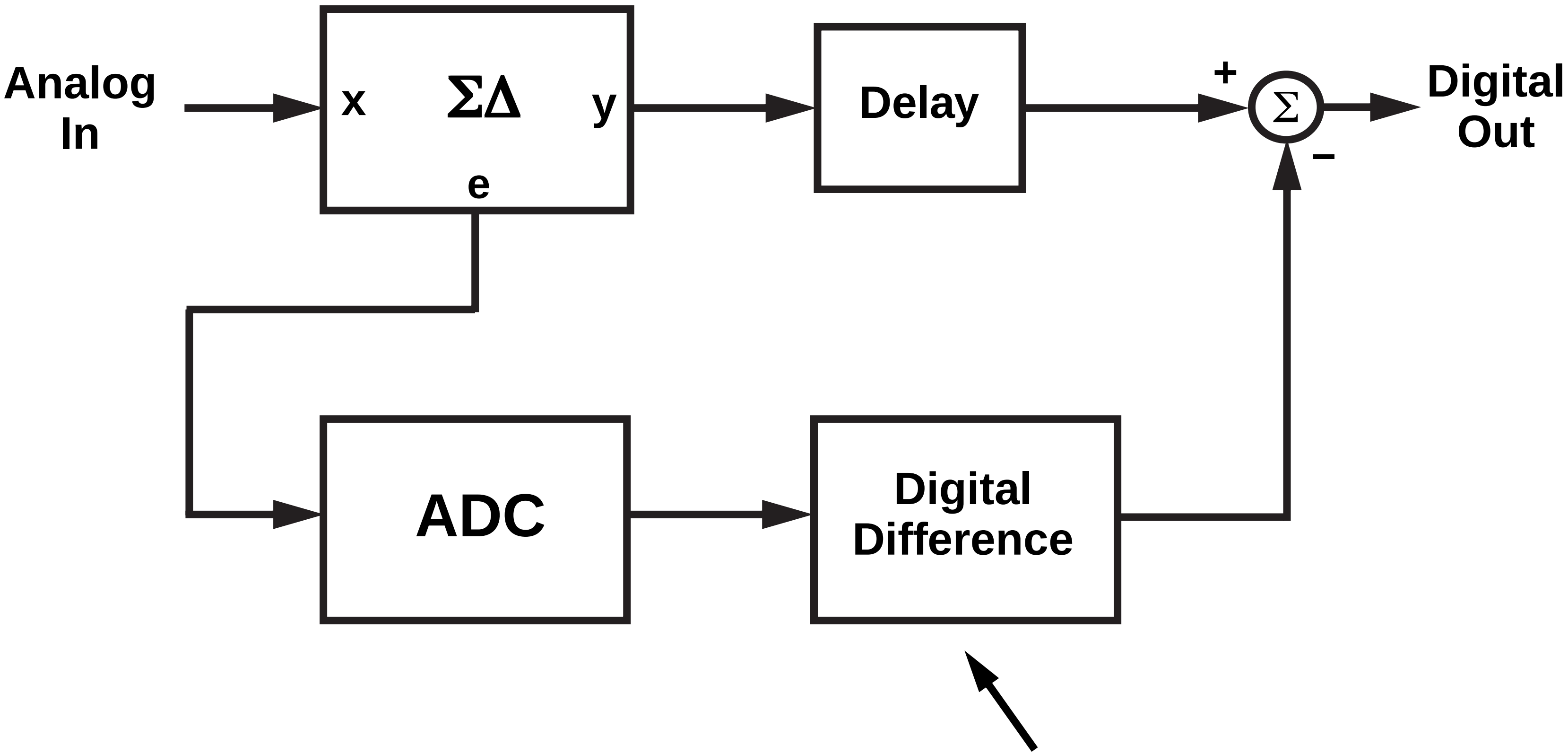

Can scale only w/ 1-bit quantizer

- Key building block for cascaded modulators

Cascaded ΣΔ Modulators

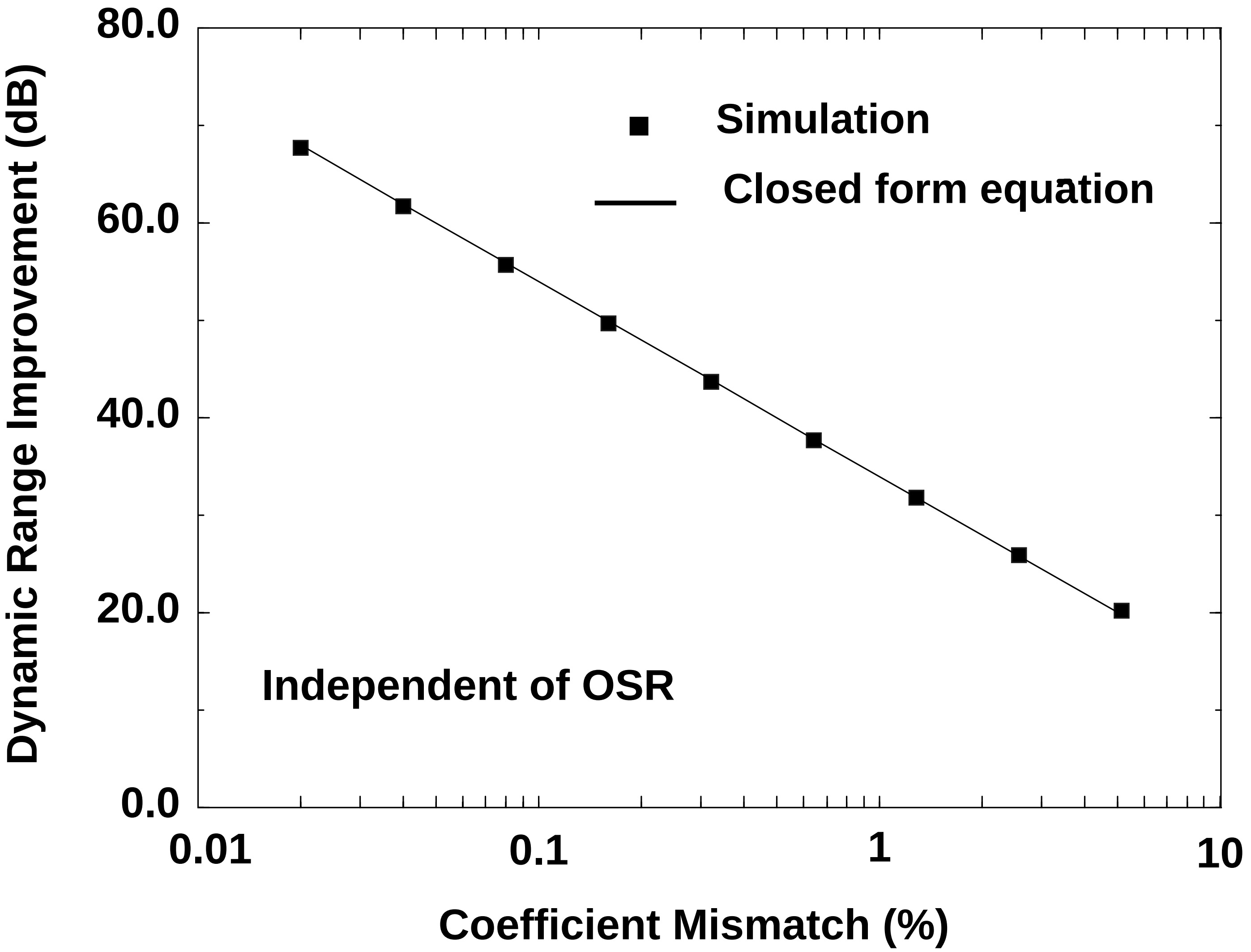
- Quantizer error quantized by subsequent stage and then filtered and subtracted digitally
- Cancellation of lower-order noise shaping terms depends on matching of analog and digital paths
- No potential instability

Cascaded Oversampling ADC

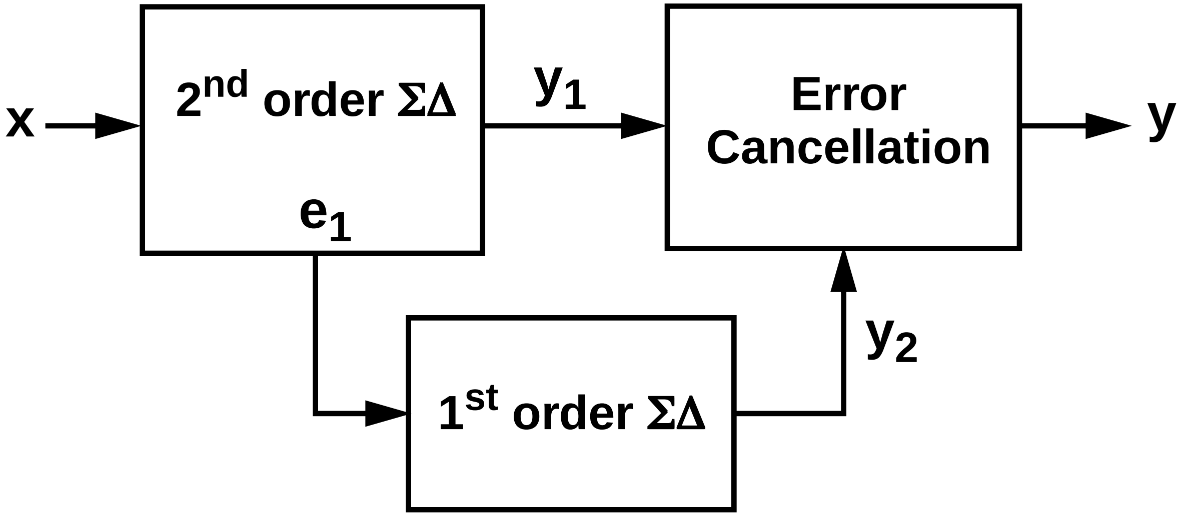


Matches noise shaping of
quantization error in first stage

Maximum Dynamic Range Improvement



Third-Order (2-1) Cascaded Modulator

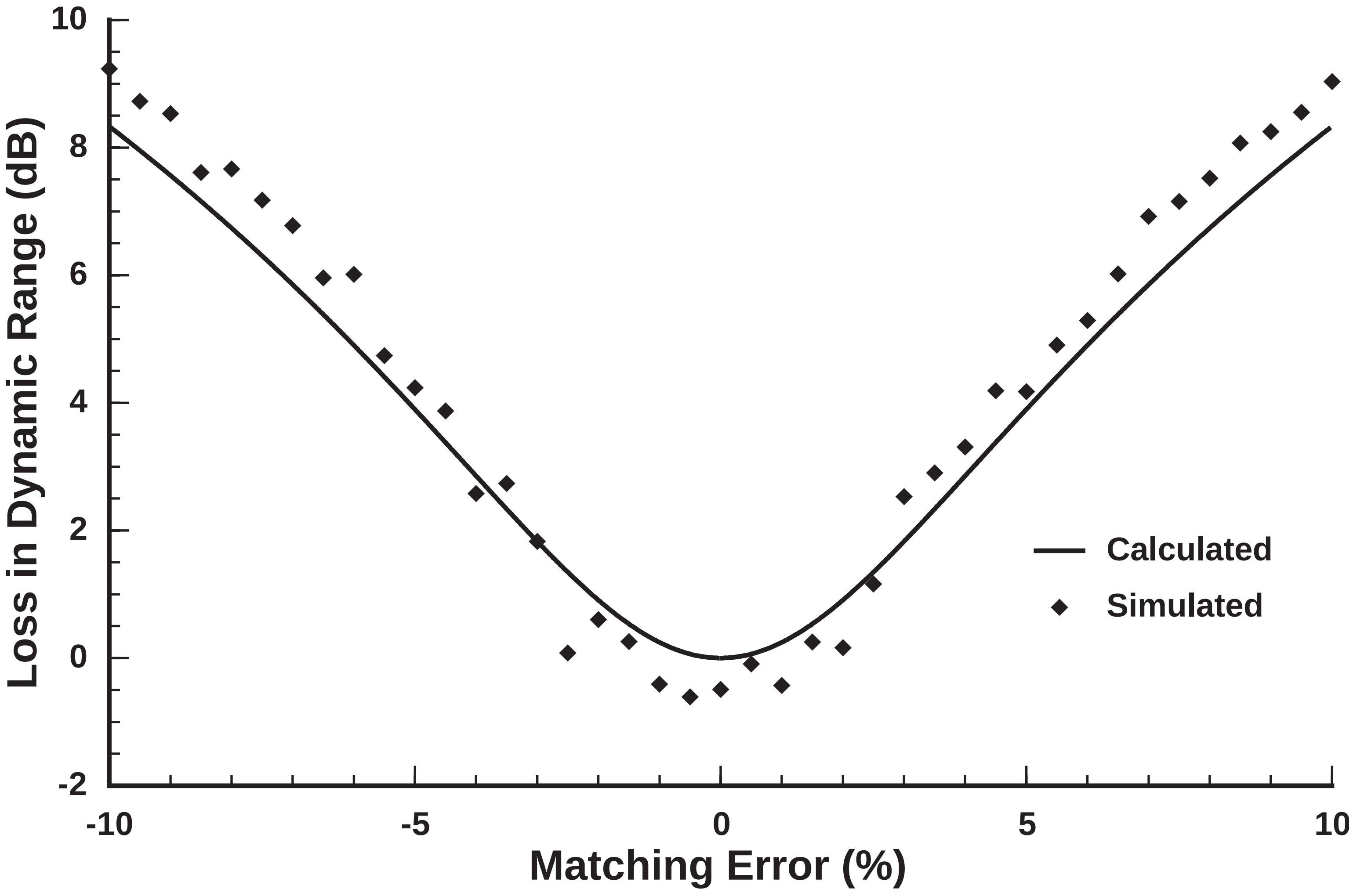


$$Y_1 = z^{-2}X + (1 - z^{-1})^2 E_1$$

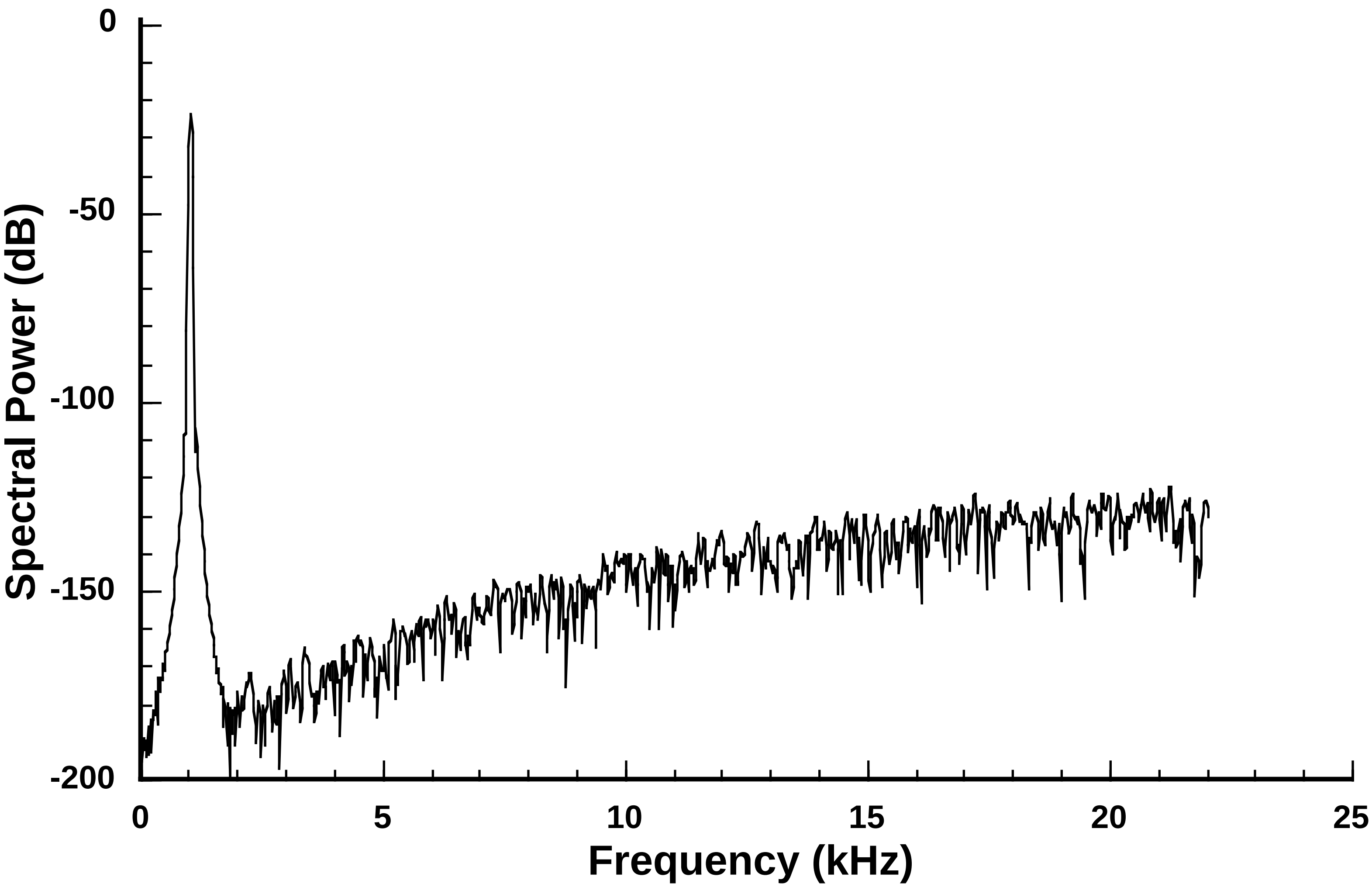
$$Y_2 = z^{-1}E_1 + (1 - z^{-1}) E_2$$

$$Y = z^{-1}Y_1 - (1 - z^{-1})^2 Y_2 = z^{-3}X - (1 - z^{-1})^3 E_2$$

Matching Error in 2-1 Cascade



2-1 Spectrum with Mismatch



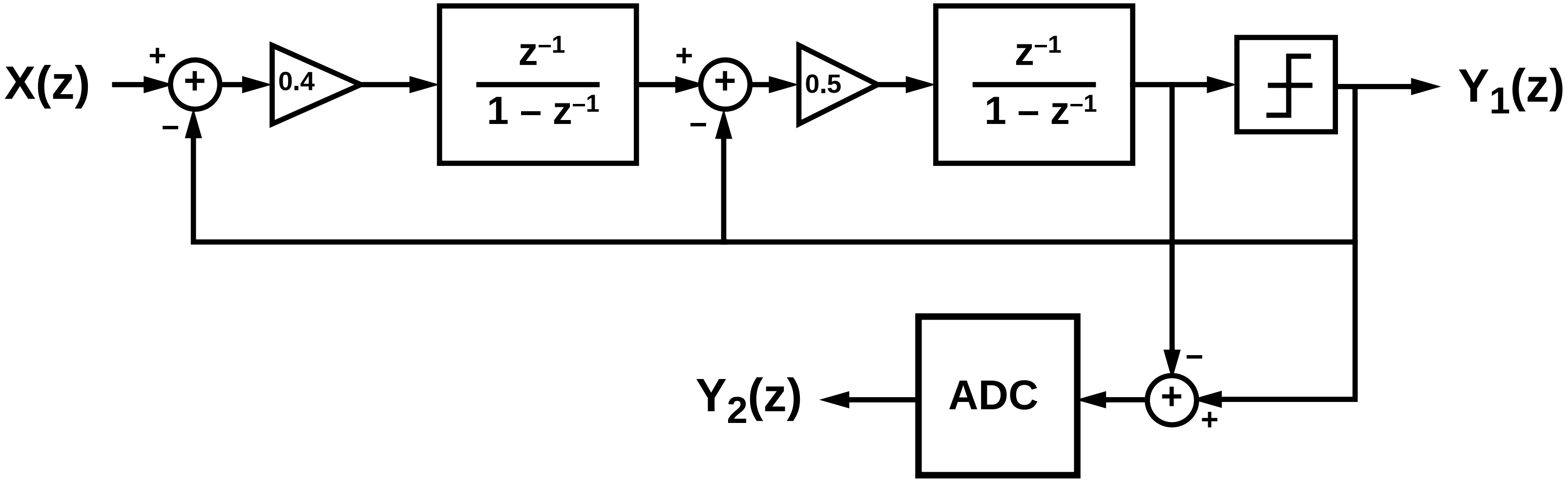
Advantages of 2-1 Cascade

- Low sensitivity to precision of analog path
- Suppression of spurious noise tones resulting from correlation of quantization noise with input
- Considerable design flexibility
- No potential instability

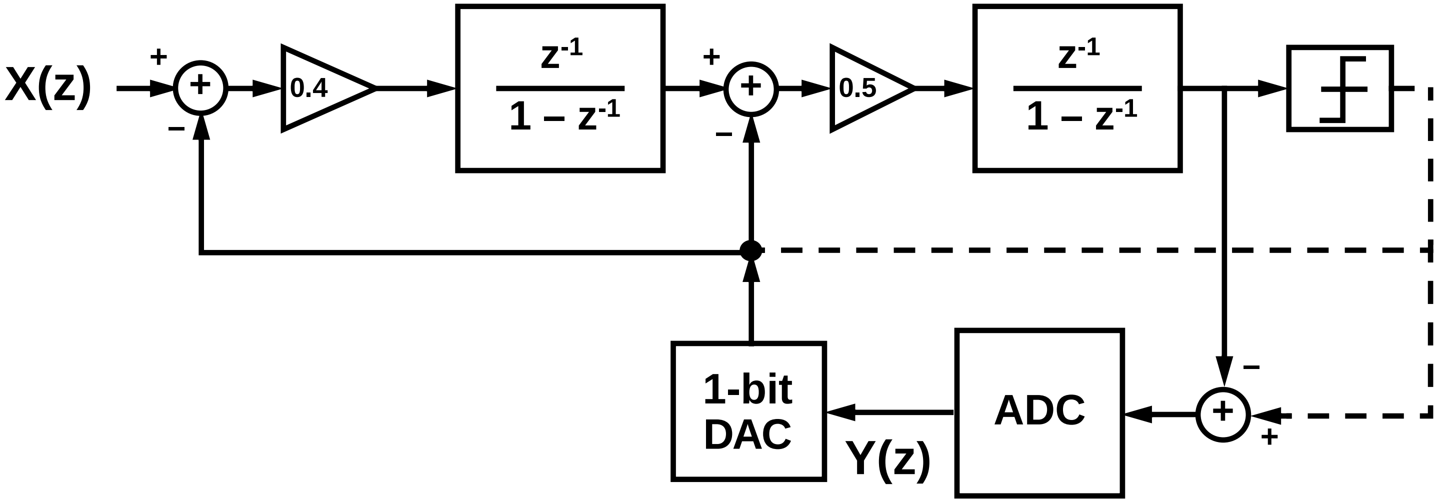
Digitizing MHz-Bandwidth Signals

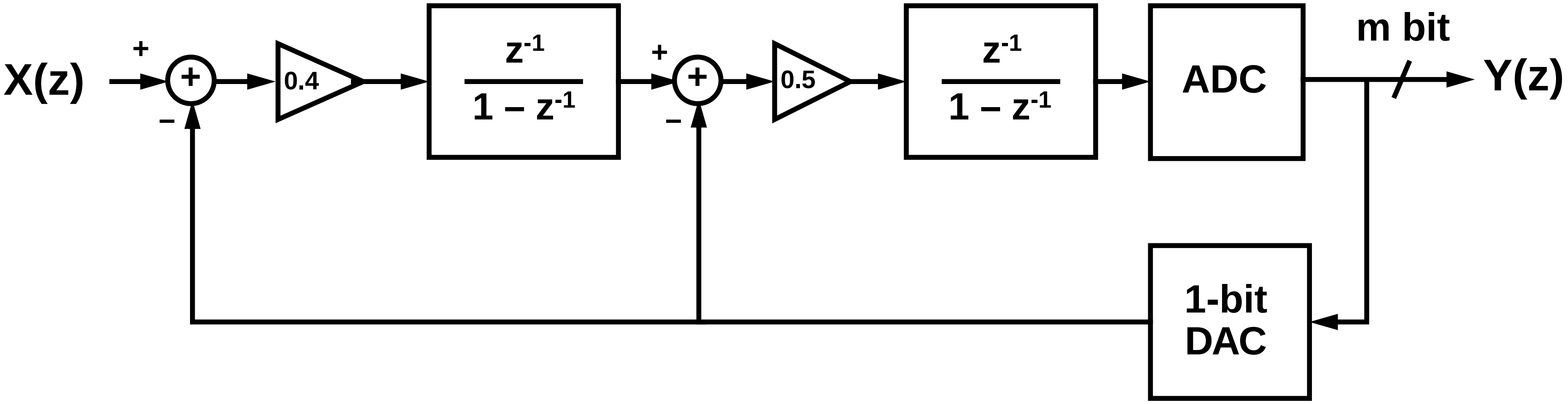
- Oversampling modulators dominate precision A/D conversion at modest signal bandwidths
  - e.g. digital audio
  - exploit large oversampling ratios
- Challenge is to digitize signals where the oversampling ratio is constrained by technology (MHz bandwidths)
- Possible approaches based on cascade architectures include
  - multibit quantization in second stage
  - merged quantization
  - distributed noise shaping zeros
  - multilevel quantization with digital linearization

Lowpass Cascade Architecture



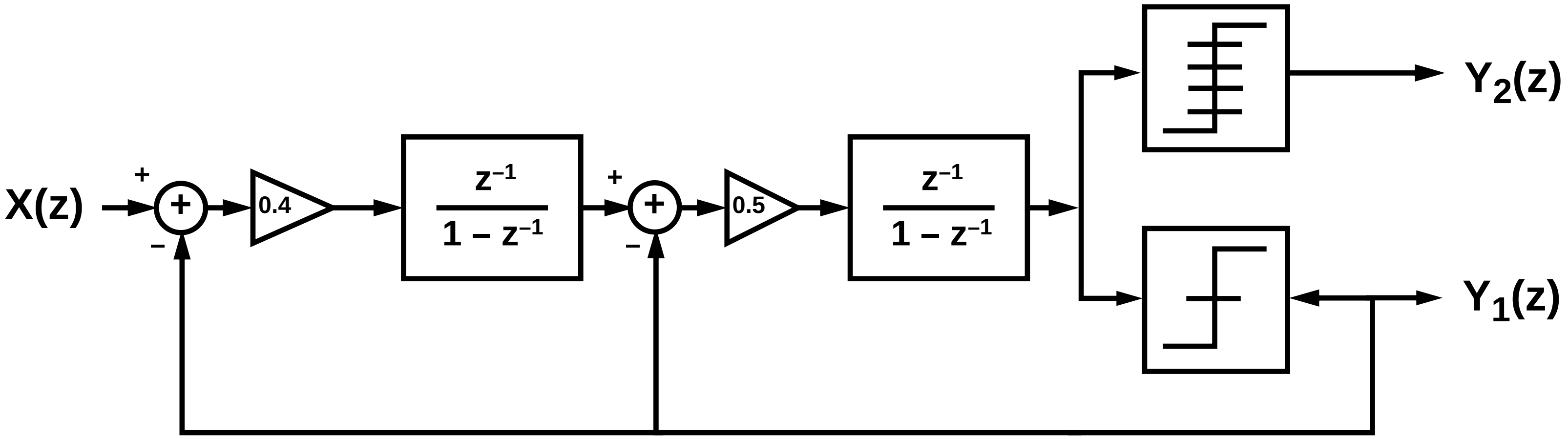
Embedded Quantization *



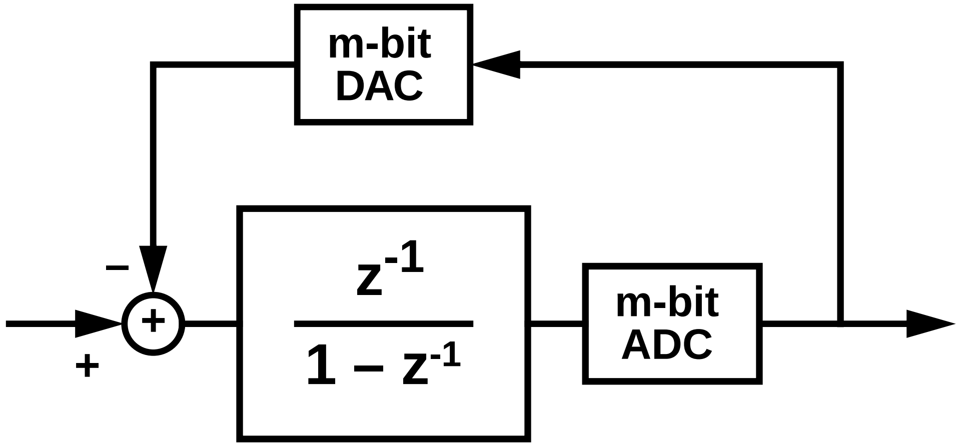


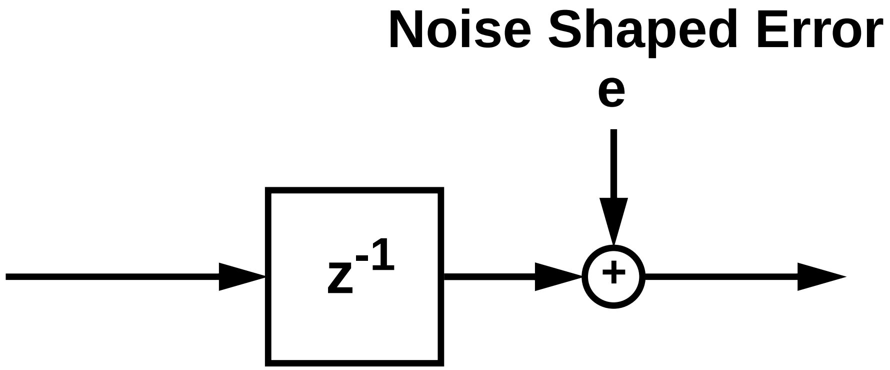
* A. Tabatabaei, 99 VLSI Ckt Symp

Model of Merged Quantizer Modulator

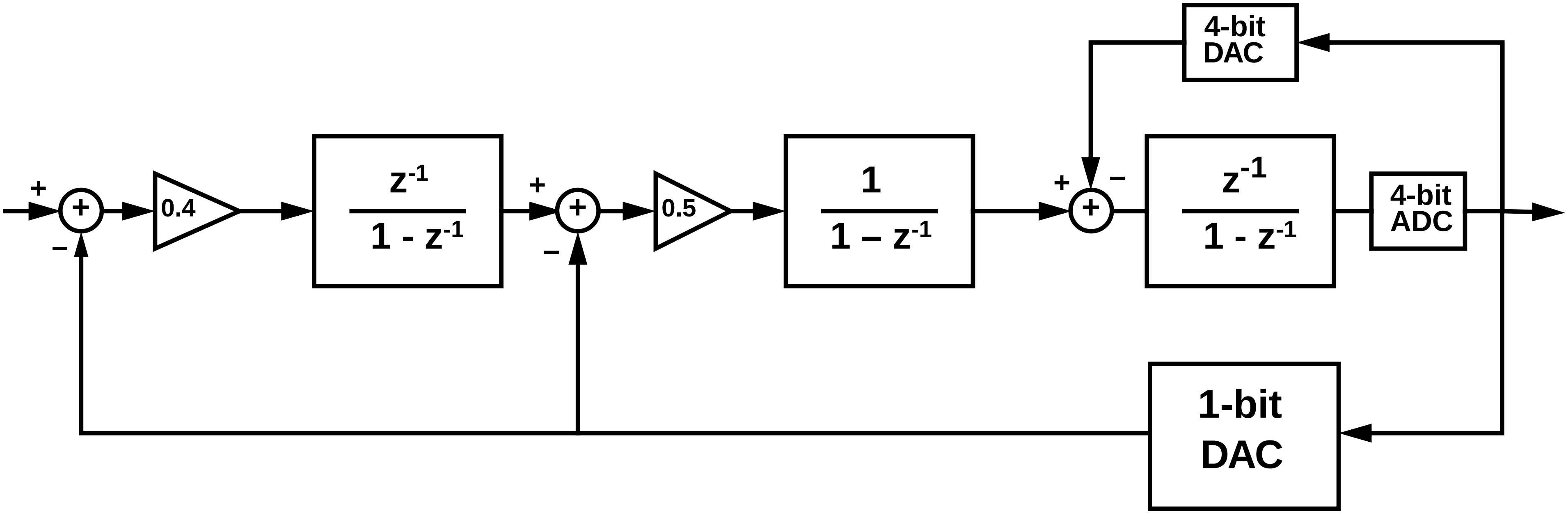


Implementation of Embedded ADC



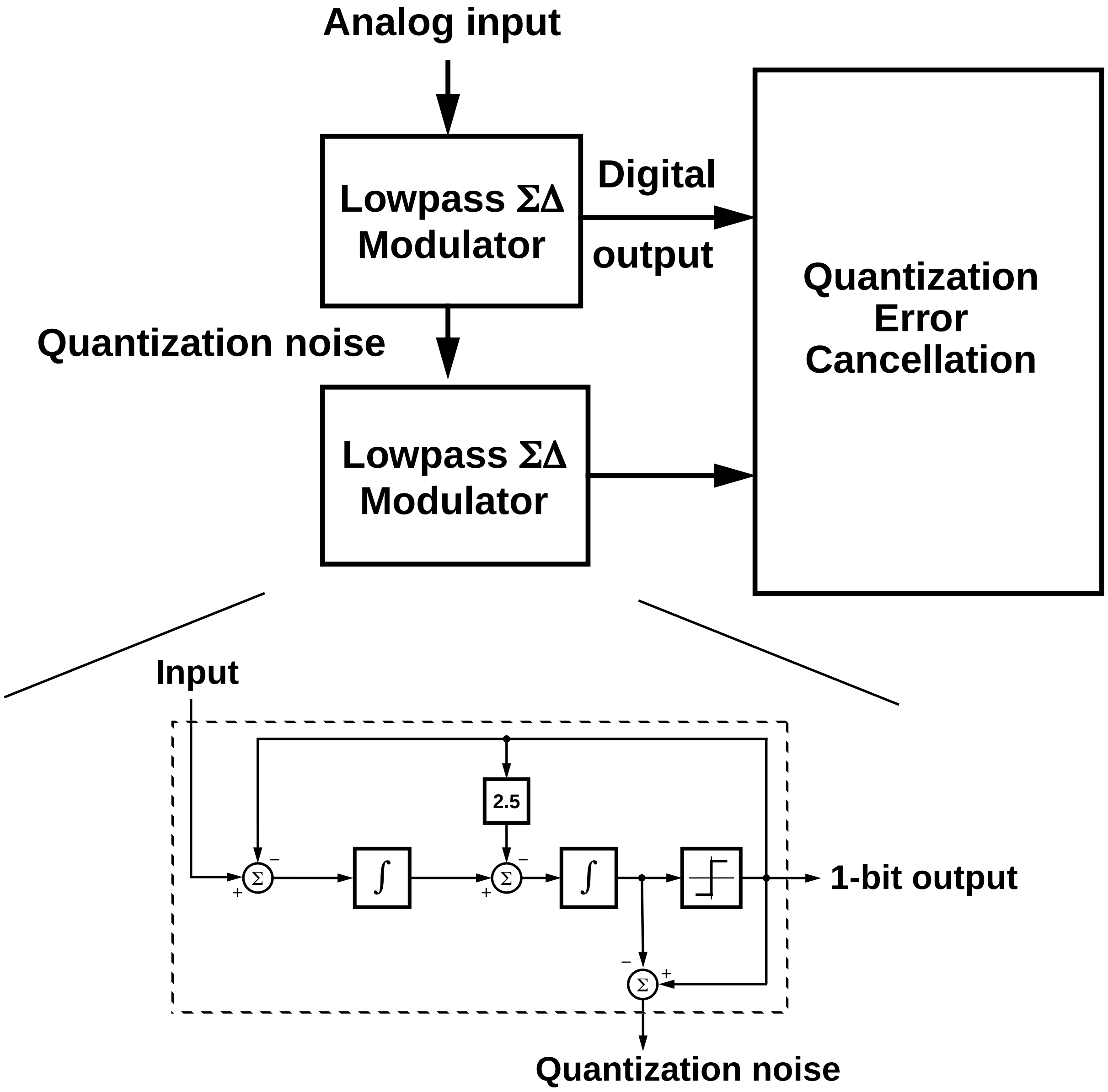


Lowpass Modulator with Embedded ADC *



- Delay unaltered
- Stability maintained by 4-bit quantization

2-2 Cascaded Modulator



Analog input

Lowpass ΣΔ Modulator

Digital output

Quantization noise

Lowpass ΣΔ Modulator

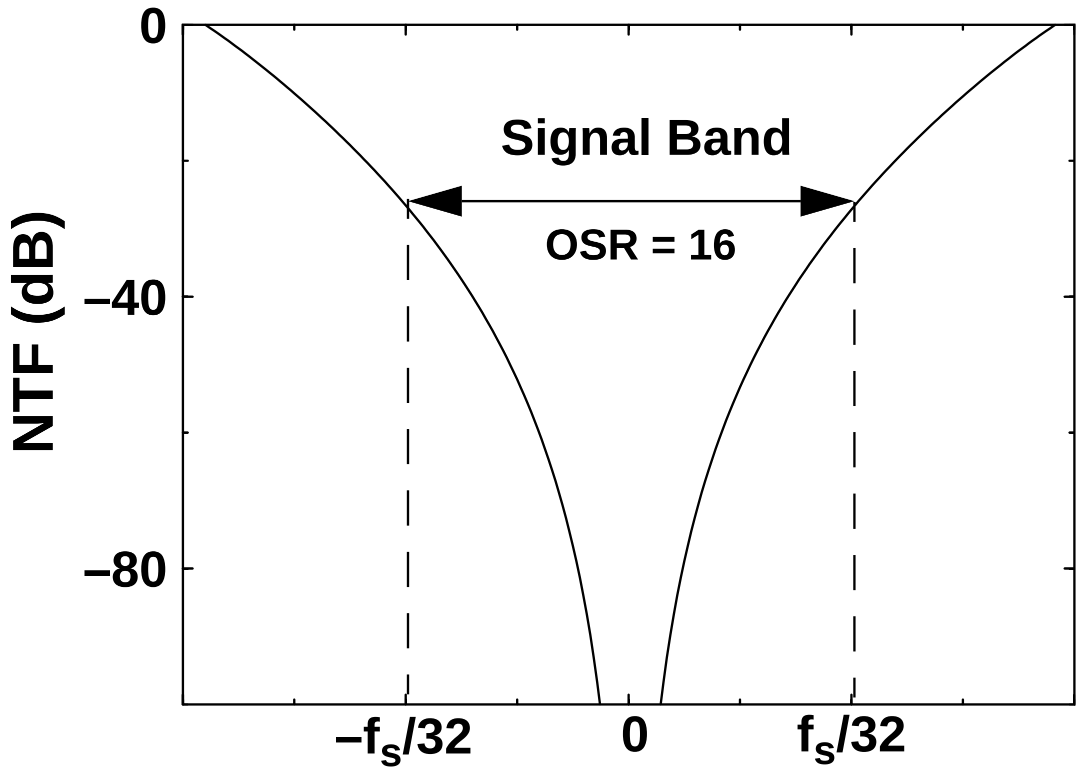
Quantization Error Cancellation

Input

1-bit output

Quantization noise

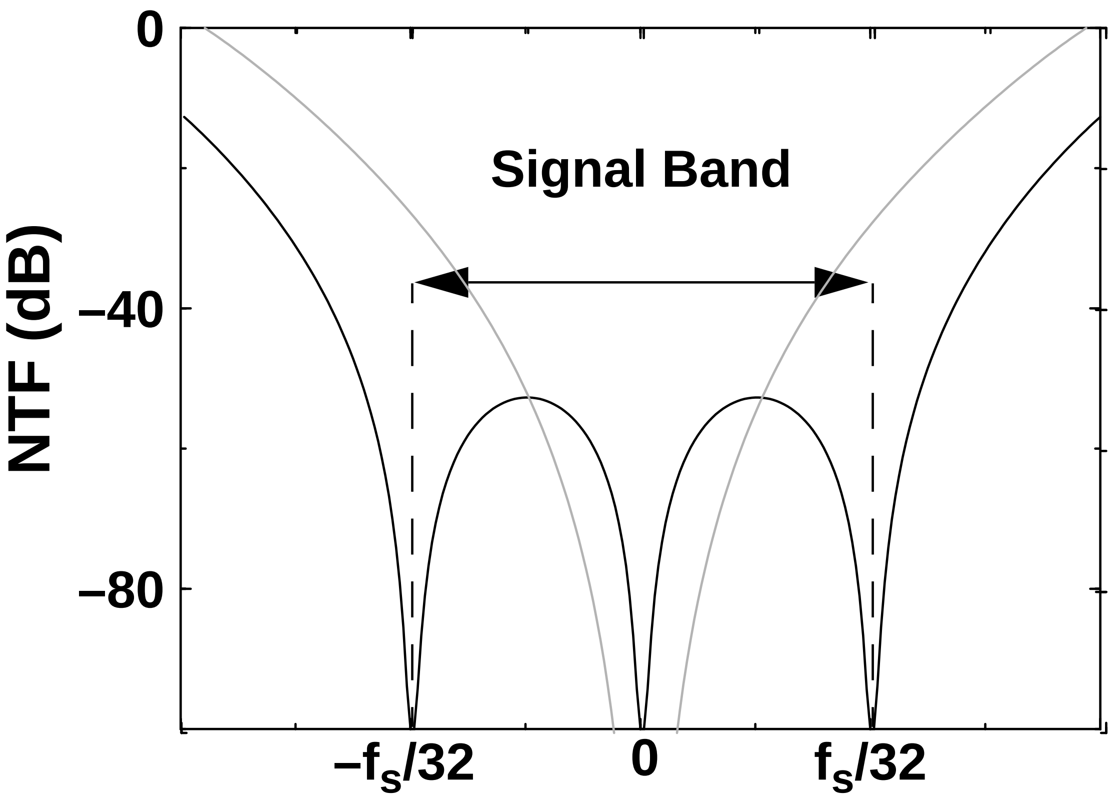
Noise Transfer Function for 2-2 Lowpass Cascade



NTF for a two-stage lowpass cascade (2-2):

- $\text{NTF} = N_1 \times N_2 = (N_{LP})^2$
- $|\text{NTF}| > 0$ dB for OSR < 8

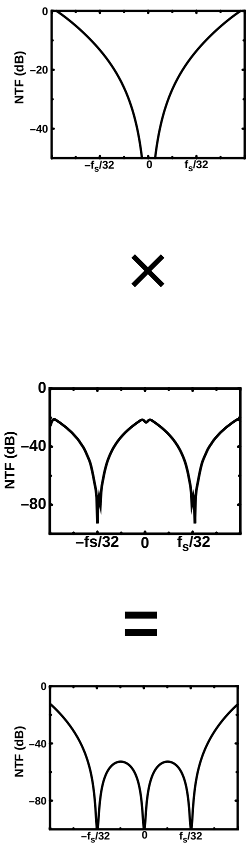
Distribute Zeros Across the Signal Band

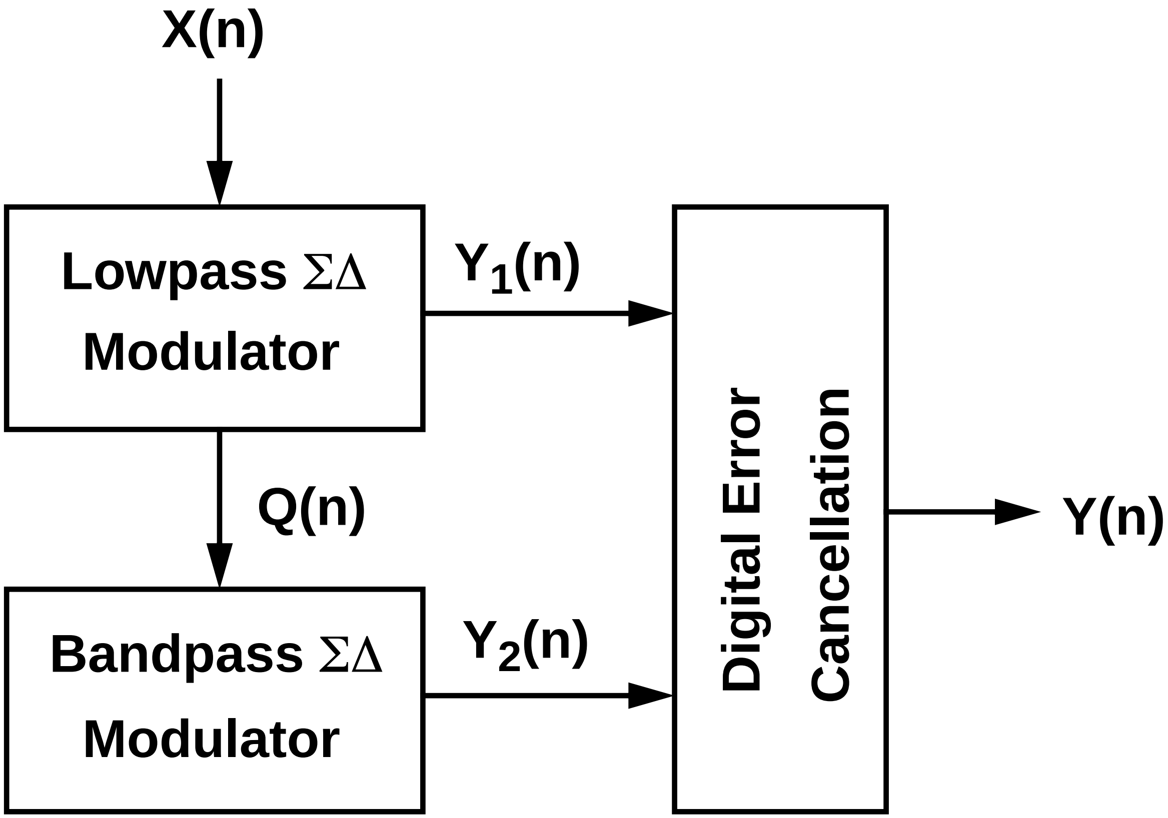


Cascade lowpass and bandpass stages:

- **Reduces inband noise**
- **NTF = $N_1 \times N_2 = N_{LP} \times N_{BP}$**

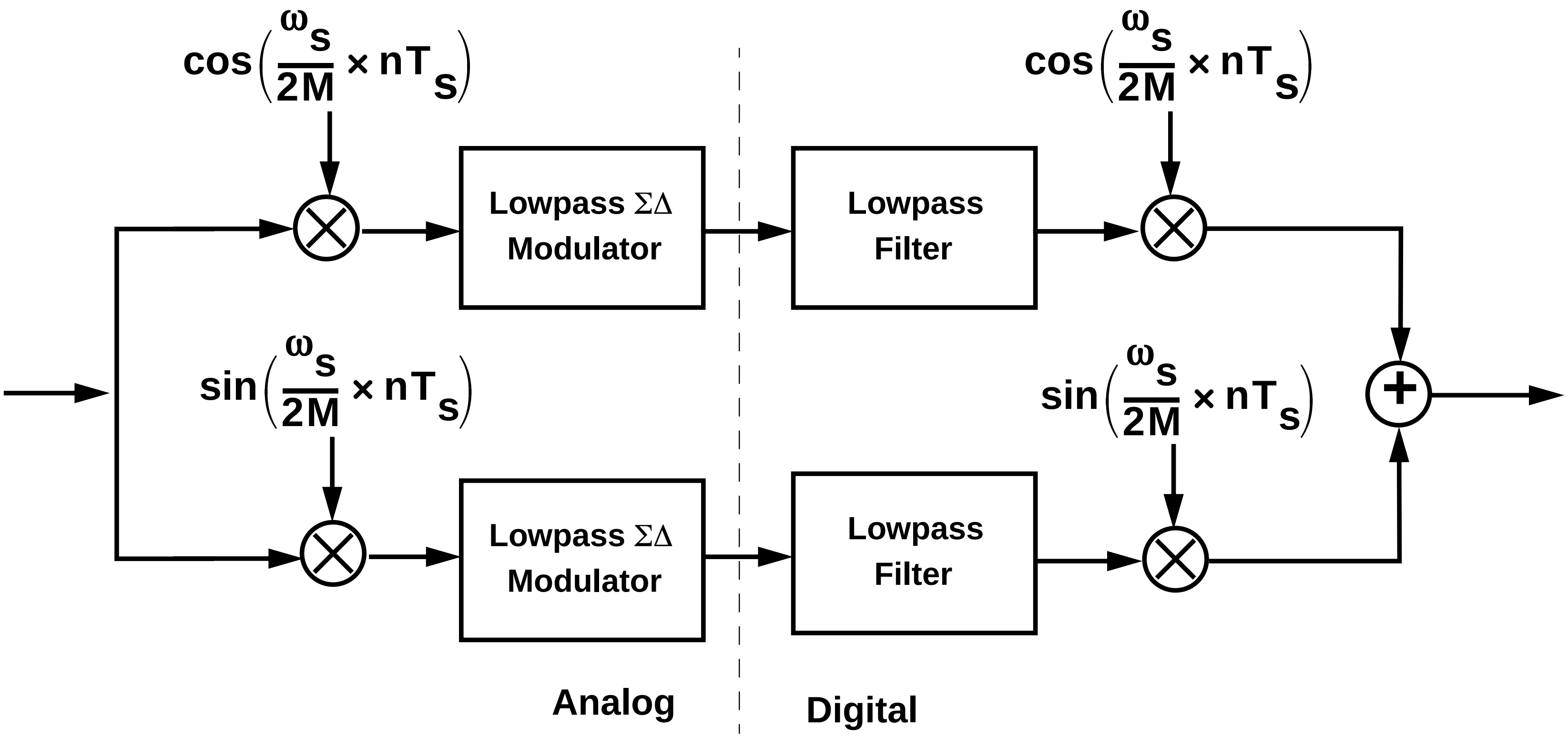
Lowpass-Bandpass Cascaded Modulator *





* A. Tabatabaei, JSSC 12/00

Bandpass Second Stage



$\cos\left(\frac{\omega_s}{2M} \times nT_s\right)$

$\sin\left(\frac{\omega_s}{2M} \times nT_s\right)$

Lowpass ΣΔ Modulator

Lowpass ΣΔ Modulator

Lowpass Filter

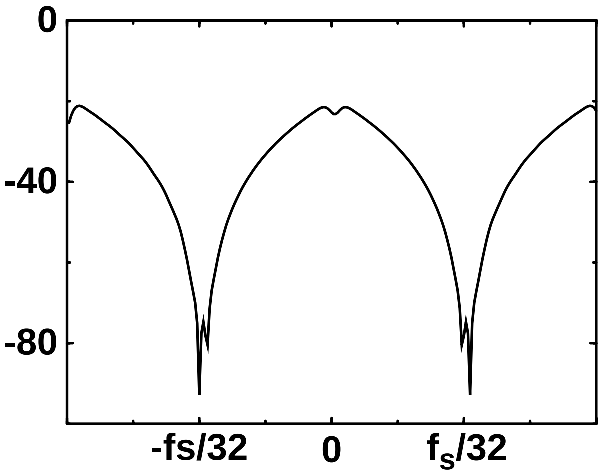
Lowpass Filter

$\cos\left(\frac{\omega_s}{2M} \times nT_s\right)$

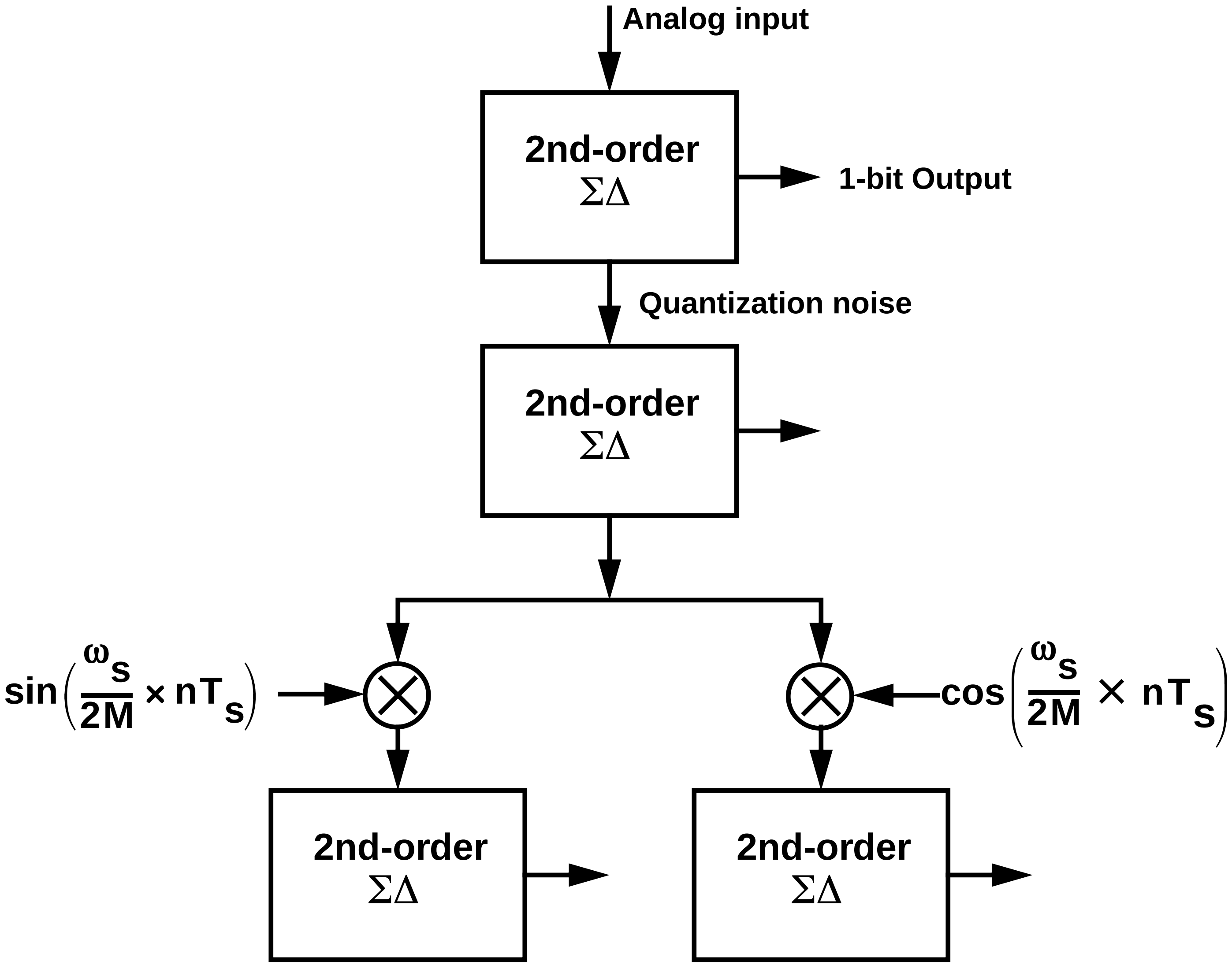
$\sin\left(\frac{\omega_s}{2M} \times nT_s\right)$

Analog

Digital



2-2-2/2 Lowpass Modulator



Analog input

2nd-order ΣΔ

1-bit Output

Quantization noise

2nd-order ΣΔ

$\sin\left(\frac{\omega_s}{2M} \times nT_s\right)$

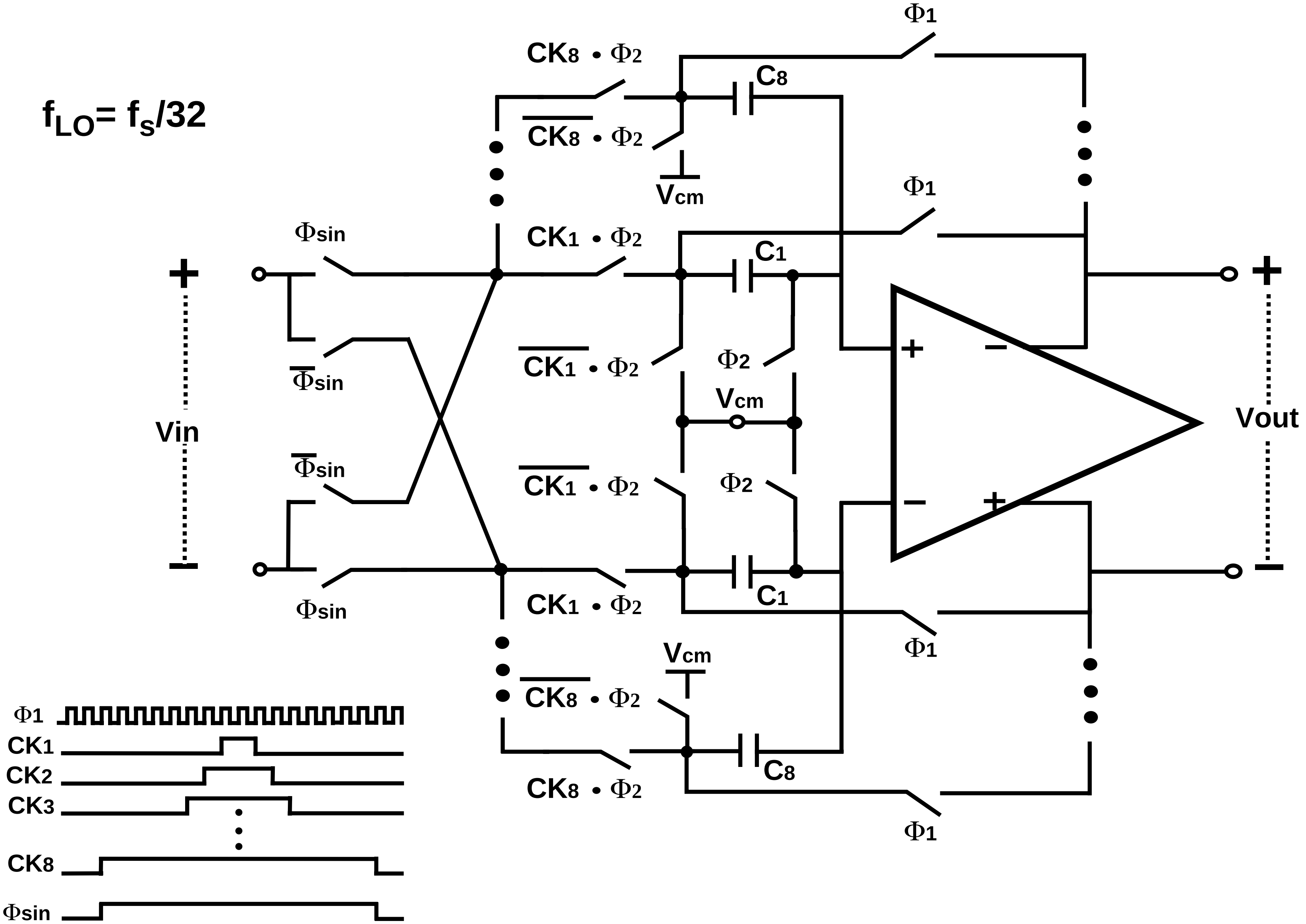
$\cos\left(\frac{\omega_s}{2M} \times nT_s\right)$

2nd-order ΣΔ

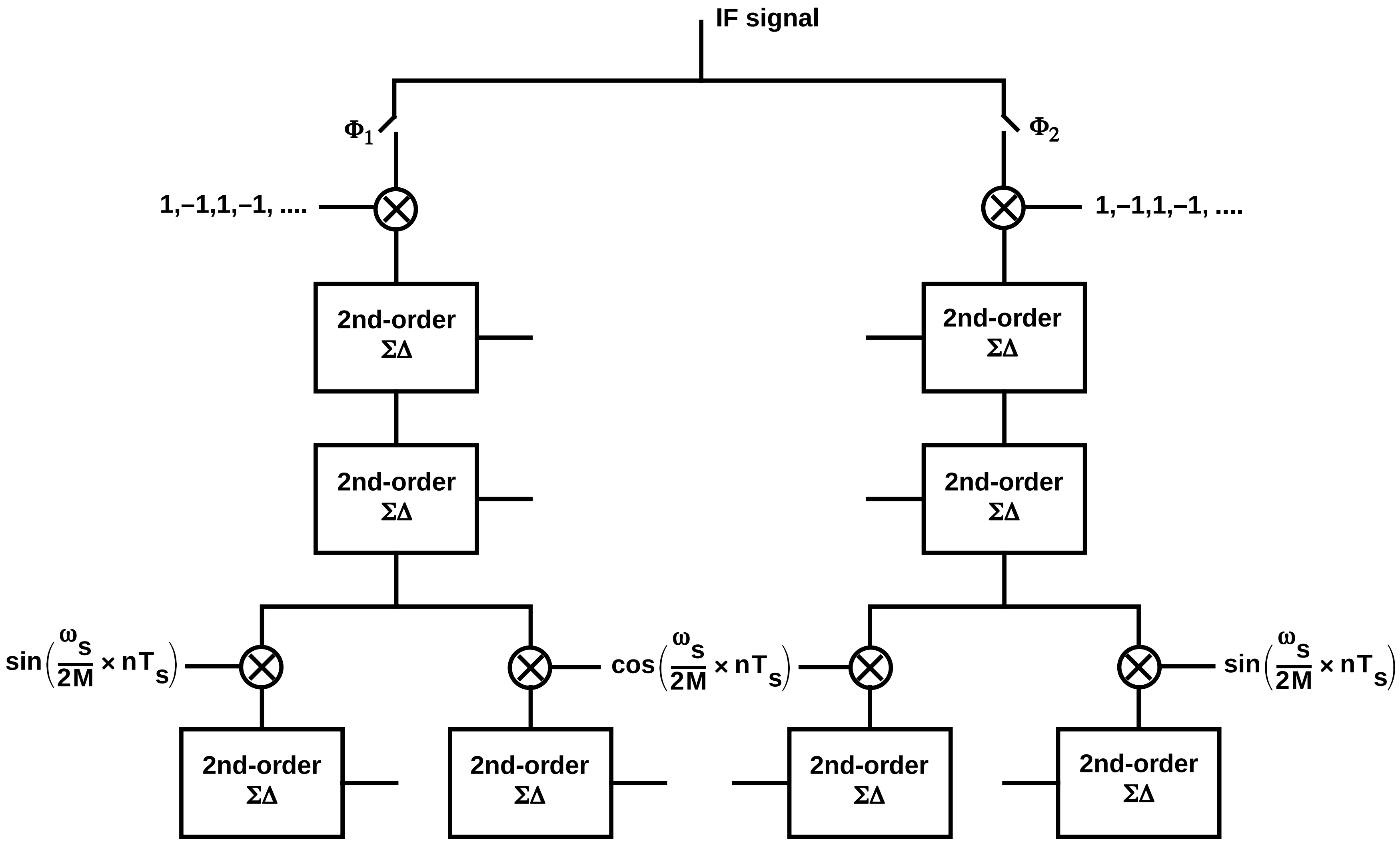
2nd-order ΣΔ

Bandpass Stage Mixer

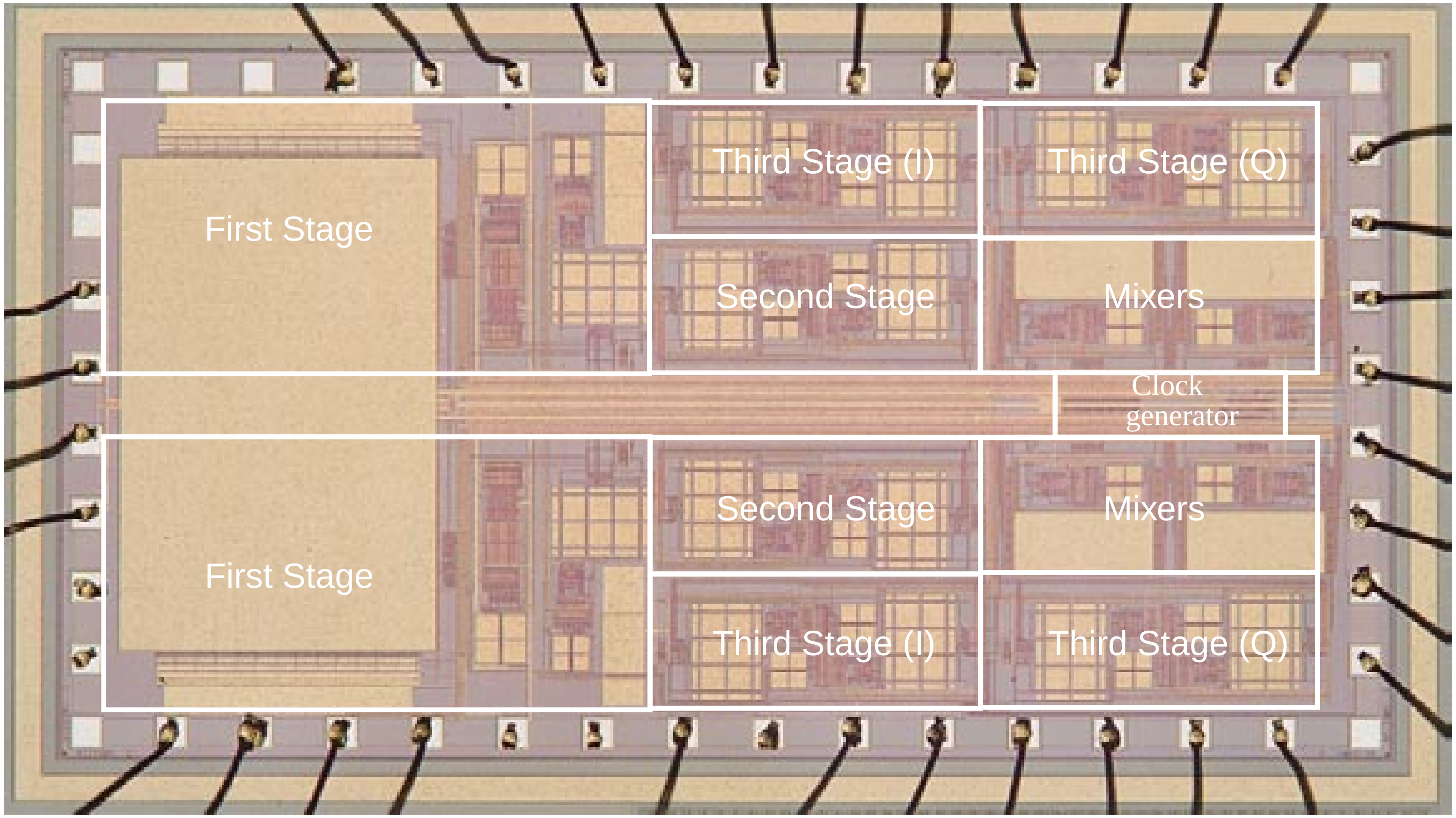
$f_{LO} = f_s/32$



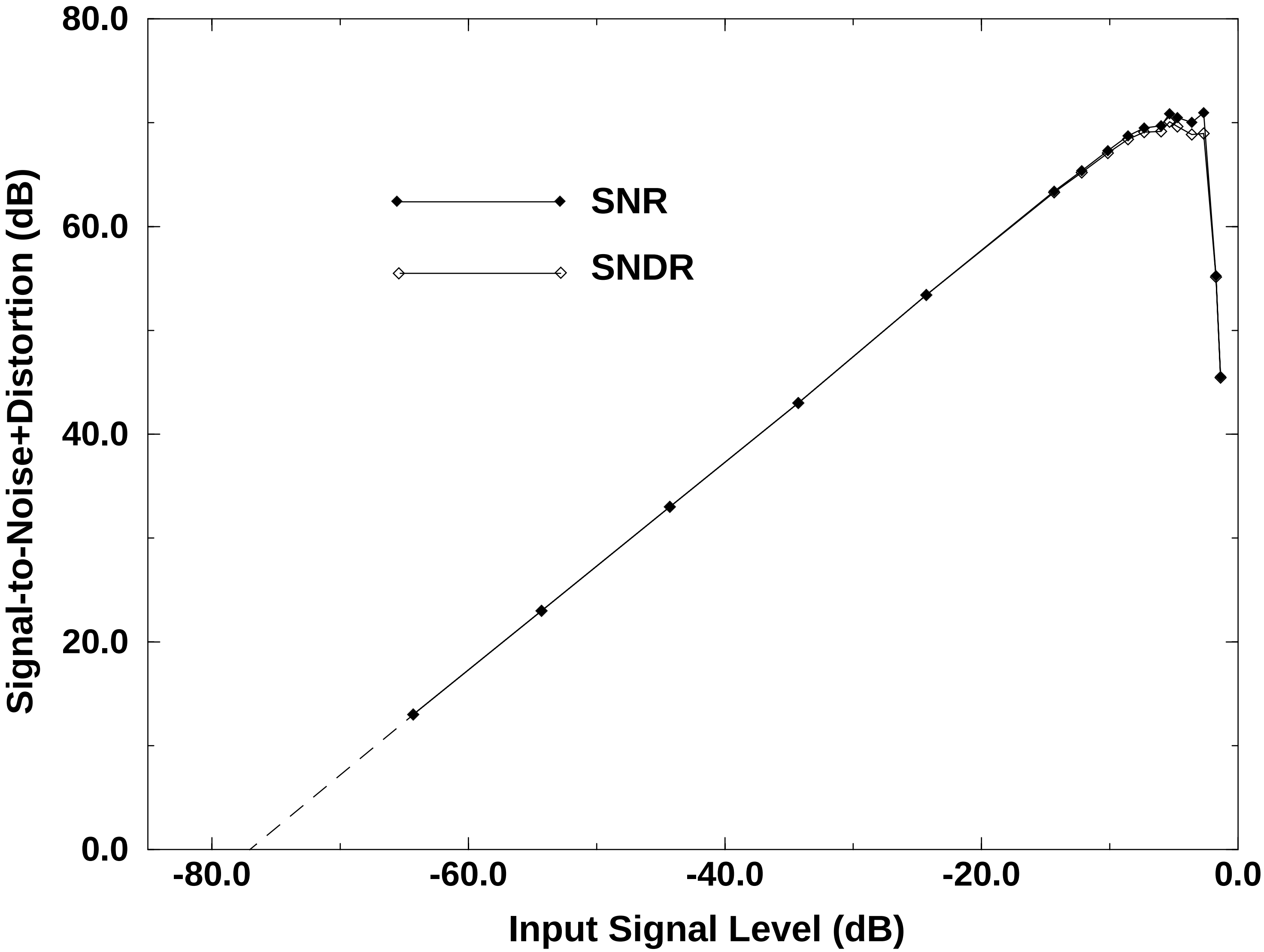
$f_s/4$ Two-Path Bandpass Modulator



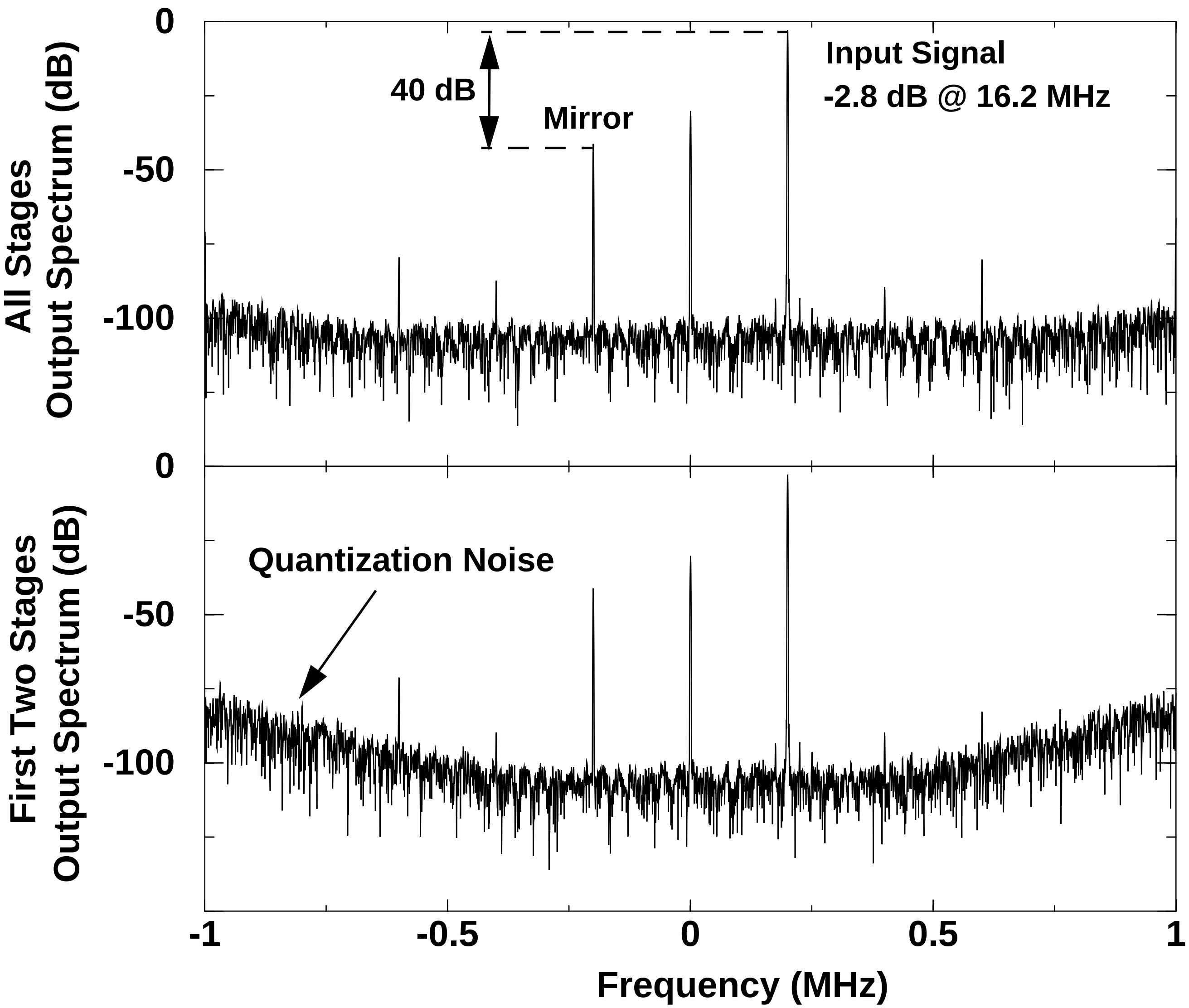
Chip Micrograph



Measured SNR and SNDR



Measured Output Spectrum



Performance Summary

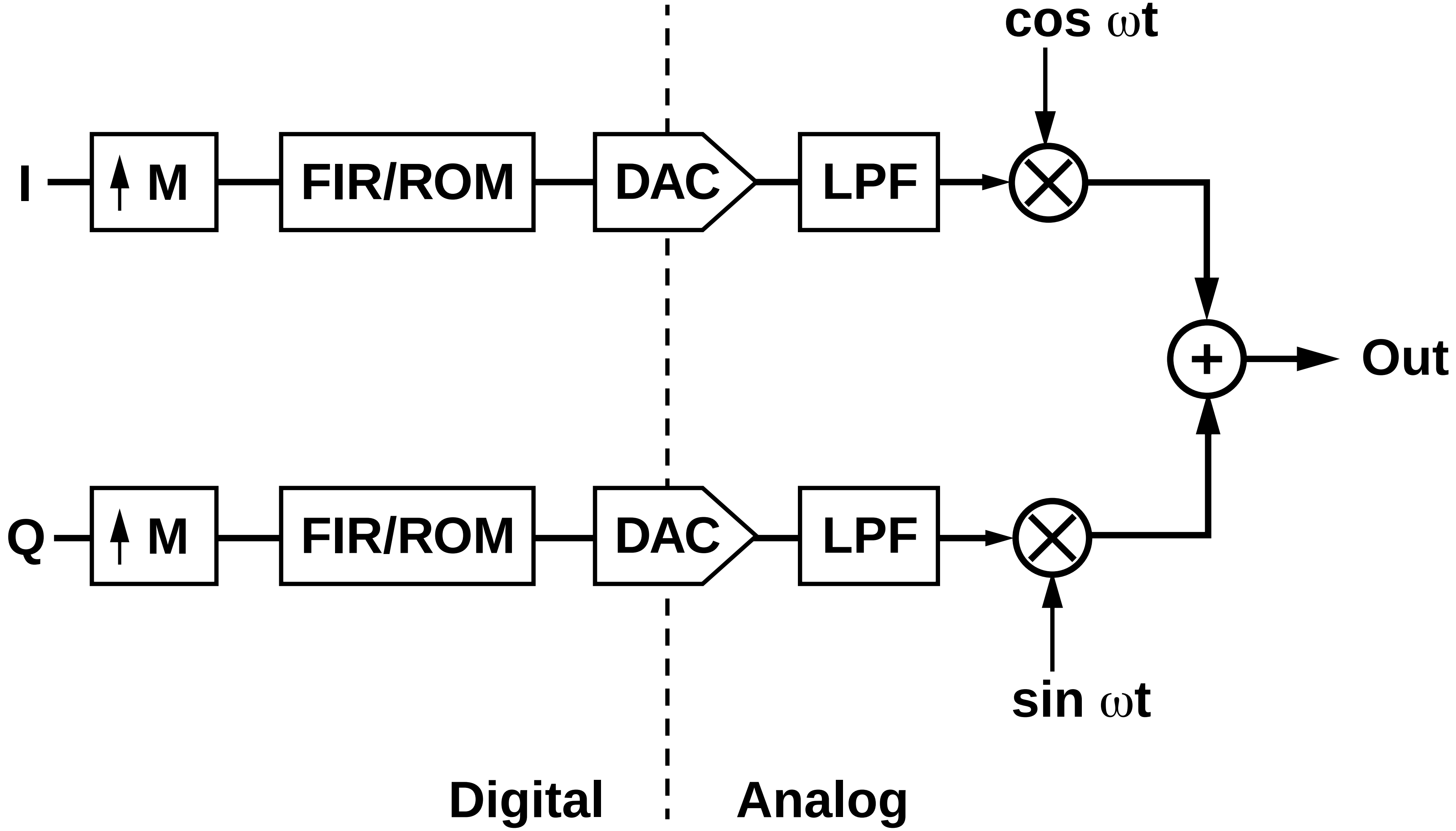
Sampling rate	64 MS/s
Passband	2 MHz centered at 16 MHz
Oversampling ratio	16
Dynamic range	75 dB
Max SNDR	70 dB
Power supply	2.5 V
Power dissipation	140 mW
Technology	0.24-μm, 5-metal CMOS
Area	2 mm × 1.7 mm

Bandpass Oversampling D/A Conversion *

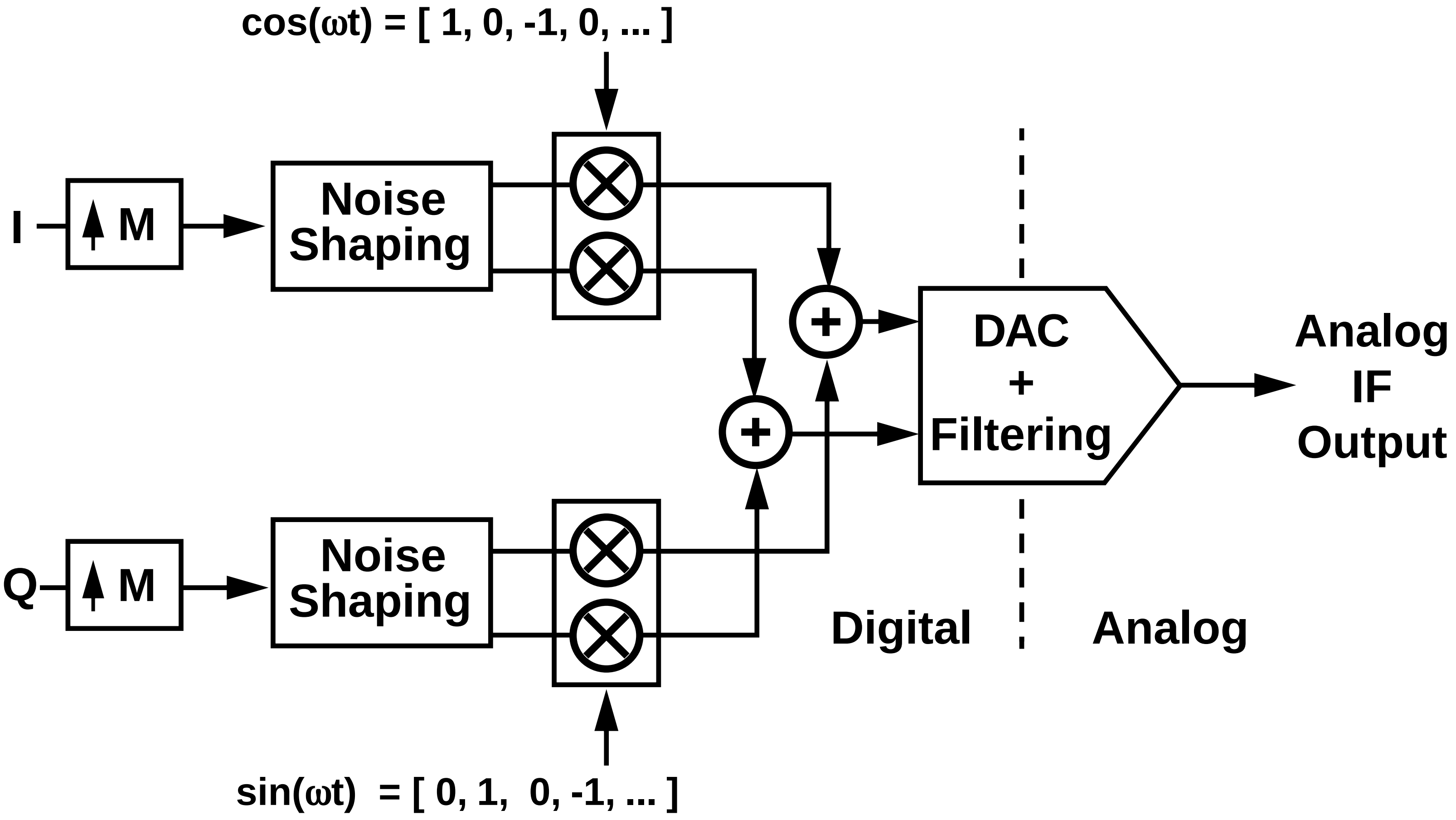
- **Consider the use of bandpass oversampling D/A conversion in wireless communications transmitters**
- **Move IF into the digital domain to eliminate**
  - **dc offset**
  - **I & Q mismatch**
- **Merge D/A conversion, noise shaping, reconstruction and IF mixing**
- **Explore the use of cascaded digital noise shaping for D/A conversion**

*** D. Barkin, 03 VLSI Ckt Symp**

Traditional Transmitter Architecture

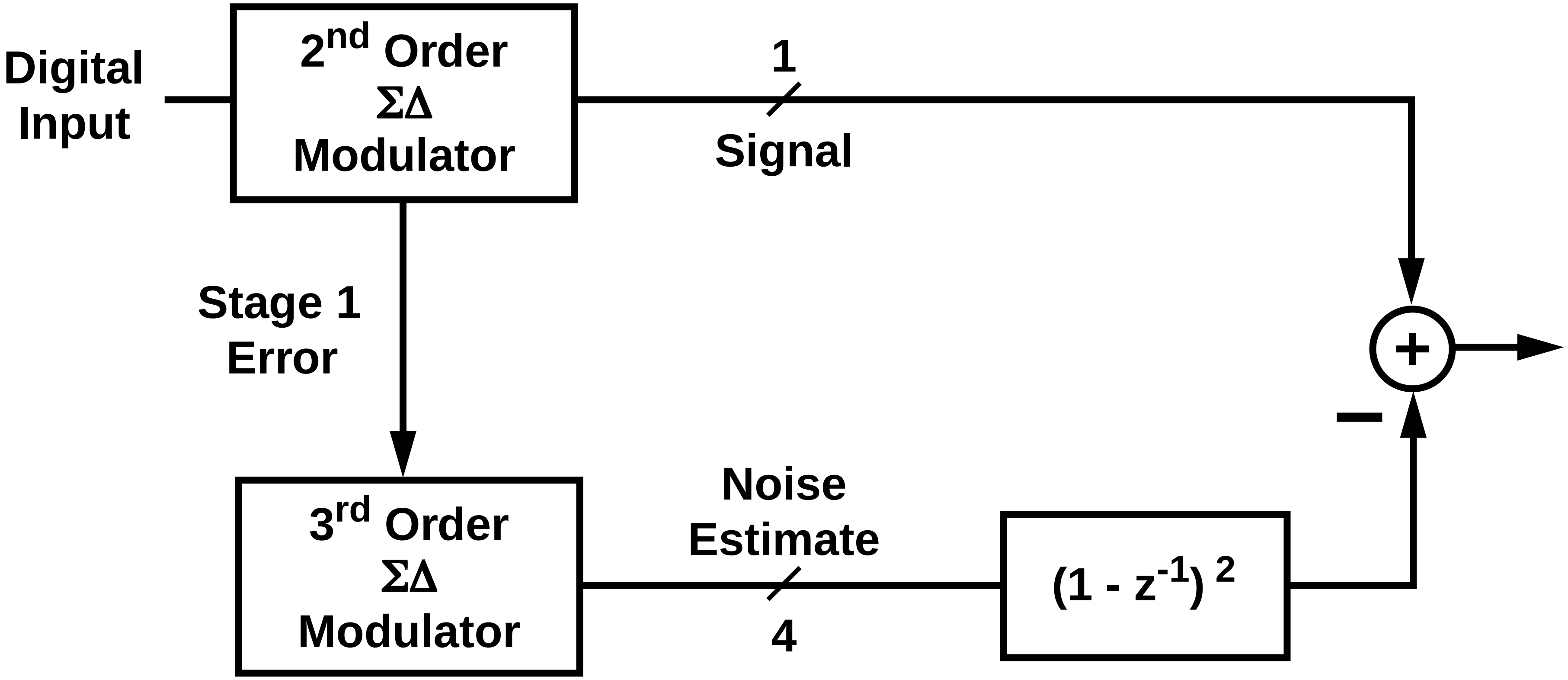


Cascaded Bandpass Oversampling DAC



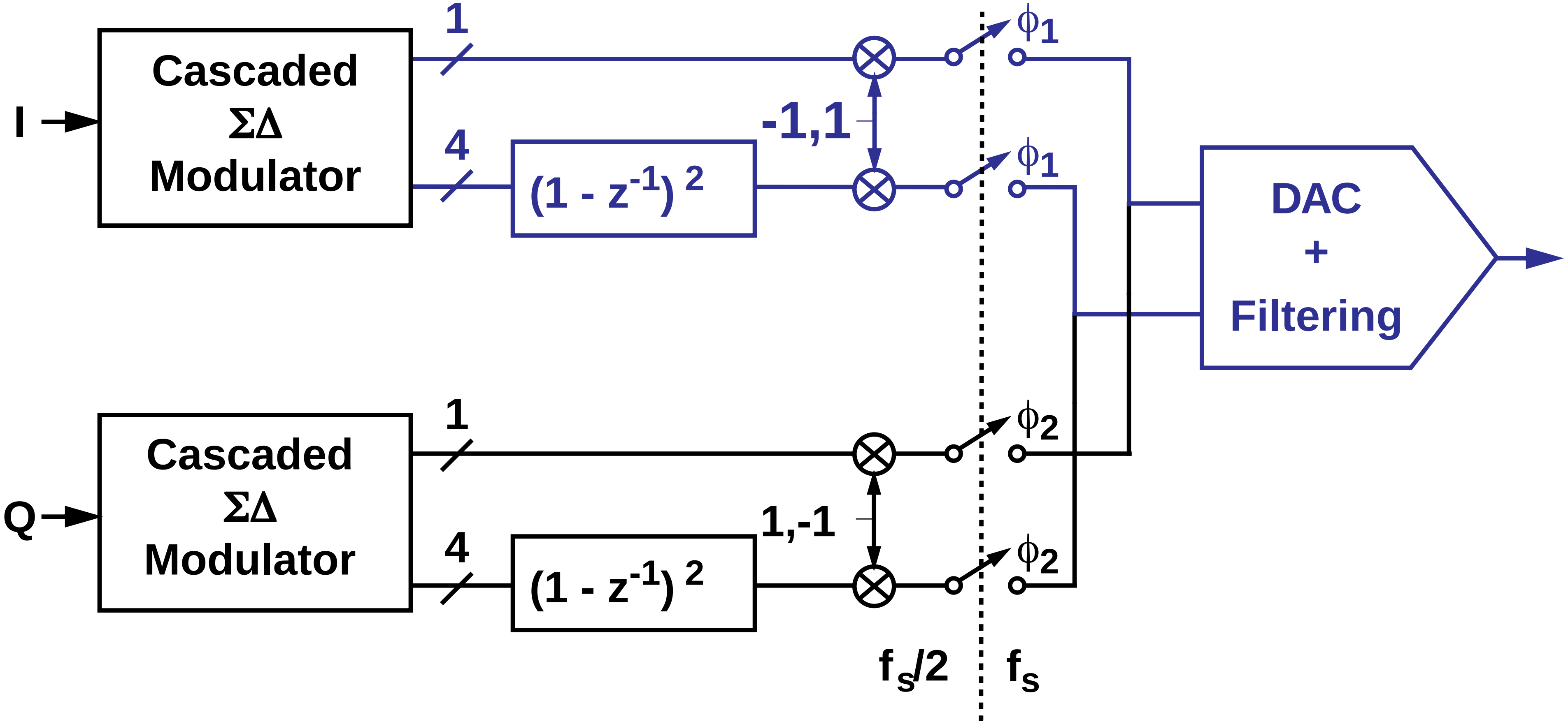
- Mix to IF (at $f_s/4$) following cascaded noise shapers
- Error cancellation performed at IF

Lowpass Cascaded Noise Shaping



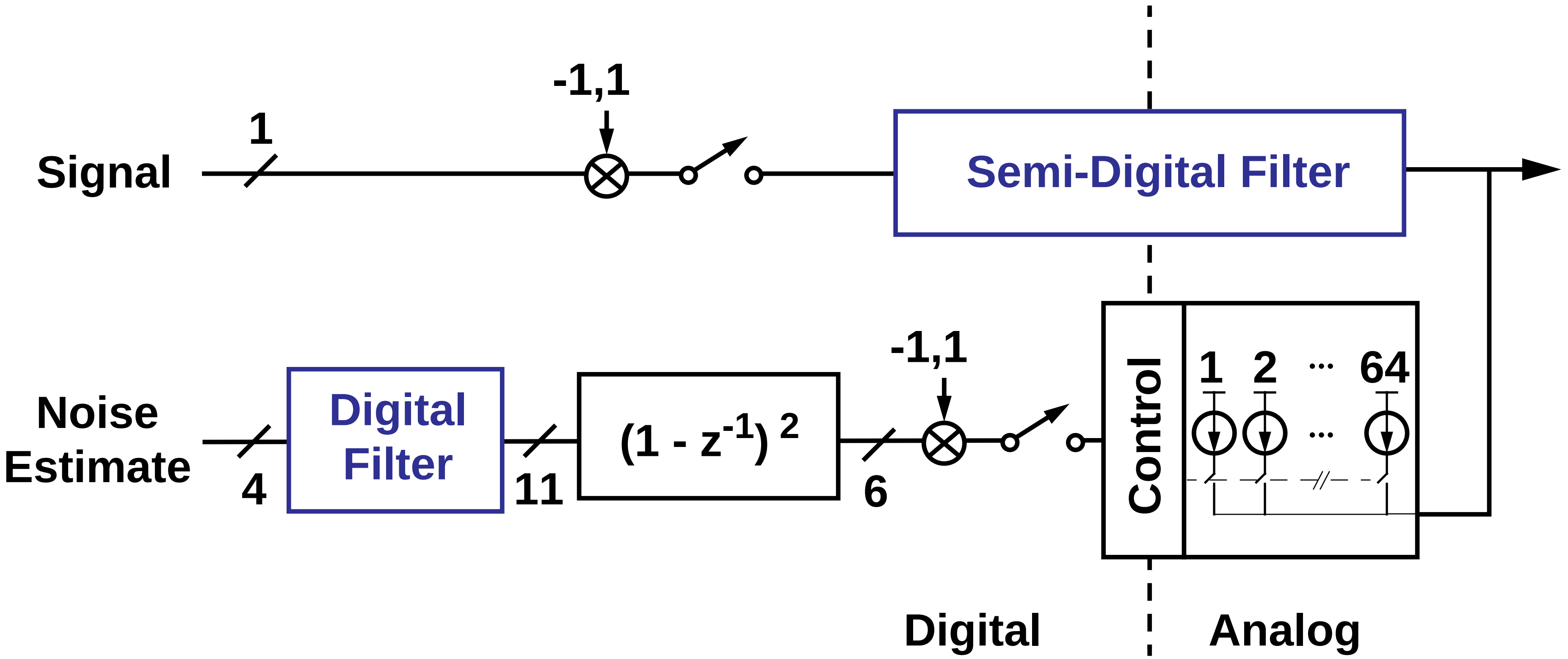
- 2^{nd} order differentiator matches noise shaping in first stage

DAC Architecture



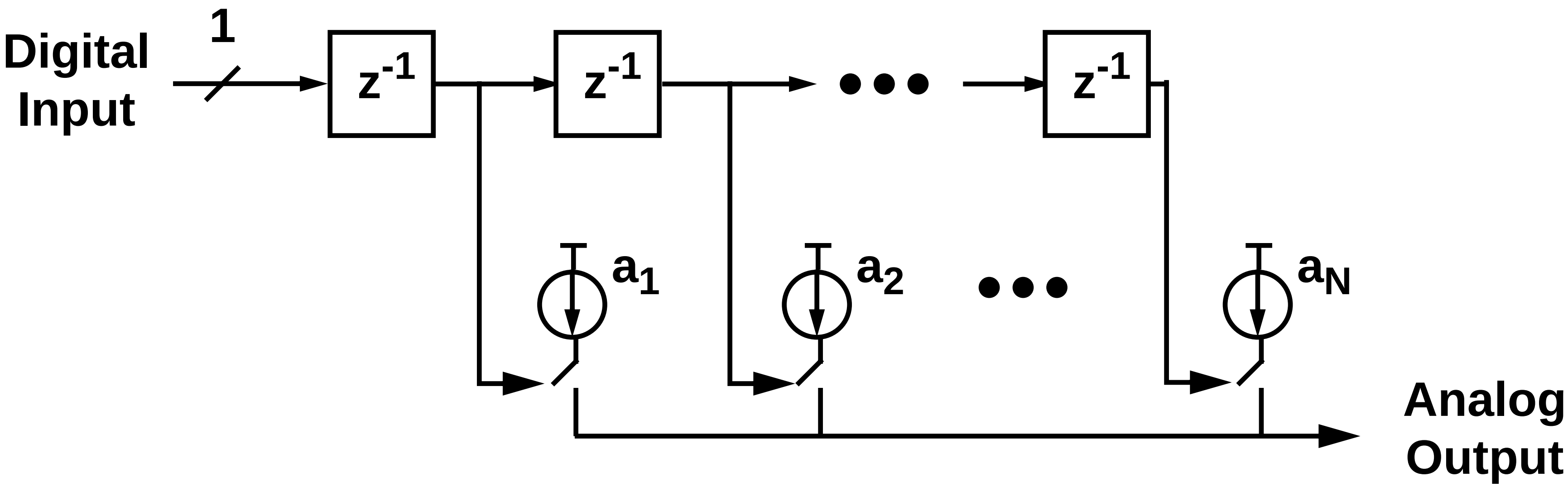
- I & Q modulators operate at $f_s/2$, saving power

Discrete Time – Continuous Time Interface



- Semi-digital filtering for reconstruction of signal
- Digital filter reduces out-of-band quantization noise
- Digital filter transfer function matches that of semi-digital filter

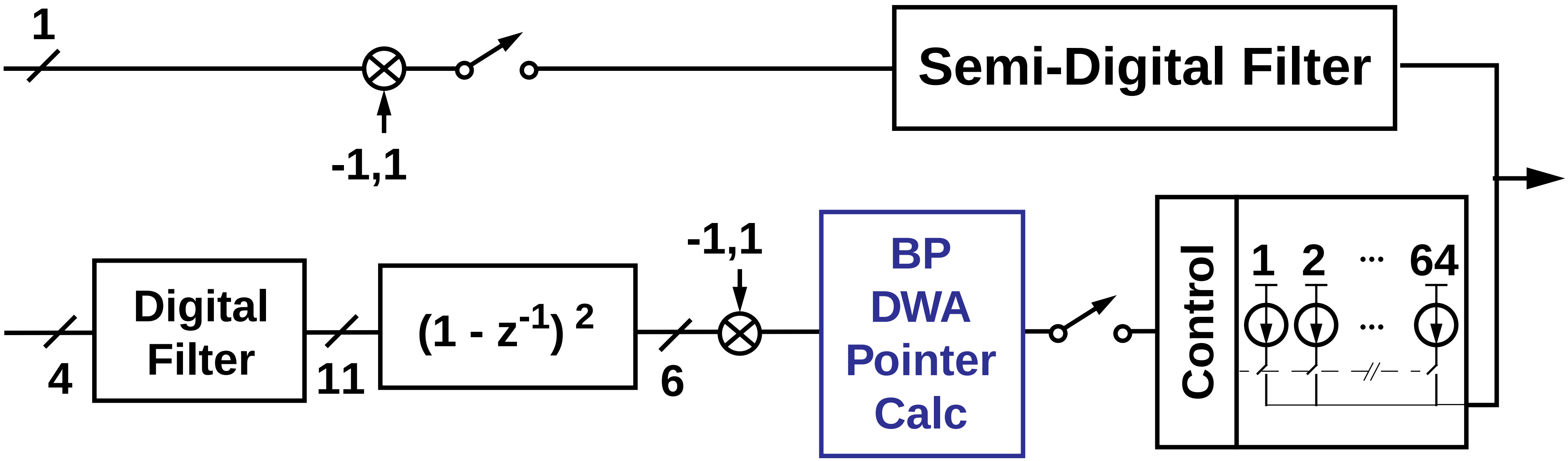
Semi-Digital Filter *



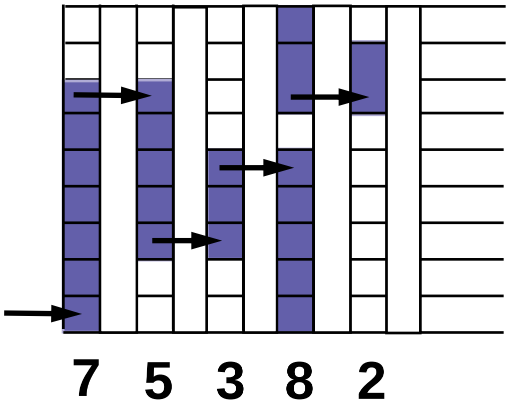
- Mismatch among current sources alters the transfer function but doesn’t introduce nonlinearity
- Area limits number of taps and precision of coefficients
- Good for 1-bit signal path but not multi-bit noise estimation path

* D.Su, et al., JSSC, 1993

Bandpass Data Weighted Averaging

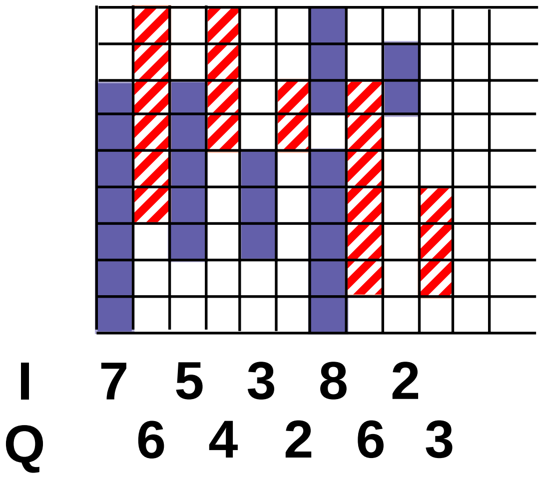


$f_s/2$ Notch



Time -->

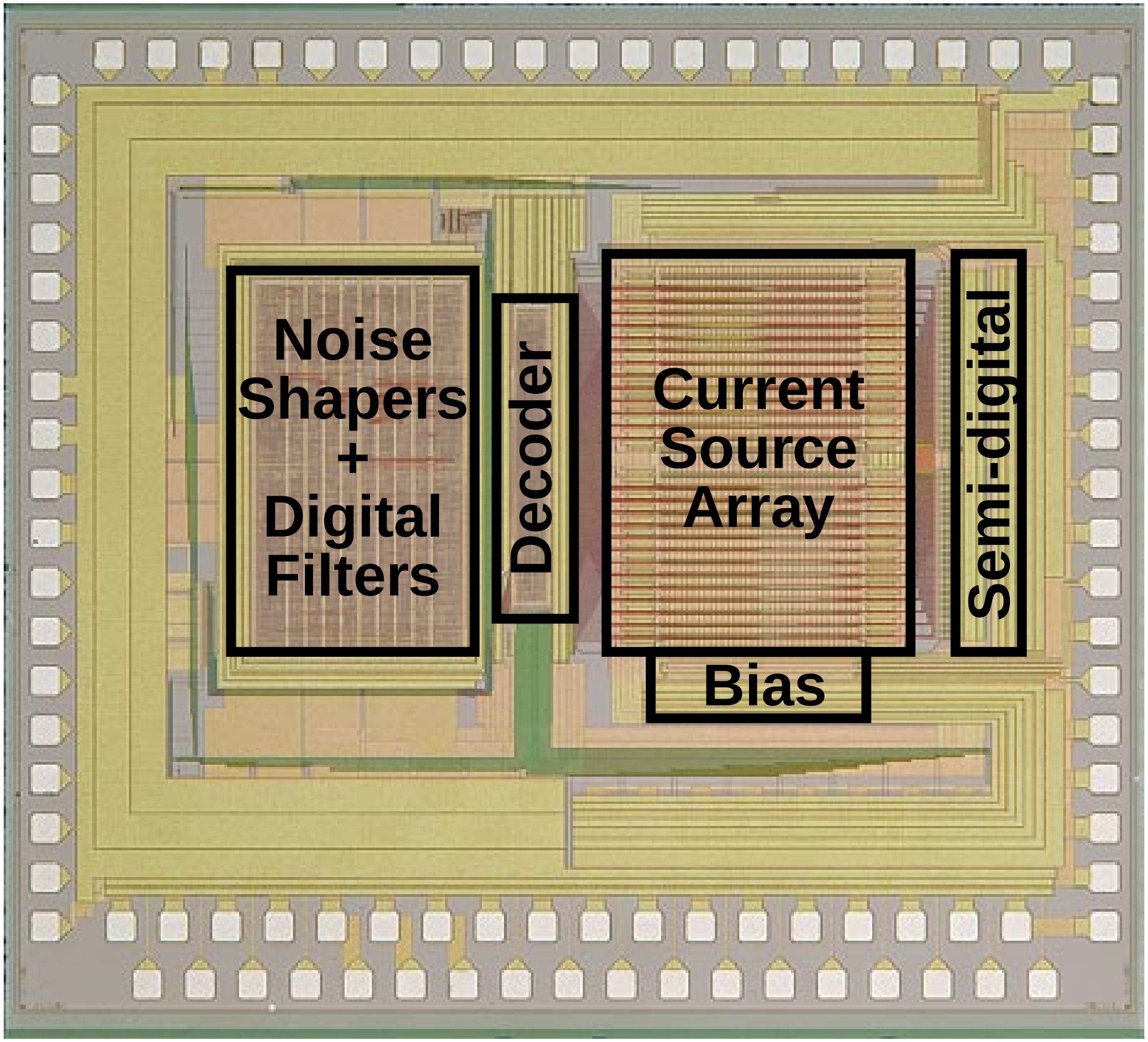
$f_s/4$ Notch



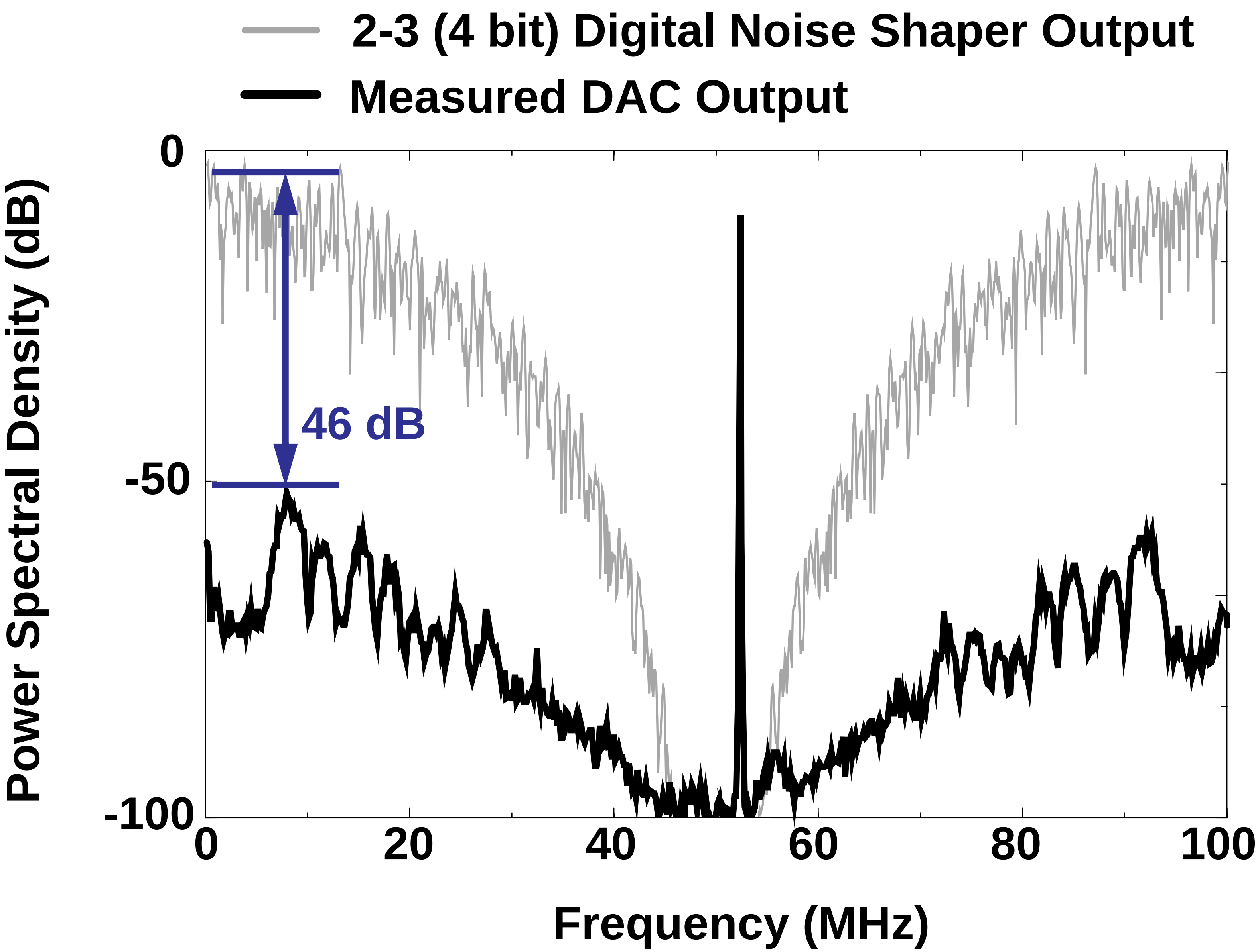
- **I & Q pointer calculations are independent**

* T. Shui, et al., ISCAS, 1998

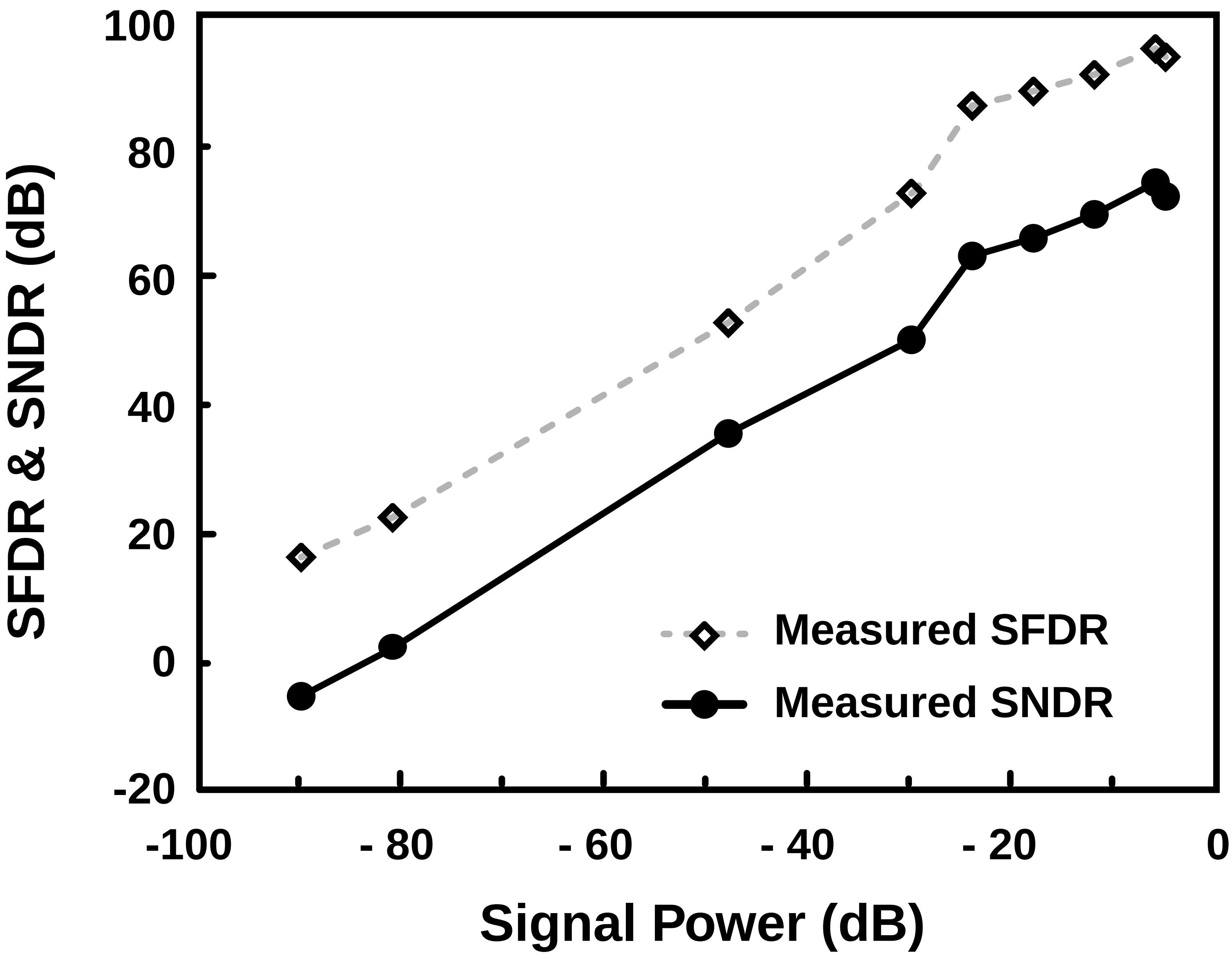
Bandpass DAC Die Photo



DAC Output Spectrum



SNDR and SFDR



DAC Performance

Technology	1-p, 5-m 0.25-µm CMOS
Supply: Modulators and Filters Other	1.5 V 2.5 V
Center Frequency	50 MHz
Bandwidth	6.25MHz
Peak SNDR	76 dB
Dynamic Range	85 dB
Peak Out-of-Band Noise Suppression	46 dB
Mirror for -6dB input	90 dB down
Active Area	2.1 mm^2
Power: Modulators and Filters Thermometer Decoders Current Source Array	30 mW 70 mW 115 mW

Summary

- Cascades of first- and second-order noise-shaping modulator stages can be used for
  - A/D and D/A conversion
  - lowpass and bandpass data conversion
  - dynamic range enhancement at low oversampling ratios
- If properly designed, advantages include
  - no potential instability
  - decorrelation of quantization noise and input
  - low sensitivity to analog precision

What's ahead?

- Challenges in data conversion interface design
  - Impact of technology scaling on analog; ITRS presents problems w.r.t. leakage, output resistance and gain
  - Demand for increased performance driven by increased use of digital signal processing
  - Still room for innovation in architecture and circuits
- Some ongoing research
  - Very low voltage ($V_{DD} \leq 1.2V$) oversampling A/D and D/A conversion
  - Oversampling D/A conversion for DSL
  - Continuous-time oversampling modulators for broadband applications