Signals and Systems

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Review

Signals As Functions of Time

- **Continuous-time signals are functions of a real argument**

  $x(t)$ where time, t, can take any real value

  $x(t)$ may be 0 for a given range of values of t

- **Discrete-time signals are functions of an argument that takes values from a discrete set**

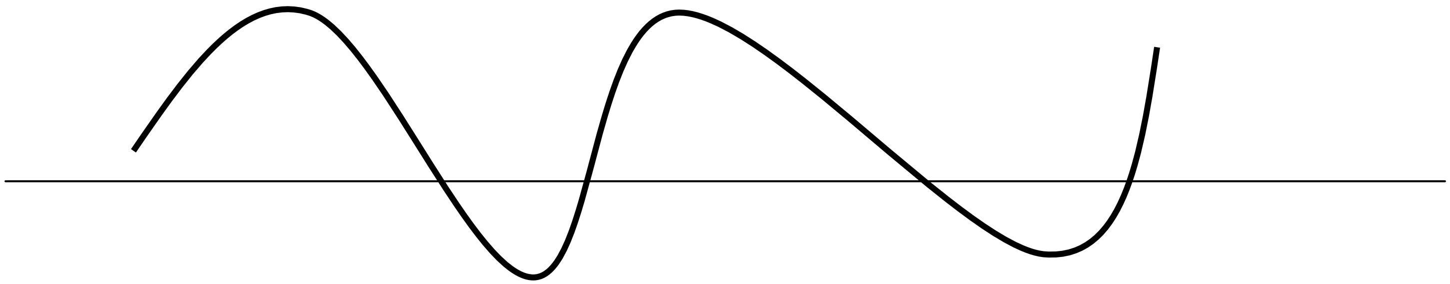
  $x[k]$ where $k \in \{...-3,-2,-1,0,1,2,3...\}$

  Integer time index, e.g. k, for discrete-time systems

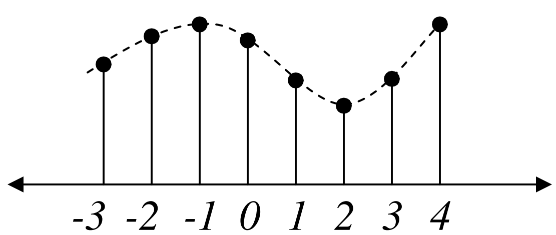
- **Values for x may be real or complex**

Analog vs. Digital Signals

- **Analog:**
  - Continuous in both time and amplitude



- **Digital:**
  - Discrete in both time and amplitude



The Many Faces of Signals

- A function, e.g. $\cos(t)$ or $\cos(\pi k)$, useful in analysis
- A sequence of numbers, e.g. {1,2,3,2,1} or a sampled triangle function, useful in simulation
- A collection of properties, e.g. even, causal, stable, useful in reasoning about behavior
- A piecewise representation, e.g.
- A generalized function, e.g. $\delta(t)$

$$u(t) = \begin{cases} 1 & \text{for } t > 0 \\ \frac{1}{2} & \text{for } t = 0 \\ 0 & \text{for } t < 0 \end{cases}$$

$$u[k] = \begin{cases} 1 & \text{for } k \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What everyday device uses two sinusoids to transmit a digital code?

Telephone Touchtone Signal

- **Dual-tone multiple frequency (DTMF) signaling**

  Sum of two sinusoids: one from low-frequency group and high-frequency group

  On for 40-60 *ms* and off for rest of signaling interval (symbol duration):

  100 *ms* for AT&T

  80 *ms* for ITU Q.24 standard

	1209 Hz	**1336 Hz**	**1477 Hz**	**1633 Hz**
697 Hz	1	2	3	A
770 Hz	4	5	6	B
852 Hz	7	8	9	C
941 Hz	*	0	#	D

Alphabet of 16 DTMF symbols, with symbols A-D for military and radio signaling applications

- **Maximum dialing rate**

  AT&T: 10 symbols/s (40 bits/s)

  Q.24: 12.5 symbols/s (50 bits/s)

ITU is the International Telecommunication Union

Review

Unit Impulse

- **Mathematical idealism for an instantaneous event**
- **Dirac delta as generalized function (a.k.a. functional)**
- **Selected properties**

Unit area: $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

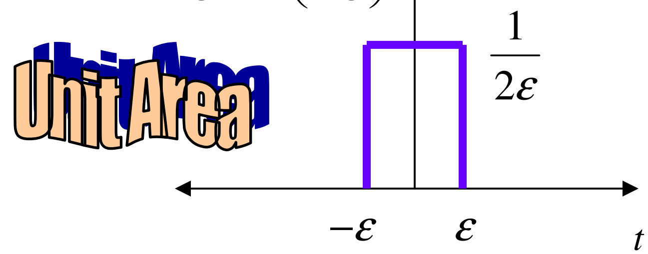
Sifting $\int_{-\infty}^{\infty} g(t)\delta(t)\, dt = g(0)$

provided g(t) is ***defined*** *at t*=0

Scaling: $\int_{-\infty}^{\infty} \delta(at)\, dt = \frac{1}{|a|}$ if $a \neq 0$

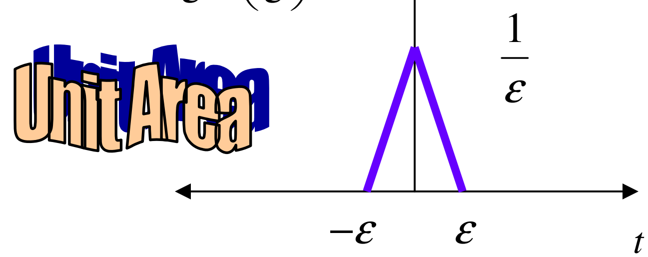
- **Note that** $\delta(0)$ is undefined

$P_\varepsilon(t) = \frac{1}{2\varepsilon}\operatorname{rect}\left(\frac{t}{2\,\varepsilon}\right)$



$\delta(t) = \lim_{\varepsilon \to 0} P_\varepsilon(t)$

$P_\varepsilon(t) = \frac{1}{\varepsilon}\Delta\left(\frac{t}{\varepsilon}\right)$

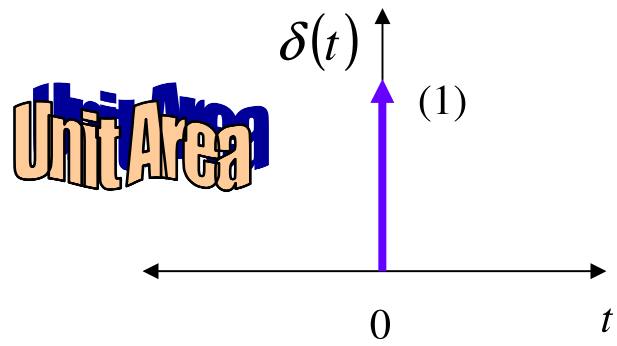


$\delta(t) = \lim_{\varepsilon \to 0} P_\varepsilon(t)$

Review

Unit Impulse

- **By convention, plot Dirac delta as arrow at origin**
  - Undefined amplitude at origin
  - Denote area at origin as (*area*)
  - Height of arrow is irrelevant
  - Direction of arrow indicates sign of area



- **With $\delta(t) = 0$ for $t \neq 0$, it is tempting to think**
  - $\cancel{\phi(t)\,\delta(t) = \phi(0)\,\delta(t)}$
  - $\cancel{\phi(t)\,\delta(t-T) = \phi(T)\,\delta(t-T)}$

Simplify unit impulse under integration only

Review

Unit Impulse

- **We can simplify** δ(t) **under integration**

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)dt = \phi(0)$$

Assuming ϕ(t) is defined at t=0

- **What about?**

$$\int_{-\infty}^{-1} \phi(t)\delta(t)dt = ?$$

- **What about?**

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)dt = ?$$

By substitution of variables,

$$\int_{-\infty}^{\infty} \phi(t+T)\delta(t)dt = \phi(T)$$

- **Other examples**

$$\int_{-\infty}^{\infty} \delta(t)\, e^{-j\varpi t} dt = 1$$

$$\int_{-\infty}^{\infty} \delta(t-2)\cos\left(\frac{\pi\, t}{4}\right) dt = 0$$

$$\int_{-\infty}^{\infty} e^{-2(x-t)}\, \delta(2-t)\, dt = e^{-2(x-2)}$$

- **What about at origin?**

$$\int_{-\infty}^{0} \delta(t)\, dt = ?$$

$$\int_{-\infty}^{0^-} \delta(t)\, dt = 0$$ Before Impulse

$$\int_{-\infty}^{0^+} \delta(t)\, dt = 1$$ After Impulse

Unit Impulse Functional

- **Relationship between unit impulse and unit step**

$$\int_{-\infty}^{t} \delta(\tau)\, d\tau = \begin{cases} 0 & t < 0 \\ ? & t = 0 \\ 1 & t > 0 \end{cases} \quad \Longleftrightarrow \quad \frac{du}{dt} = \delta(t)$$

$$= u(t)$$

- **What happens at the origin for $u(t)$?**

  $u(0^-) = 0$ and $u(0^+) = 1$, but $u(0)$ can take any value

  Common values for $u(0)$ are 0, ½, and 1

  $u(0) = ½$ is used in impulse invariance filter design:

  L. B. Jackson, "A correction to impulse invariance," *IEEE Signal Processing Letters*, vol. 7, no. 10, Oct. 2000, pp. 273-275.

Review

Systems

- **Systems operate on signals to produce new signals or new signal representations**

$$x(t) \rightarrow \boxed{T\{\bullet\}} \rightarrow y(t) \qquad\qquad x[k] \rightarrow \boxed{T\{\bullet\}} \rightarrow y[k]$$

$$y(t) = T\{\, x(t) \,\} \qquad\qquad y[k] = T\{\, x[k] \,\}$$

- **Continuous-time examples**

$y(t) = ½\, x(t) + ½\, x(t\text{-}1)$

$y(t) = x^2(t)$

Squaring function can be used in sinusoidal demodulation

- **Discrete-time system examples**

$y[n] = ½\, x[n] + ½\, x[n\text{-}1]$

$y[n] = x^2[n]$

Average of current input and delayed input is a simple filter

System Properties

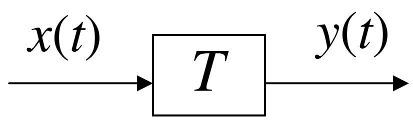
- **Let** $x(t)$**,** $x_1(t)$**, and** $x_2(t)$ **be inputs to a continuous-time linear system and let** $y(t)$**,** $y_1(t)$**, and** $y_2(t)$ **be their corresponding outputs**
- **A linear system satisfies**

  Additivity: $x_1(t) + x_2(t) \Rightarrow y_1(t) + y_2(t)$

  Homogeneity: $a\, x(t) \Rightarrow a\, y(t)$ for any real/complex constant a
- **For a time-invariant system, a shift of input signal by any real-valued** τ **causes same shift in output signal, i.e.** $x(t - \tau) \Rightarrow y(t - \tau)$ **for all** τ

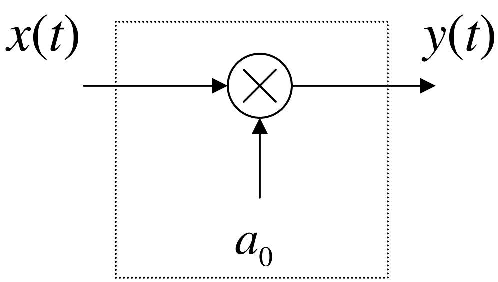
System Properties

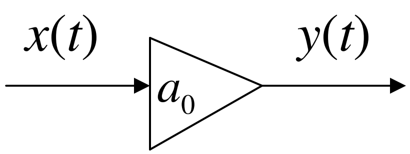
- **Ideal delay by *T* seconds. *Linear*?**



$$y(t) = x(t - T)$$

- **Scale by a constant (a.k.a. gain block)**
  - Two different ways to express it in a block diagram



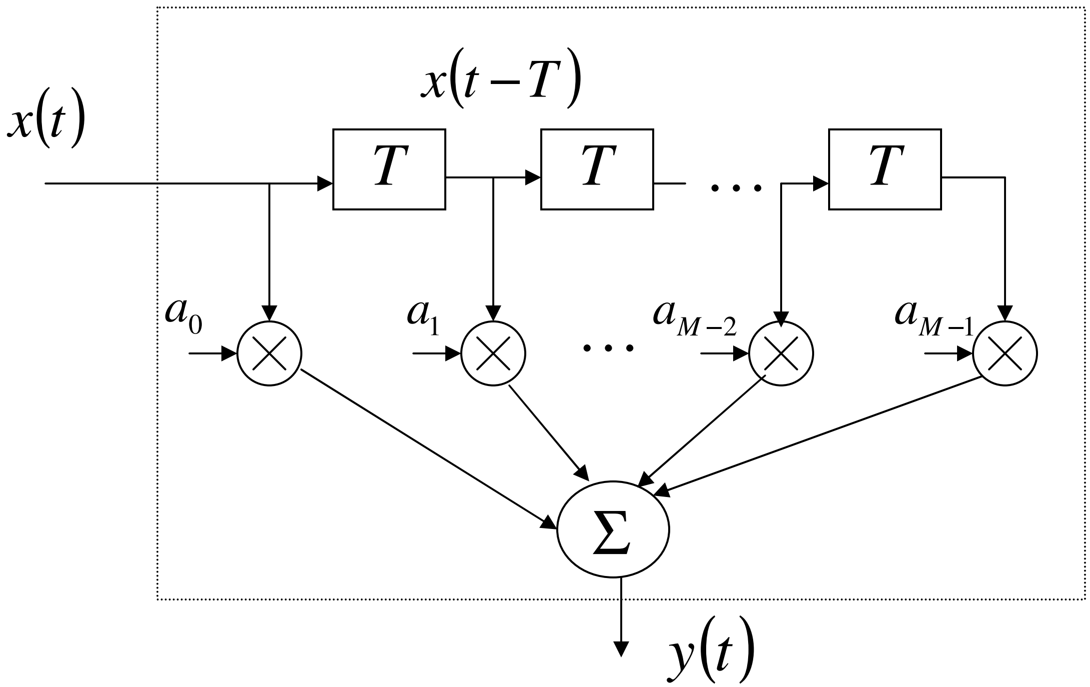


$$y(t) = a_0 \, x(t)$$

  - Linear?

System Properties

- **Tapped delay line**



Each T represents a delay of T time units

There are M-1 delays

$$y(t) = \sum_{m=0}^{M-1} a_m \, x(t - mT)$$

Coefficients (or taps) are a_0, a_1, $\ldots a_{M\text{-}1}$

- ***Linear? Time-invariant?***

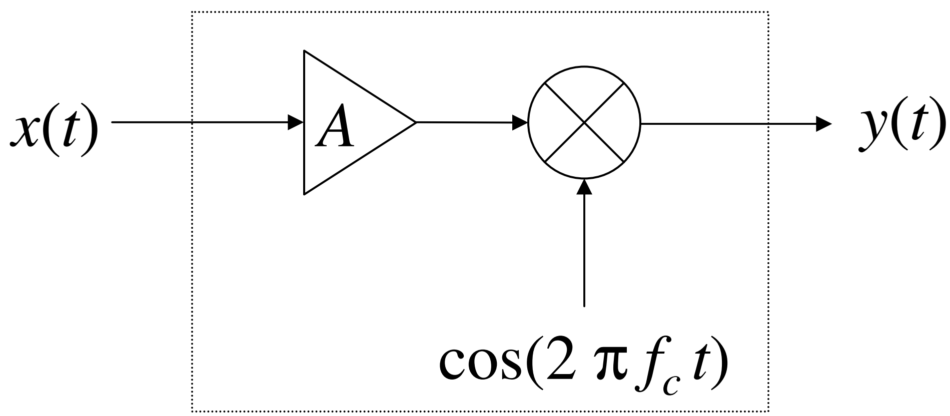
System Properties

- **Amplitude Modulation (AM)**

  $y(t) = A\, x(t) \cos(2\pi f_c t)$

  f_c is the carrier frequency (frequency of radio station)

  A is a constant



- ***Linear? Time-invariant?***
- **AM modulation is AM radio if** $x(t) = 1 + k_a\, m(t)$ **where** $m(t)$ **is message (audio) to be broadcast**

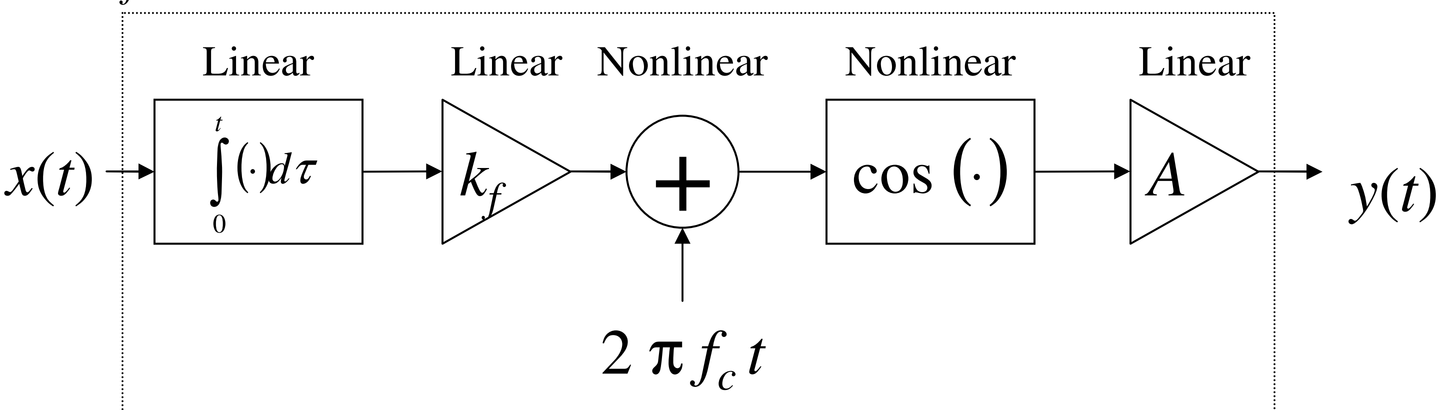
System Properties

- **Frequency Modulation (FM)**

FM radio:
$$y(t) = A\cos\left(2\pi f_c t + k_f \int_0^t x(t)\,dt\right)$$

f_c is the carrier frequency (frequency of radio station)

A and k_f are constants



- ***Linear? Time-invariant?***

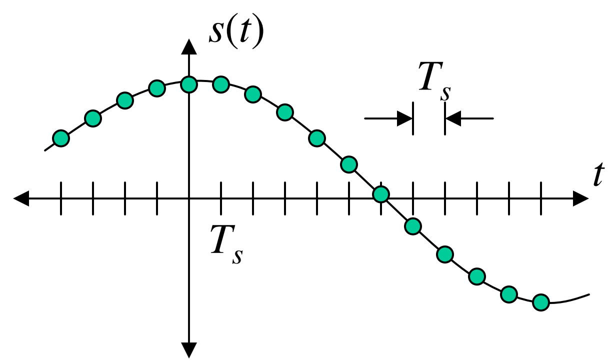
Sampling

- **Many signals originate as continuous-time signals, e.g. conventional music or voice.**
- **By sampling a continuous-time signal at isolated, equally-spaced points in time, we obtain a sequence of numbers**

$$s[k] = s(k\,T_s)$$

$k \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

T_s is the sampling period.



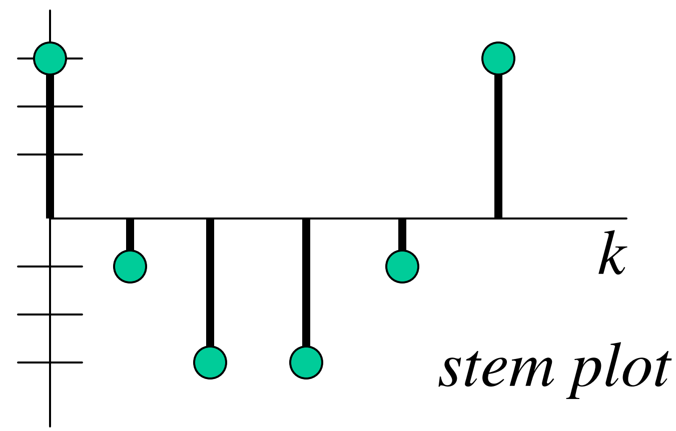
Sampled analog waveform

Generating Discrete-Time Signals

- **Uniformly sampling a continuous-time signal**
  - Obtain $x[k] = x(T_s\, k)$ for $-\infty < k < \infty$.
  - How to choose T_s?
- **Using a formula**

  $x[k] = k^2 - 5k + 3$, for $k \geq 0$
  would give the samples
  $\{3, -1, -3, -3, -1, 3, ...\}$

  What does the sequence look like in continuous time?



System Properties

- **Let** $x[k]$, $x_1[k]$, **and** $x_2[k]$ **be inputs to a linear system and let** $y[k]$, $y_1[k]$, **and** $y_2[k]$ **be their corresponding outputs**
- **A linear system satisfies**

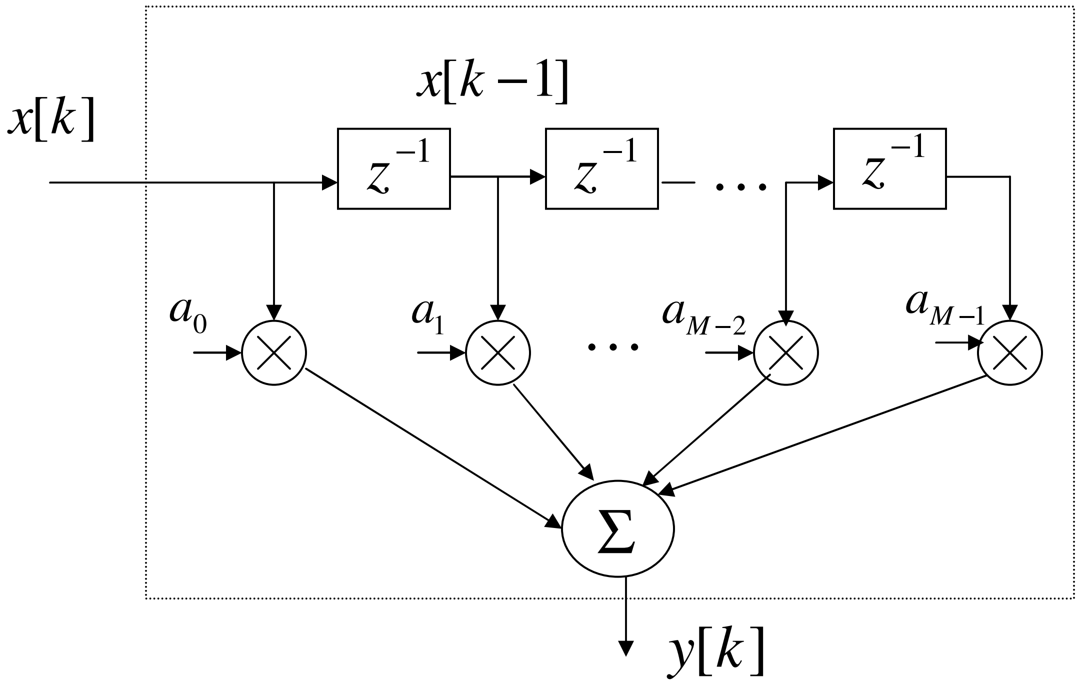
  Additivity: $x_1[k] + x_2[k] \Rightarrow y_1[k] + y_2[k]$

  Homogeneity: $a\, x[k] \Rightarrow a\, y[k]$ for any real/complex constant a
- **For a time-invariant system, a shift of input signal by any integer-valued** m **causes same shift in output signal, i.e.** $x[k - m] \Rightarrow y[k - m]$, **for all** m

System Properties

- **Tapped delay line in discrete time**

See also slide 5-3



Each z^{-1} represents a delay of 1 sample

There are *M-1* delays

$$y[k] = \sum_{m=0}^{M-1} a_m \; x[k-m]$$

Coefficients (or taps) are a_0, a_1, …$a_{M\text{-}1}$

- ***Linear? Time-invariant?***

System Properties

- **Continuous time**

$$f(t) \longrightarrow \boxed{\frac{d}{dt}(\cdot)} \longrightarrow y(t)$$

$$y(t) = \frac{d}{dt}\{f(t)\}$$
$$= \lim_{\Delta t \to 0} \frac{f(t) - f(t - \Delta t)}{\Delta t}$$

- ***Linear?***
- ***Time-invariant?***

- **Discrete time**

$$f[k] \longrightarrow \boxed{\frac{\hat{d}}{dt}(\cdot)} \longrightarrow y[k]$$

$$y[k] = y(kT_s) = \frac{d}{dt}\{f(t)\}\Big|_{t=kT_s}$$
$$= \lim_{T_s \to 0} \frac{f(kT_s) - f(kT_s - T_s)}{T_s}$$
$$= f[k] - f[k-1]$$

See also slide 5-13

- ***Linear?***
- ***Time-invariant?***

Conclusion

- **Continuous-time versus discrete-time:**
  discrete means quantized in time
- **Analog versus digital:**
  digital means quantized in time and amplitude
- **A digital signal processor (DSP) is a discrete-time and digital system**

  A DSP processor is well-suited for implementing LTI digital filters, as you will see in laboratory #3.

Signal Processing Systems

- Speech synthesis and recognition
- Audio CD players
- Audio compression: MPEG 1 layer 3 audio (MP3), AC3
- Image compression: JPEG, JPEG 2000
- Optical character recognition
- Video CDs: MPEG 1
- DVD, digital cable, HDTV: MPEG 2
- Wireless video: MPEG 4 Baseline/H.263, MPEG 4 Adv. Video Coding/H.264 (emerging)
- *Examples of communication systems?*

Moving Picture Experts Group (MPEG)

Joint Picture Experts Group (JPEG)

Optional

Communication Systems

- **Voiceband modems (56k)**
- **Digital subscriber line (DSL) modems**

  ISDN: 144 kilobits per second (kbps)

  Business/symmetric: HDSL and HDSL2

  Home/asymmetric: ADSL, ADSL2, VDSL, and VDSL2
- **Cable modems**
- **Cellular phones**

  First generation (1G): AMPS — Analog

  Second generation (2G): GSM, IS-95 (CDMA)

  Third generation (3G): cdma2000, WCDMA — Digital