Modified Conjugate Gradient Method for ADSL Echo Canceller

February 12, 1999

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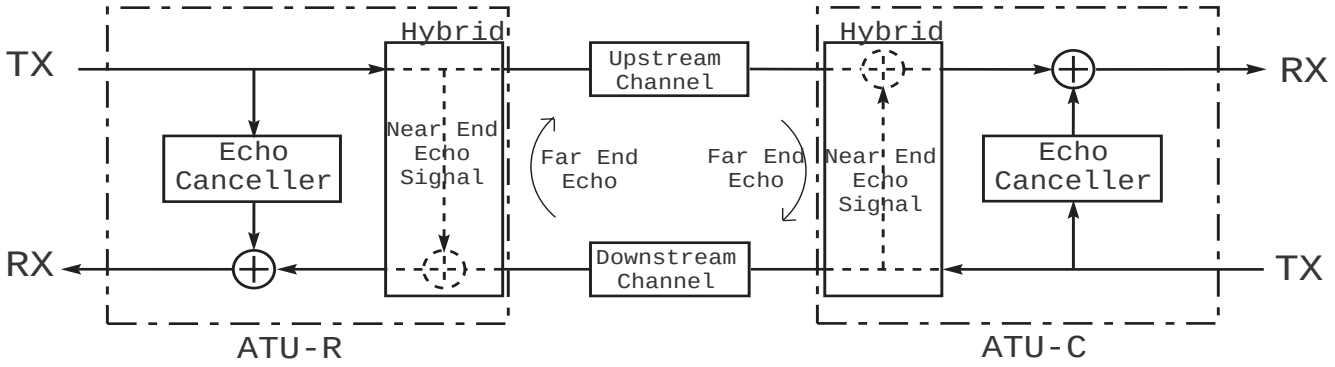
Outline of presentation

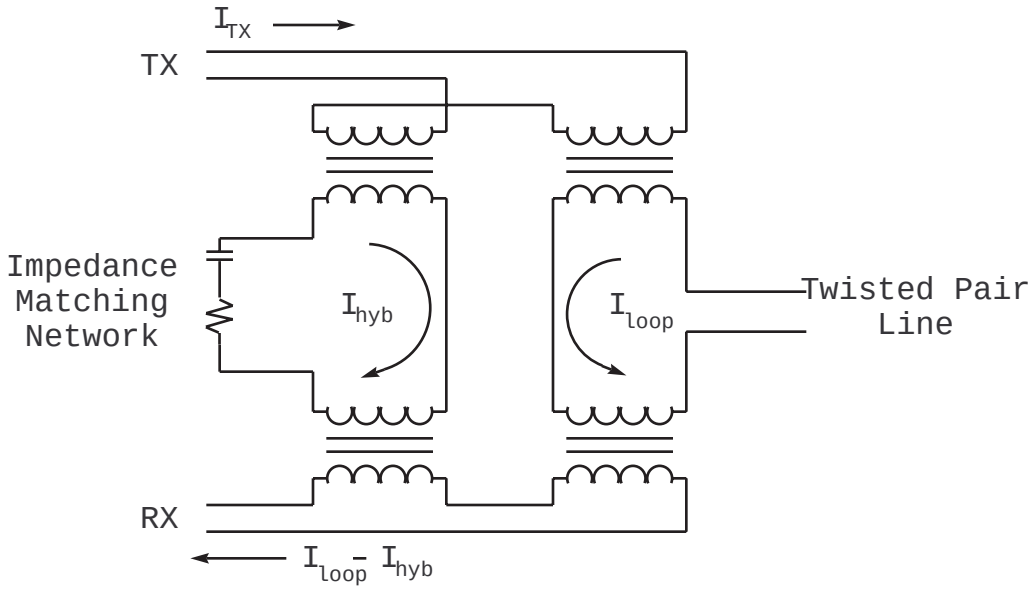
1. What is Echo?
2. General ADSL Echo Canceller Architecture
3. Overview of Conjugate Gradient Method
4. Experimental Results

What is Echo?



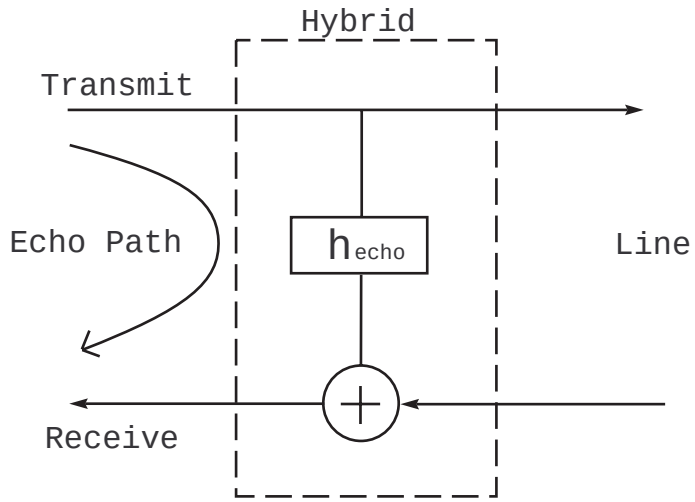
- Far end echo is attenuated twice by the channel (negligible).
- Near end echo is dominant
- Modeled as transmit signal passed through echo transfer function and added to the received signal

Cause of Echo



I_{loop} is the sum of local transmit signal and receive signal from far end.

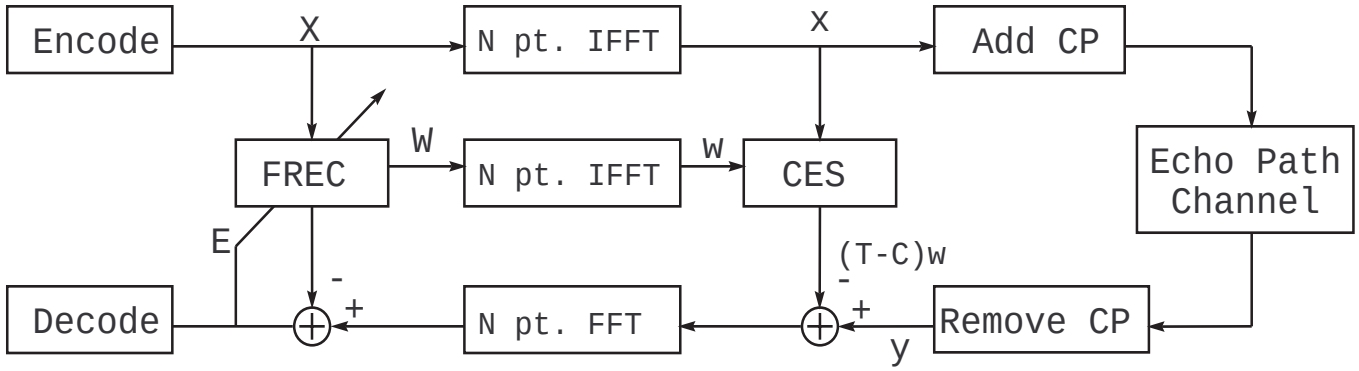
Ideally, $I_{hyb} = I_{tx}$. In reality, $I_{hyb} \cong I_{tx}$ modified as a function of frequency.



Hybrid modeled as “frequency selective channel”.

Received signal is the sum of far-end transmitted signal and echo signal.

ADSL Echo Canceller



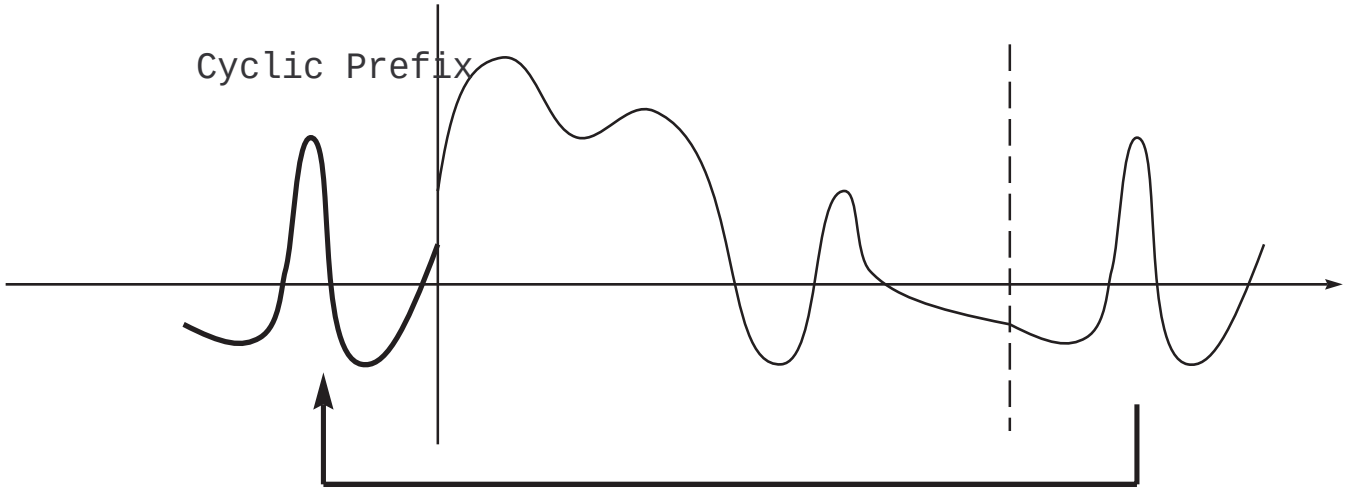
- Circular Echo Synthesis (CES) : To remove tail of impulse response that cannot be compensated by Cyclic Prefix.
- Frequency Domain Echo Canceller (FREC) : Frequency domain adaptation.
- Difference between ATU-R and ATU-C

  ATU-R : Uses inline demodulation FFT. As shown above.

  ATU-C : Requires separate FFT to update the FREC.

Cyclic Prefix

- FFT/IFFT assumes periodic sequence, or circular convolution.
- The actual channel is linear convolution.

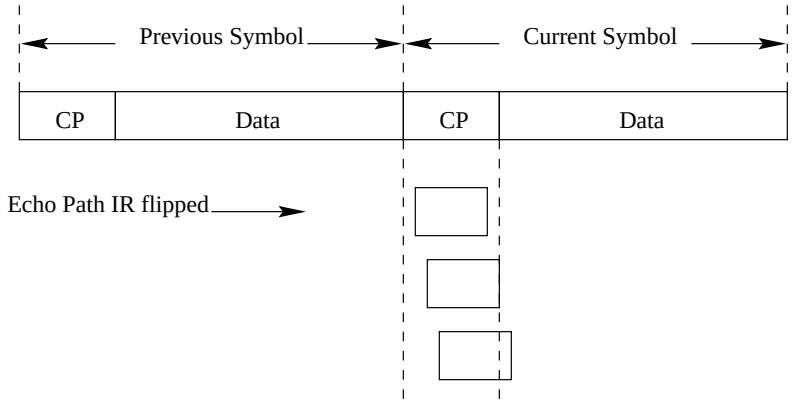


$$x(n) = \{x(0), x(1), \ldots, x(N-1)\}$$

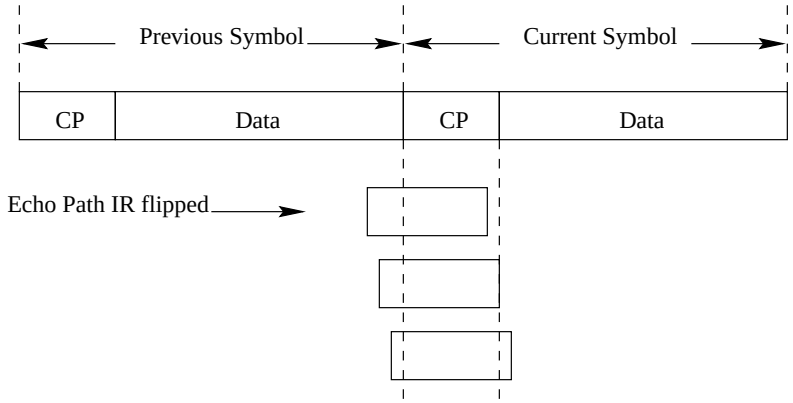
$$x_{cp}(n) = \{x(N-CP), \ldots, x(N-1), x(0), x(1), \ldots, x(N-1)\}$$

Circular Echo Synthesis

Case 1) Echo Path IR length less than CP length



Case 2) Echo Path IR length longer than CP length



Frequency Domain Echo Canceller

Frequency domain Least Mean Square (LMS) method.

- Start with W(0) found during initialization.
- Repeat the following for each frame received.

$$E_i = F_N(y_i - x_{i,\,i-1} w_i) - X_i \cdot W_i$$

$$W_{i+1} = W_i - \mu X_i \cdot E_i$$

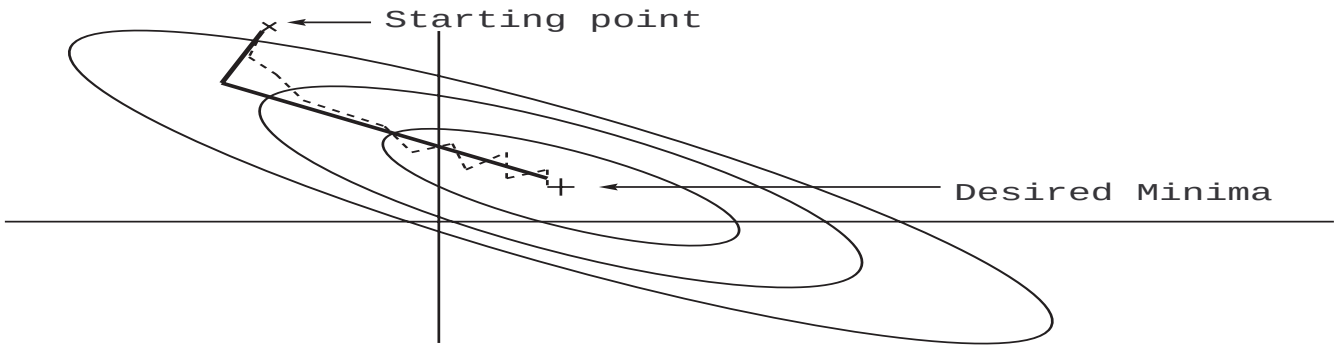
- A good value of μ must be experimentally found by trial and error.
  - Large μ : Fast convergence, large misadjustments, ill behavior.
  - Small μ : Slow convergence, small misadjustments, good numerical behavior.

General Conjugate Gradient Method

Underlying Principle : $d_i^T A d_k = 0$ for $i \neq k$. A is a symmetric positive definite matrix.

Find set of direction vectors d_i such that the optimal solution W_{opt} can be represented by a finite set of d_i's ($W_{opt} \in \{d_0, d_1, \ldots, d_{N-1}\}$)

Recursively compute and traverse in each direction d_i to approach the minima of the cost function.



Modified Conjugate Gradient Method

Initialize W(0)

Repeat the following computations :

$$E_i = F_N(y_i - x_{i,\,i-1} w_i) - X_i \cdot W_i$$

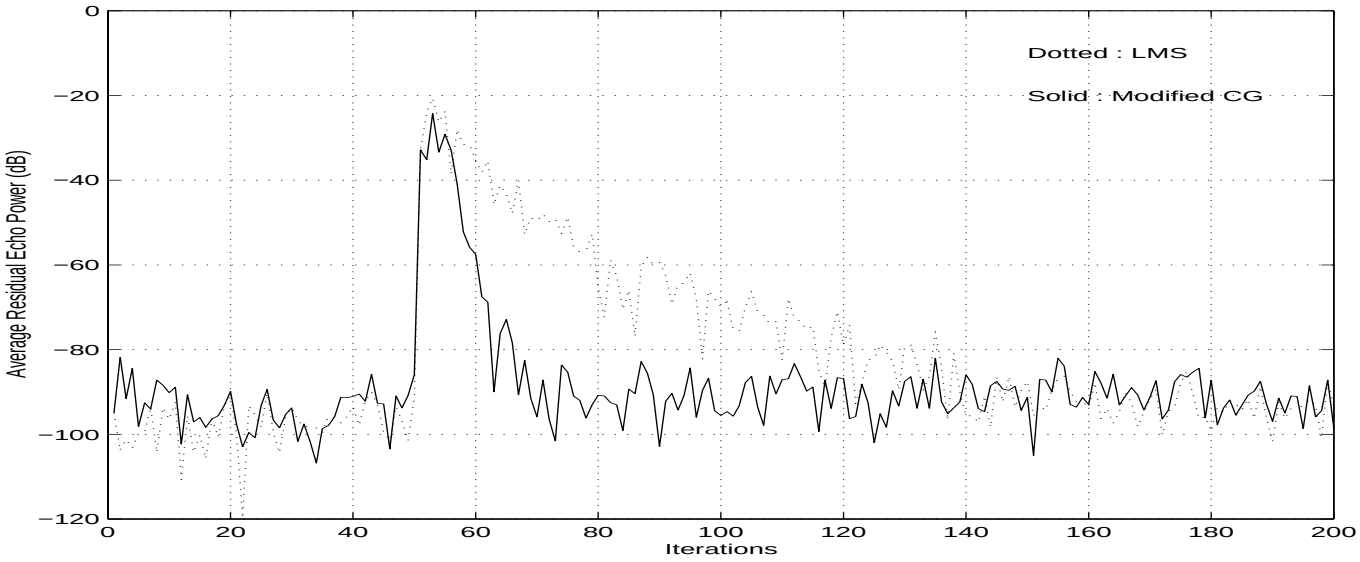
$$G_i = 2X_i^* \cdot E_i$$

$$\alpha_i = \frac{G_i^T \cdot G_i}{G_i^T (2G_i^T X_i X_i^*)}$$

$$W_{i+1} = W_i - \alpha_i G_i$$

- All values explicitly computed.
- Number of multiplications required per iteration :
  - LMS : 2N
  - Modified CG : 6N
- 3 times more multiplication than LMS.

Experimental Results



Method	LMS	MCG
Number of Multiplications per iteration	2N	6N
Number of iterations until convergence	100	20
Total number of multiplications until convergence	200N	120N

Conclusion

- Applied Modified Conjugate Gradient method to echo cancellation problem in ADSL system.
- Advantage
    - Fast Convergence
    - Compatible with LMS
- Disadvantage
    - 3 times more computation per iteration
    - One division required
- A good compromise for alternative adaptive filtering method.