Coloring Optical Packets:
How many crayons do we really need?

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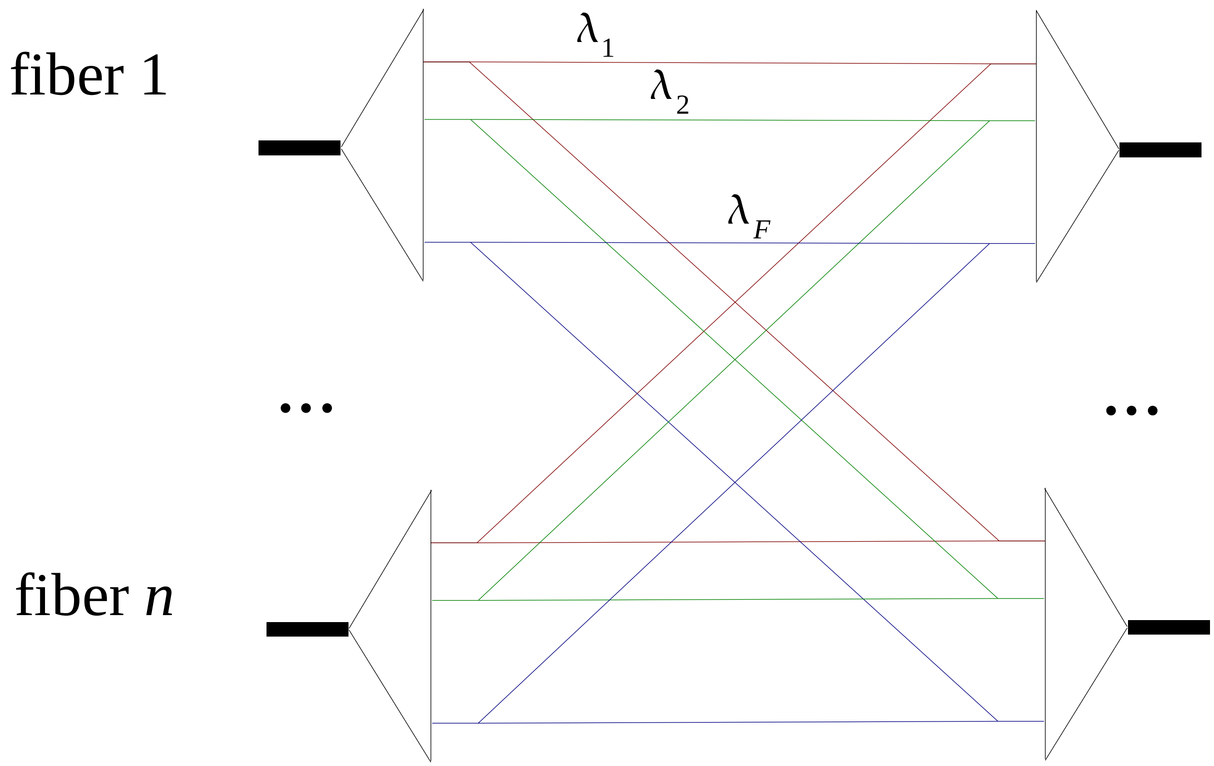
Outline

- The application
- The analysis
- The results
- The conclusions

Bufferless Optical Wavelength−Division Multiplexed Packet Switch (fixed wavelength packets)



F nxn switches

Good: switch simplicity

Bad: blocking!

Blocking analysis

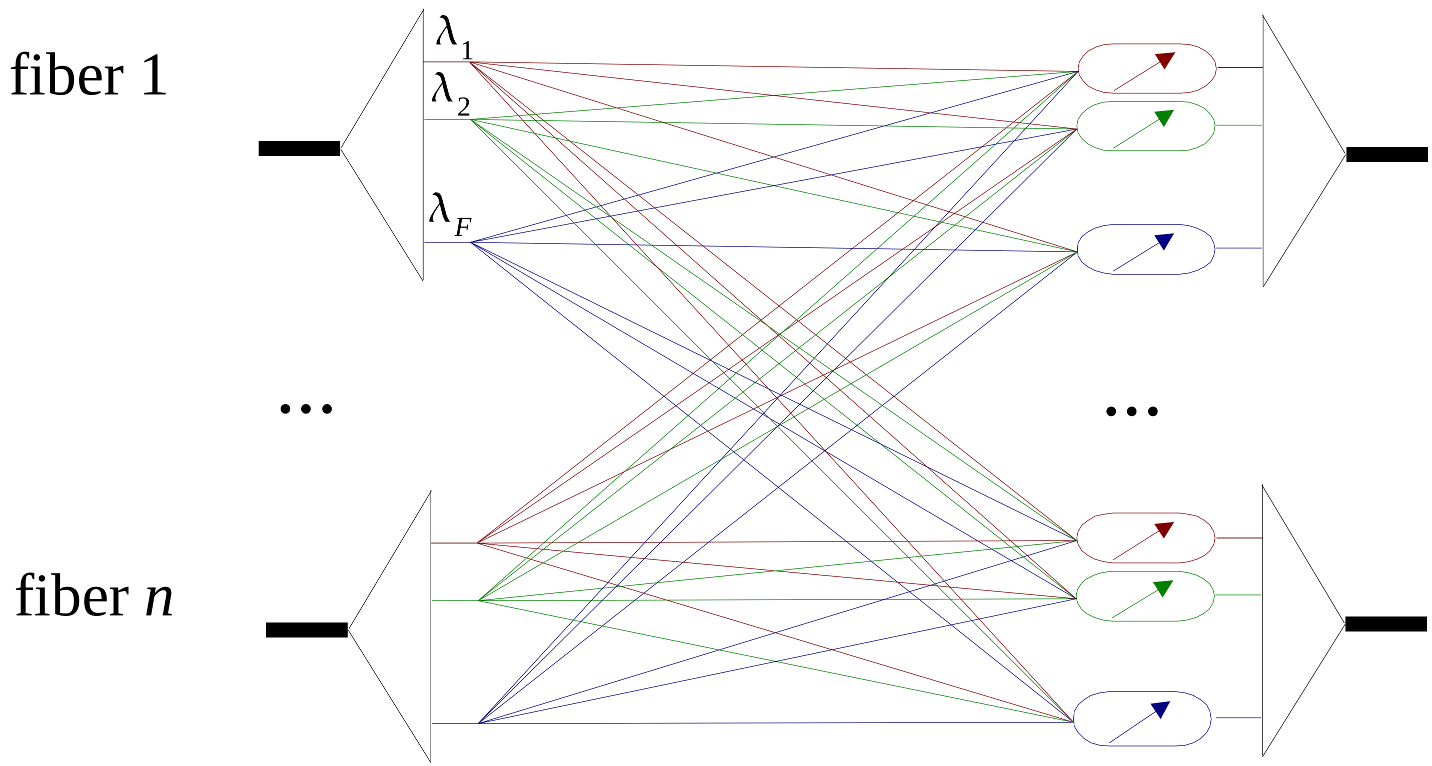
- Traffic assumptions
  - Packet present at input w.p. $0<a<1$
  - Uniform $1/n$ output routing

M = #packets desiring given port   Bin($a/n,n$) ~ Pois(a)

$$L_1 = P\{\text{loss}\} = \frac{E\left[(M-1)_+\right]}{a} \approx 1 - \frac{1}{a} + \frac{e^{-a}}{a} \approx \frac{a}{2}$$

so if packet arrives in 1/5 slots, 1/10 packets lost!

Bufferless Optical Wavelength−Division Multiplexed Packet Switch (full wavelength conversion)



one *nF*x*nF* switch

nF fixed−output all−input wavelength converters

Full conversion

- Good: Blocking falls exponentially in *F*

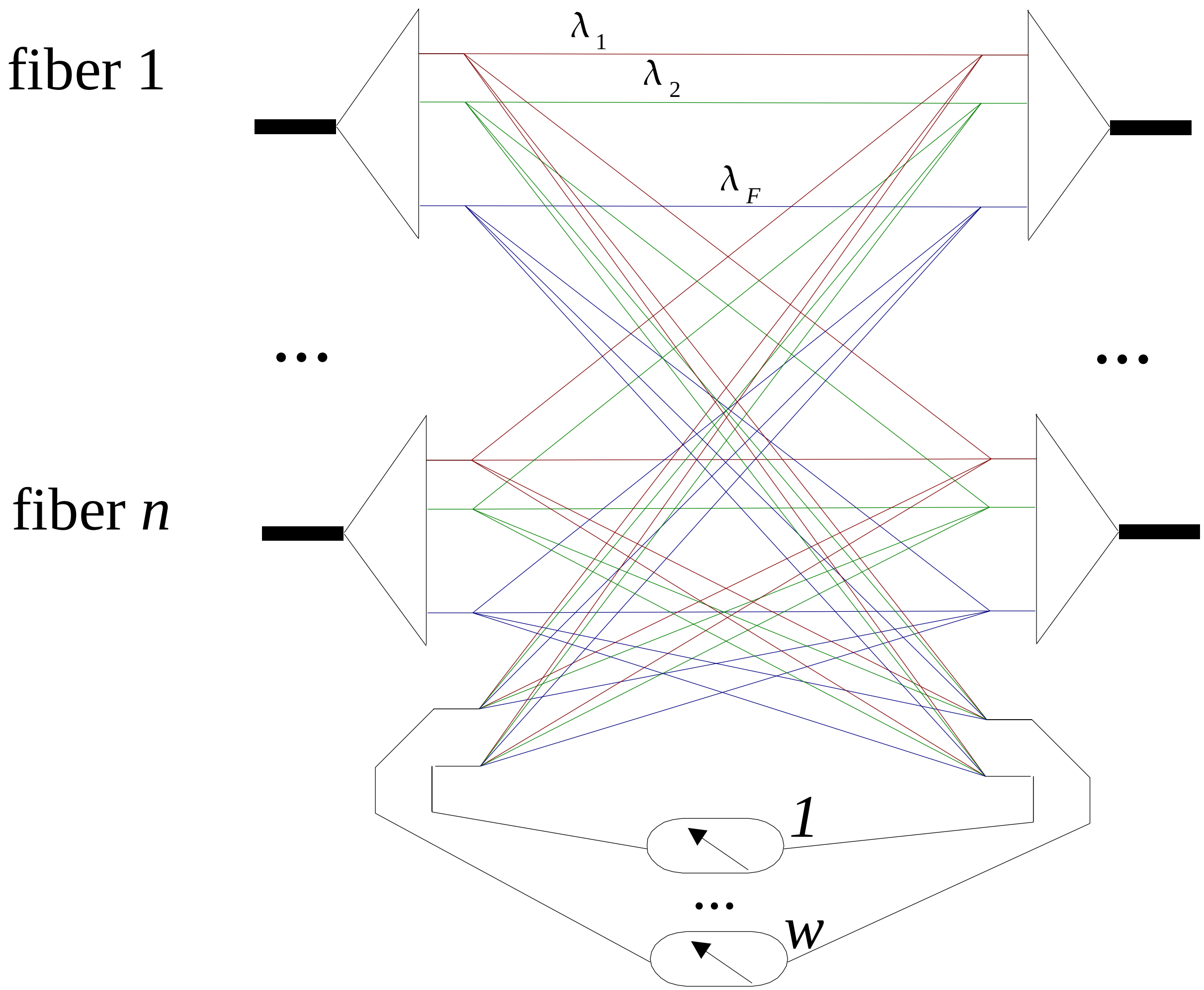
M = #packets desiring given fiber Bin(*a/n*,*nF*) ~ Pois(*aF*)

$$L_F = P\{\text{loss}\} = \frac{E\left[(M-F)_+\right]}{aF} \quad \text{falls off as} \quad \frac{e^{-F}}{F}$$

so if *a*=1/5, *F* = 20, 1 out of 10 billion packets lost!

- Bad: Complex switch
- Bad: Wavelength converters can be expensive !$

Bufferless Optical Wavelength−Division Multiplexed Packet Switch (shared wavelength conversion)



F nxn switches
one *nF*x*w* switch
one *w*x*nF* switch
w tunable wavelength converters

Packet loss (shared conversion)

- More than F packets want a given output fiber

$$L_F$$

- Not enough wavelength converters available

$$L_w^{\mathrm{c}} = \frac{E\left[\left(W-w\right)_+\right]}{anF} = \frac{1}{anF}\sum_{k=w+1}^{nF} P\left[W \geq k\right]$$

- Total loss

$$P\left[\mathrm{loss}\right] = L_F + L_w^{\mathrm{c}}$$

Occupancy problem: P[$W \geq w$]

- Exact combinatorial expression
  - good for small n,F, unwieldy for large n,F
- Simulation
  - good for small n,F, high blocking
- Large deviations approach
  - good for large n,F, low blocking
  - gives insight into how blocking occurs

Overall strategy

- Focus first on single wavelength
- Condition on number of packets present, *r*

$$P[W \geq w] = \sum_{r=1}^{F} P[W \geq w | R=r] \, P[R=r]$$

$$= \sum_{r=1}^{F} e^{-n\, Q(w/n, r/n)} \; e^{-n\, l(r/n)}$$

- As *n* increases, largest term dominates

$$P[W \geq w] \approx \exp(-n[\, Q(w/n, \bar{\beta}) + l(\bar{\beta}) \,])$$

$$\text{where } \bar{\beta} \text{ minimizes } Q(w/n, \beta) + l(\beta)$$

Exponent for number of packets, R

- R is Bin(a, n), the sum of n Bernoulli r.v.'s with success probability a.
- Classic large deviations problem when $r>na$.

$$P[R=r] \approx e^{-n\, l(r/n)}$$

$$l(\beta) = \beta \log \frac{\beta}{a} + (1-\beta) \log \frac{(1-\beta)}{(1-a)}$$

Exponent for W given R

- R = number of packets present
- U = number of occupied urns
- $W = R - U$ = number of redundant packets

$$P[W \geq w \mid R = r] \; = \; P[U \leq r - w \mid R = r] \; = \; P[R \geq r \mid U = r - w]$$

Exponent for *R* given *U*

How many balls needed to fill nα out of n urns?

$$R_n = \sum_{j=1}^{n\alpha} B_j^n$$

where B_j^n = number of balls used to fill j−th urn.
These are independent geometric r.v.'s with success rate $(n-j+1)/n$.

$$P[R \geq n\beta] \approx e^{-n\, J(\beta,\alpha)}$$

$$J(\beta,\alpha) = (\beta-\alpha)\log\rho + (1-\alpha)\log(1-\alpha) + \beta(1-\rho\alpha)$$

$$\text{where } \rho \text{ solves } \beta\rho = -\log(1-\alpha\rho)$$

In previous notation, $Q(\alpha,\beta) = J(\beta,\beta-\alpha)$.

Extensions

- Fixed−color wavelength converters
- Asynchronous operation
- Switches with buffers
- Asymetric traffic patterns

Conclusions

- We will all be happier if we learn to share our crayons.