HALFTONING AND INVERSE HALFTONING

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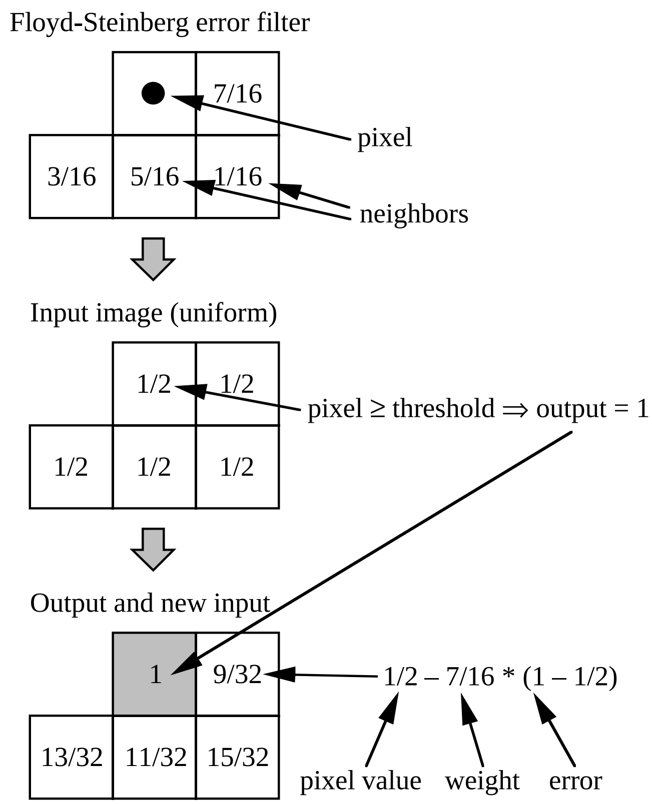
OUTLINE

- Introduction to halftoning
- Error diffusion
- Linear gain model for error diffusion
- Edge sharpening
- Human visual system model
- Quality metrics
- Inverse halftoning

OBJECTIVE

- Develop a formal mathematical framework for analysis and design of error diffusion algorithms for digital image halftoning
  - Model halftoning as two-dimensional delta-sigma modulation
  - Derive objective measures for subjective quality of edge sharpening and noise in halftoned images
  - Apply concepts to inverse halftoning
- Applications:
  - Design of optimal error diffusion filters with respect to subjective quality
  - Optimize quality of halftoned oversampled images
  - Fast inverse halftoning schemes

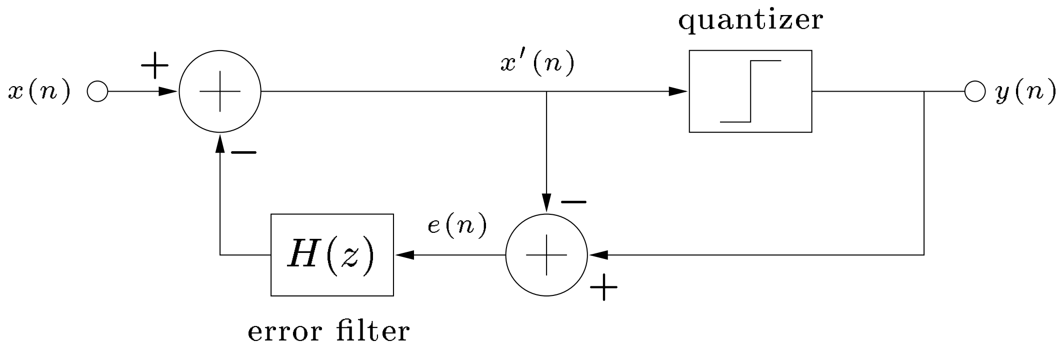
ERROR DIFFUSION



Error diffusion scheme (Floyd-Steinberg filter)

- Error ‘diffuses’ across and down the image
- Neighborhood operation requires more computation than screening
- Best results possible in reasonable time
- Artifacts (worms, idle tones) in some areas

LINEAR ANALYSIS



Equivalent circuit of error diffusion

- Assume quantizer adds white noise uncorrelated with input
- Output given by

$$Y(z) = X(z) + N(z)(1 - H(z))$$

- Signal transfer function (STF) is flat, noise transfer function (NTF) is high-pass
- Circuit is equivalent in form to a noise-shaping feedback coder

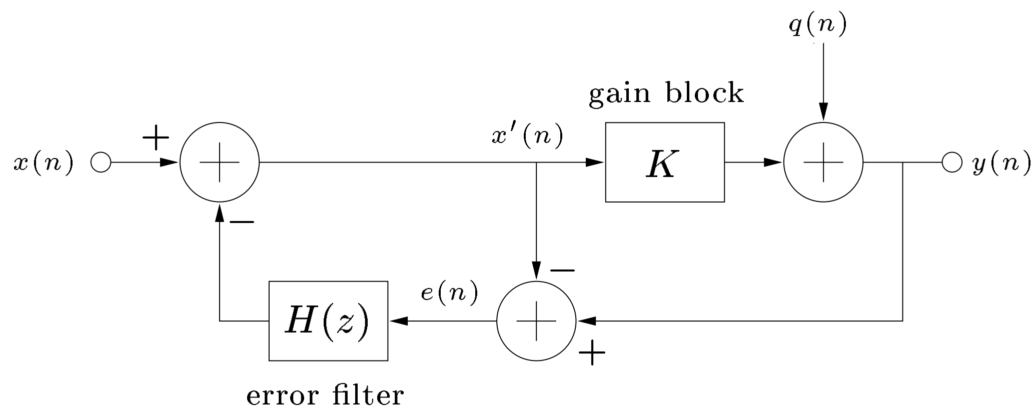
ERROR IMAGE

Original image

Error image

- Error image is highly correlated with input (Knox, 1992)
- Correlation is higher for larger error filters
- Degree of image sharpening increases with correlation
- Suggests *linear gain model* for quantizer
- Signal and noise paths modeled separately (Ardalan and Paulos, 1987)

LINEAR GAIN MODEL



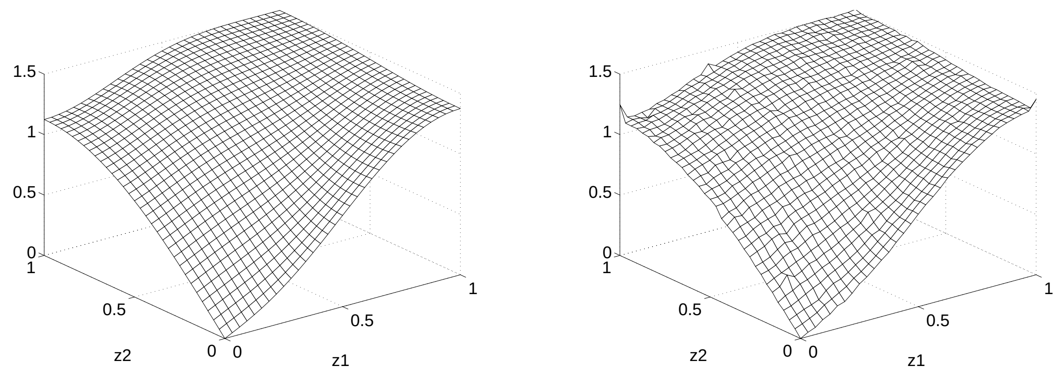
Equivalent circuit with linearized quantizer

- Output given by

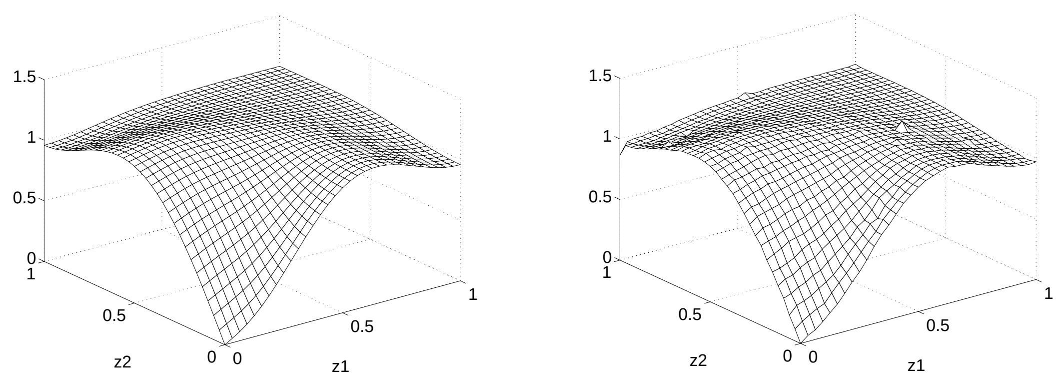
$$Y(z) = \underbrace{\frac{K}{1+(K-1)H(z)}}_{\text{STF}} X(z) + \underbrace{\frac{1-H(z)}{1+(K-1)H(z)}}_{\text{NTF}} N(z)$$

- K is measured empirically; varies with image and error filter
- Accounts for image sharpening
- Noise treated separately ($K = 1$)

NOISE TRANSFER FUNCTION



Predicted Measured

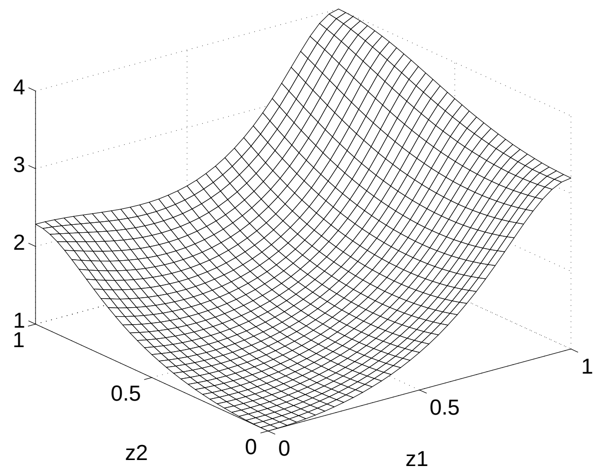


Predicted Measured

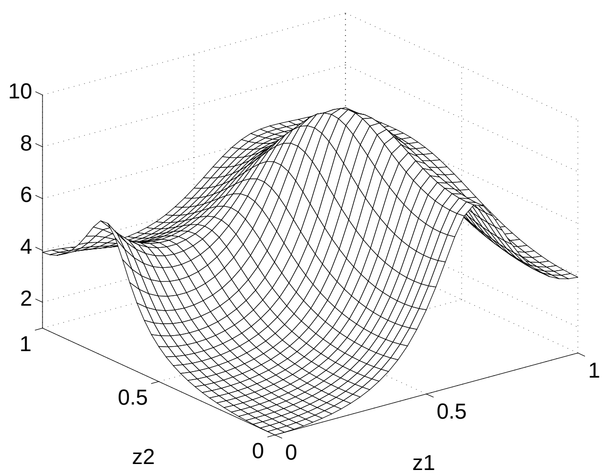
Top: Floyd-Steinberg. Bottom: Jarvis *et al*.

- Similar results for 1-D delta-sigma modulators

SIGNAL TRANSFER FUNCTION



Floyd-Steinberg STF, $K = 2.0$

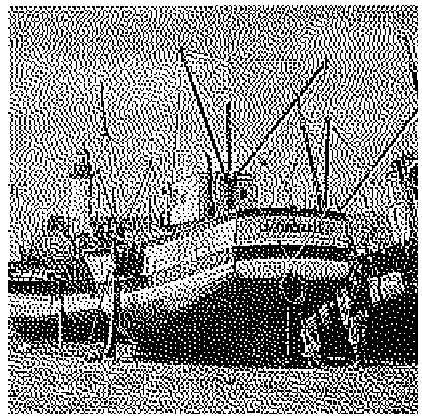


Jarvis *et al*. STF, $K = 4.5$

- Linear gain model accounts for sharpening seen with large error filters

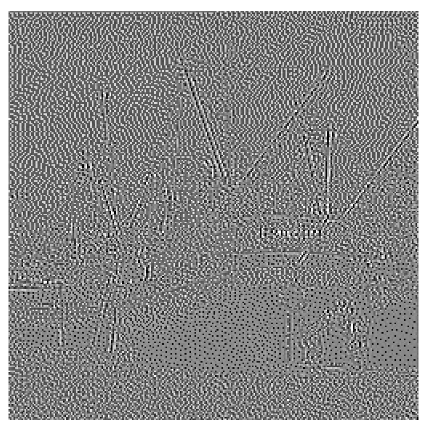
RESULTS OF LINEAR MODEL

Original image



Jarvis halftoned

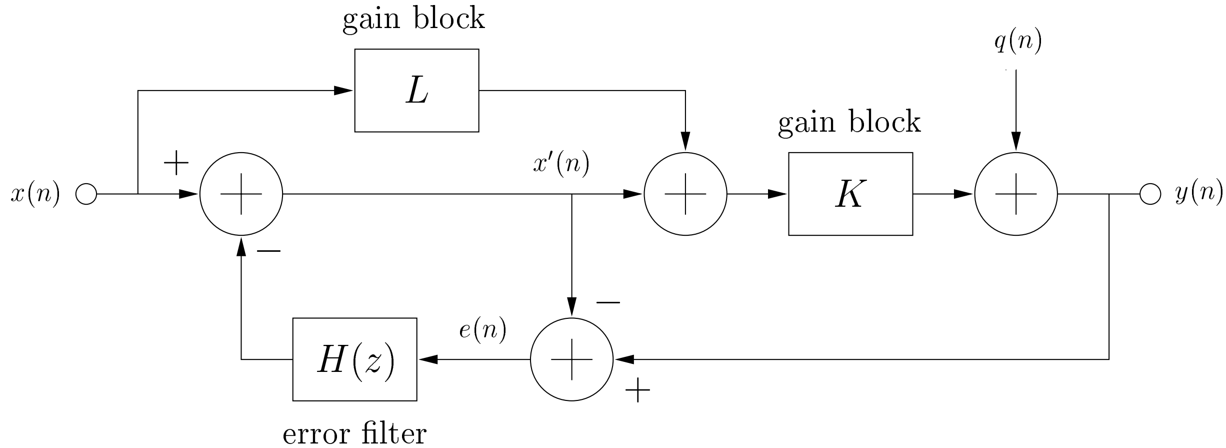
Gain model output



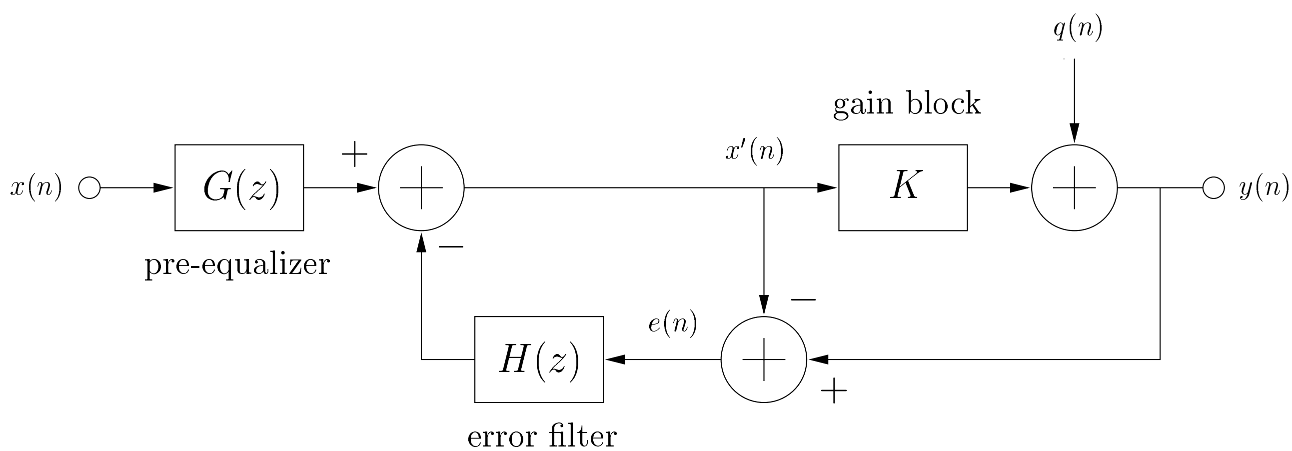
Difference

- Sharpening is decoupled from noise
- Effect of noise shaping can be quantified

SHARPNESS MANIPULATION I



- Knox and Eschbach (1992) showed that image sharpness could be manipulated by changing gain L
- Sharper if $L > 0$; less sharp if $L < 0$
- Exactly equivalent to pre-equalizer scheme



SHARPNESS MANIPULATION II

- Pre-equalization is given by

$$G(z) = 1 + L(1 - H(z))$$

- Error diffusion transfer function:

$$\mathrm{STF} = \frac{K}{1 + (K-1)H(z)}$$

- For flat response, choose

$$L = \frac{1-K}{K}$$

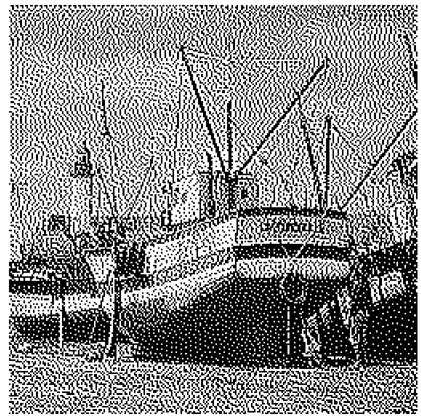
since pre-equalization is then

$$G(z) = \frac{1 + (K-1)H(z)}{K} = \frac{1}{\mathrm{STF}}$$

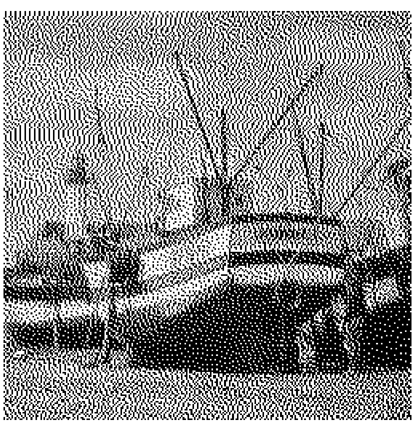
- Gives an easy way to show that computed value of K is correct
- Also allows flat signal transfer function, regardless of error filter

RESULTS

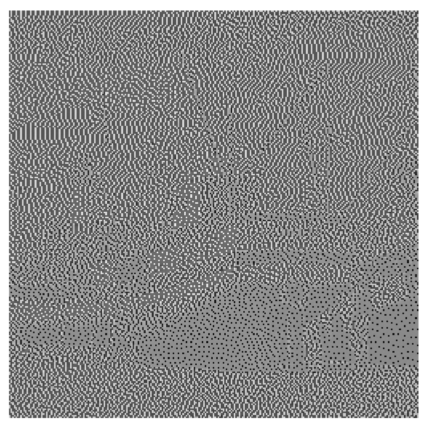
Original image



Jarvis halftoned



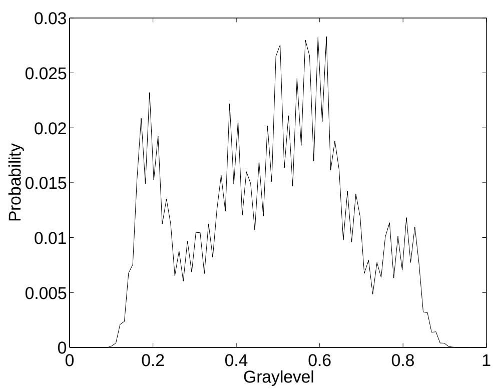
Halftoned, flat STF



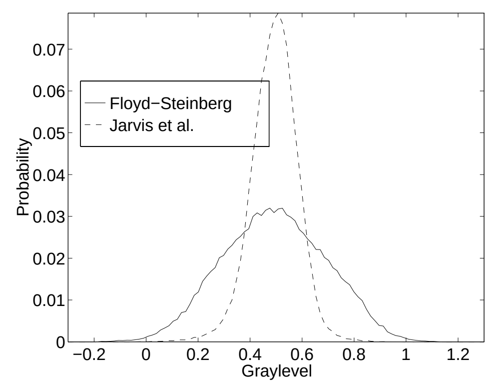
Difference

- Difference is close to zero, as expected
- Further corroboration of linear gain model

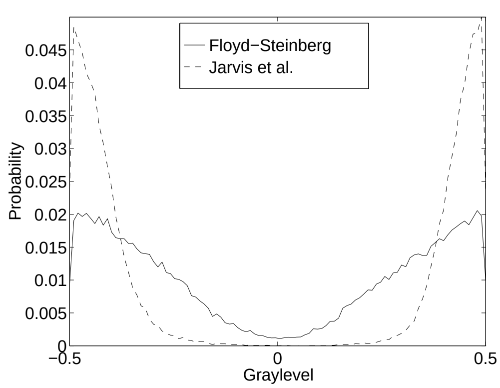
HISTOGRAMS



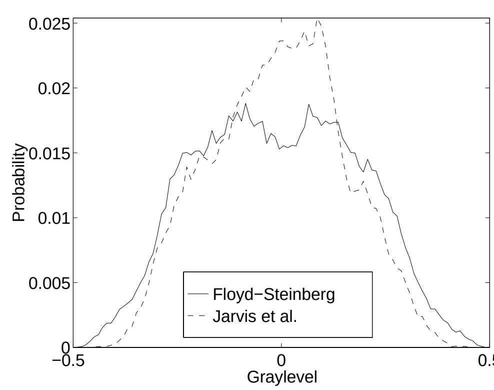
Original image



Quantizer input



Quantizer error



Filter output

- Narrow histogram at quantizer input leads to higher effective quantizer gain, K
- Quantizer error bounded by ± 0.5

SMALL ERROR FILTERS I

- Can small error filters be designed to sharpen as much as large filters?
  - Design large sharpening filter
  - Construct smaller filter whose frequency response is closest to the large filter in a mean square sense (Wong, 1996):

$$g_n = h_n + \alpha$$

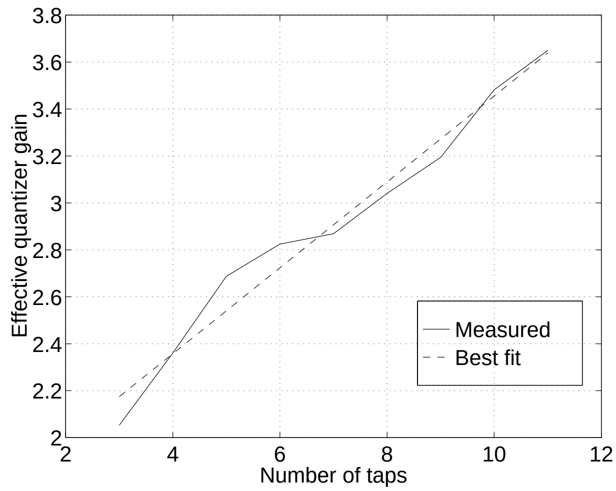
where:

h_n, g_n are the coefficients of the large and small error filters, respectively

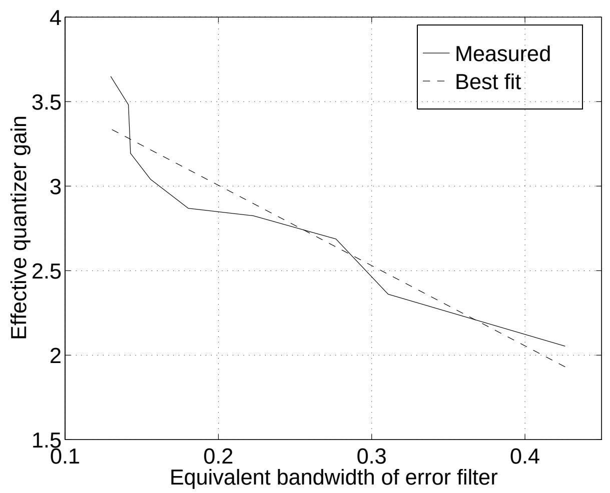
α is a constant chosen to satisfy the gain constraint at DC

- Result: sharpening ability falls off linearly as number of filter taps decreases
- Degree of sharpening related to bandwidth of error filter

SMALL ERROR FILTERS II



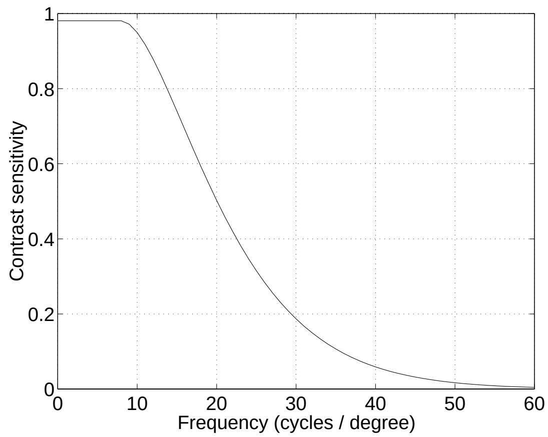
Variation of quantizer gain with filter size



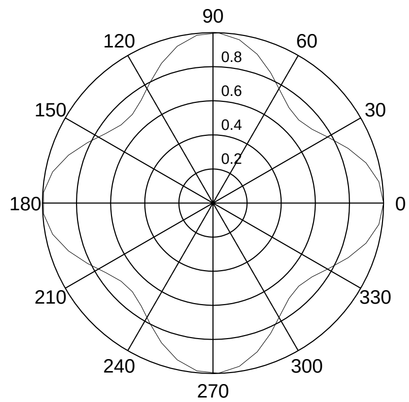
Variation of quantizer gain with filter bandwidth

- Sharpening correlated with bandwidth

VISUAL SYSTEM MODEL



Radial



Angular

Sensitivity of human visual system

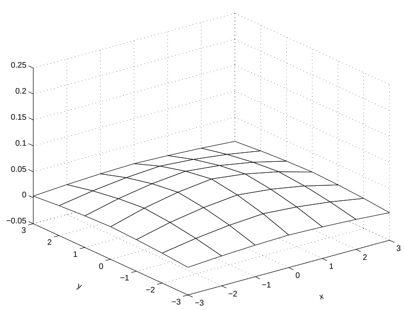
- Noise isolated by subtracting sharpened, noiseless image from halftoned image, or by re-halftoning with appropriate *L* factor to halftone without sharpening
- Weighted noise figure computed using visual system model (Mannos and Sakrison, 1976)
- Good correlation between visual quality and WSNR (Mitsa and Varkur, 1993)

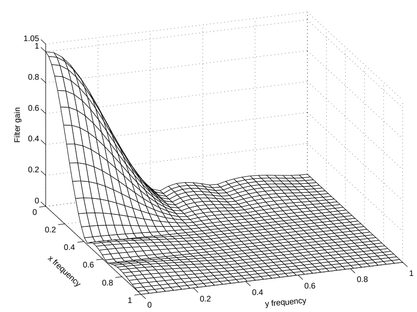
INVERSE HALFTONING

- Inverse halftoning attempts to recreate the original image, given its halftone
- Halftoning discards 7 bits per pixel for an 8-bit image ⇒ perfect reconstruction is not possible
- Applications and reasons:
  - Processing scanned images
  - Data reduction for fax/printers
  - ~~Graduation~~ Pedagogical interest
- Halftoning scheme must be taken into account—artifacts vary greatly
- Error diffused images give the best results when inverse halftoned:
  - Artifacts are mostly noise
  - Noise is highpass
  - Inherent sharpening improves quality

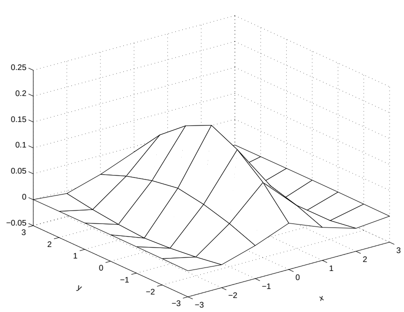
PROPOSED ALGORITHM

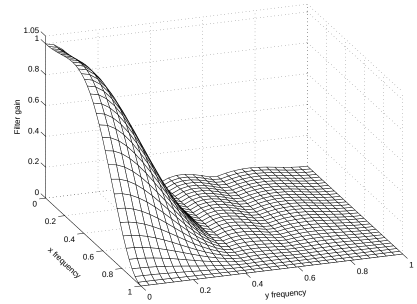
- Inverse halftoning is a form of **wordlength expansion**—from one bit to 8 bits
- Wordlength is increased by filtering (averaging) over a neigborhood
- Tradeoff: grayscale and spatial resolution
- We use an averaging filter whose effective support depends on local edge content





Space Smooth region Freq.

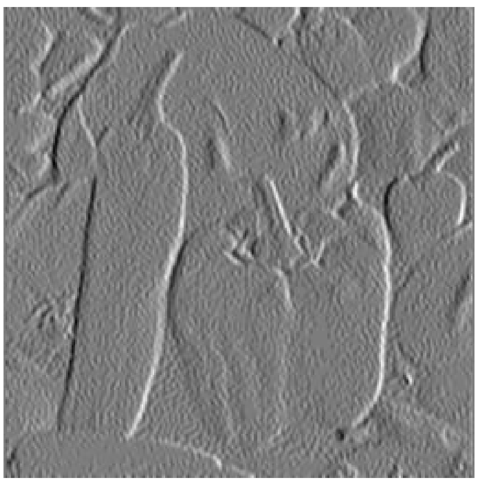




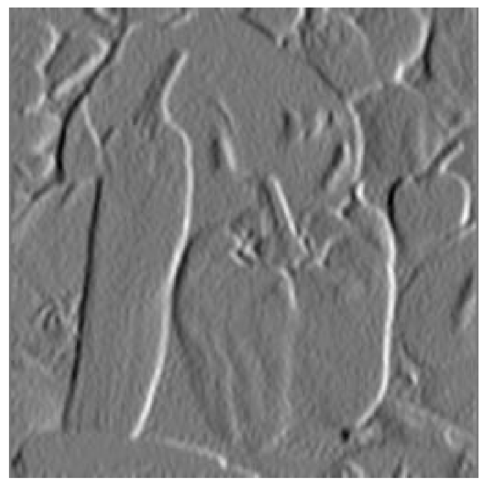
Space Horizontal edge Freq.

EDGE DETECTION

- Multiscale, bi-directional edge detection improves noise performance



Small-scale *x* edges



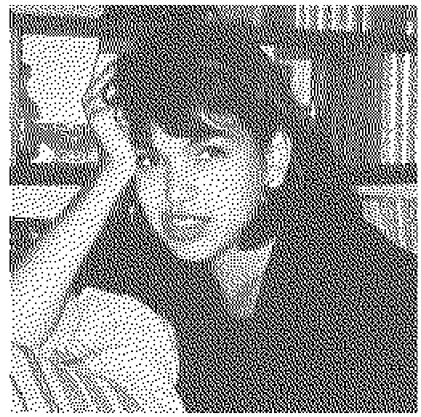
Large-scale *x* edges

Image	SNR (dB)
Small-scale *x*	21.7
Large-scale *x*	28.4
Composite *x*	**26.3**
Small-scale *y*	22.3
Large-scale *y*	28.0
Composite *y*	**26.3**

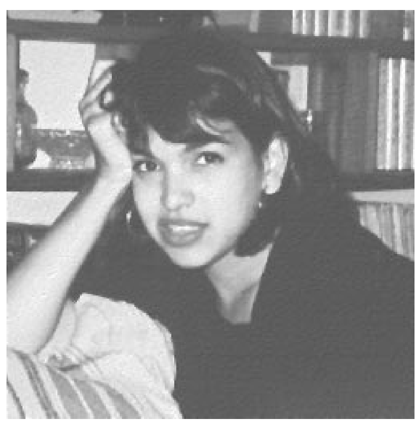
SNR results for edge images

RESULTS

Original image



Halftone



Inverse halftone

- Inverse halftone loses some clarity, but sharp edges remain sharp
- Processing time: 3 seconds on Ultra Sparc
- Less than 4K memory allocated

CONCLUSION

- Halftoning
  - Linear gain model of quantizer produces good modeling results in error diffusion
  - Objective measures of subjective quality by decoupling edge sharpening and noise effects:
    - Edge sharpening proportional to gain
    - Weight noise by perceptual SNR measure
  - Recent results may reveal how gain is related to error filter
- Inverse halftoning
  - Simple method gives state-of-the-art results at low computational cost
  - Modeling of inverse halftoning in similar way to halftoning is being investigated, allowing comparison of schemes
  - Model halftoning as varying spatial blur and additive shaped noise