

*Signal and Image Processing Seminar*

**DESIGN AND QUALITY  
ASSESSMENT OF FORWARD  
AND INVERSE ERROR  
DIFFUSION HALFTONING  
ALGORITHMS**

Ph.D. Defense

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# ***OUTLINE***

- Introduction to halftoning
- Visual quality metrics for forward and inverse halftoning
  - ▶ Human visual system
  - ▶ Weighted noise metric (WSNR)
  - ▶ Modeling other distortions
- Halftoning by error diffusion
  - ▶ Linear gain model
  - ▶ Modified error diffusion
  - ▶ Noise metric
  - ▶ Tonality metric
- Inverse halftoning
  - ▶ Algorithm design and results
  - ▶ Modeling inverse halftoning
  - ▶ Quality metrics
- Rehalftoning and interpolation
- Contributions

# ***INTRODUCTION: HALFTONING***

- Was analog, now digital image processing
- Wordlength reduction for images
  - ▶ 8-bit to 1-bit for grayscale
  - ▶ 24-bit RGB to 8-bit for color displays
  - ▶ 24-bit RGB to CMY for color printers
- Applications
  - ▶ Printers
  - ▶ Digital copiers
  - ▶ Liquid crystal displays
  - ▶ Video cards
- Halftoning methods
  - ▶ Screening
  - ▶ Error diffusion
  - ▶ Direct binary search
  - ▶ Hybrids

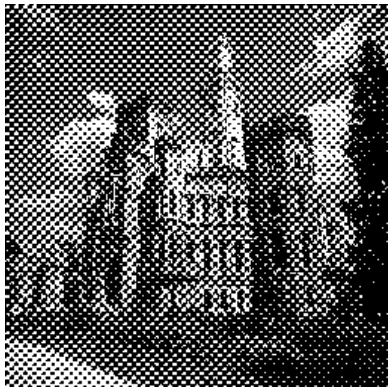
# ***EXAMPLE HALFTONES***



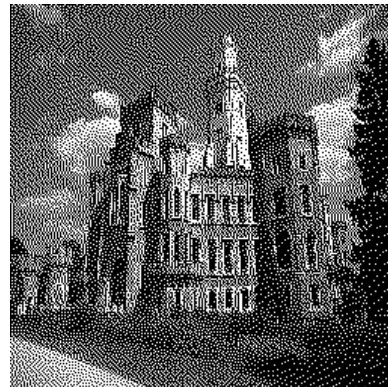
**Original image**



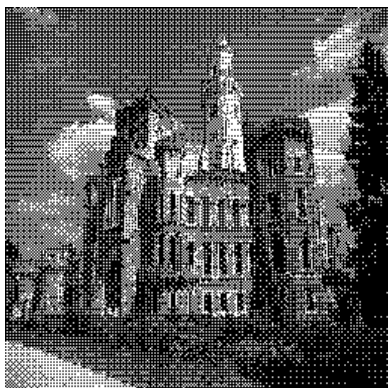
**Direct binary search**



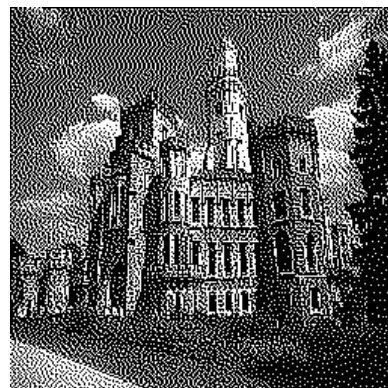
**Clustered dot screen**



**Error diffusion I**

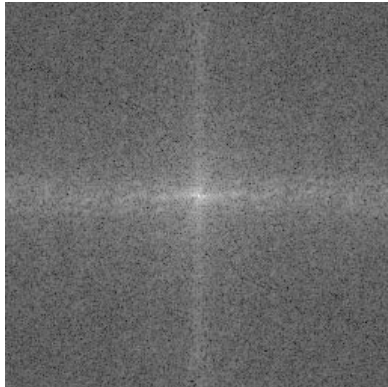


**Dispersed dot screen**

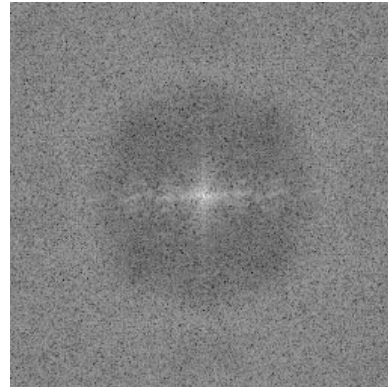


**Error diffusion II**

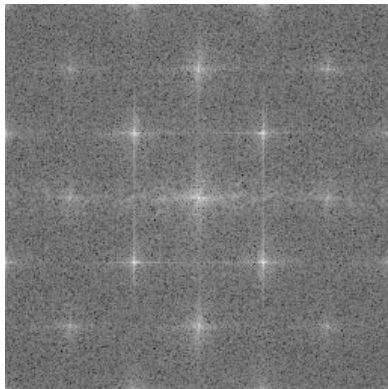
# ***FOURIER TRANSFORMS***



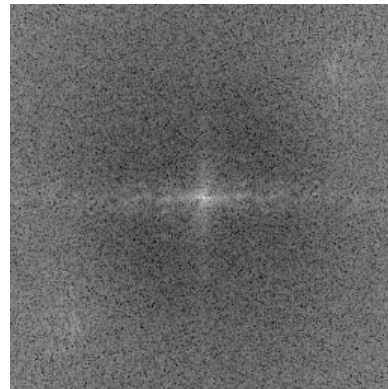
**Original image**



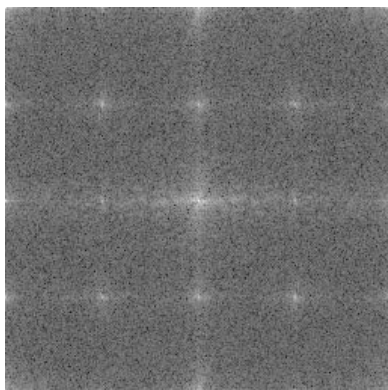
**Direct binary search**



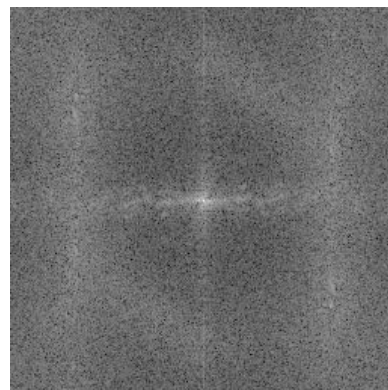
**Clustered dot screen**



**Error diffusion I**



**Dispersed dot screen**



**Error diffusion II**

## ***PROBLEMS TO BE SOLVED***

- **Visual quality metrics for forward and inverse halftones**
  - ▶ Quantify frequency distortion
  - ▶ Quantify artifacts
  - ▶ Quantify quantization noise
- **Modeling error diffusion**
  - ▶ Develop tractable model
  - ▶ Demonstrate accuracy of model
  - ▶ Use model to design applications
- **Inverse halftoning**
  - ▶ Develop efficient algorithm
  - ▶ Develop model for inverse halftoning

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  - ▶ Linear gain model
  - ▶ Modified error diffusion
  - ▶ Noise metric
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# ***HUMAN VISUAL SYSTEM (HVS)***

- Non-linear, spatially varying
- Assuming linearity, spatial invariance explains [Cornsweet 1970]
  - ▶ Mach band effect
  - ▶ Apparent brightness vs. intensity



White noise  
SNR = 10 dB

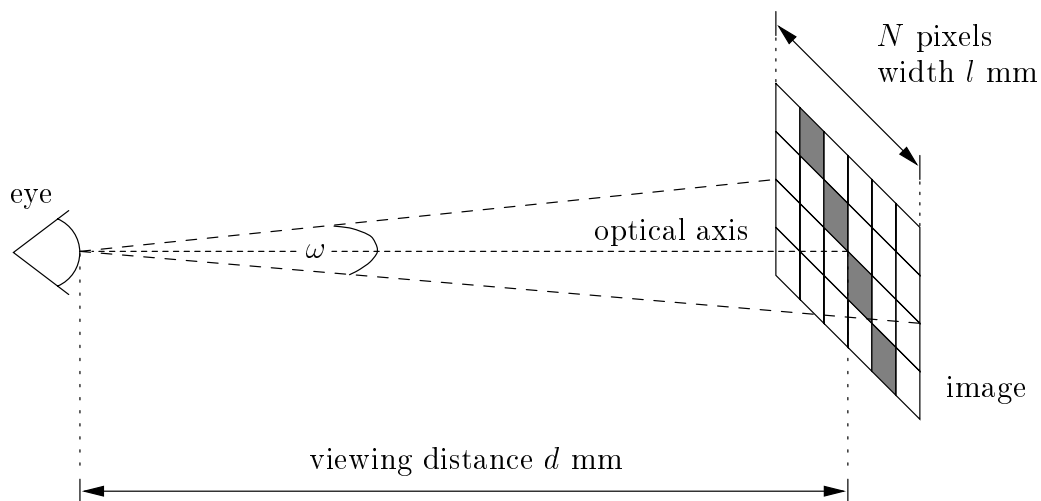


Blue (highpass) noise  
SNR = 10 dB

- Weight by spatial frequency to quantify visual impact of noise

# ANGULAR FREQUENCY

- Sensitivity depends on angular frequency subtended at eye
- Compute angular frequency from image size (pixels), printed image size (mm), viewing distance (mm)



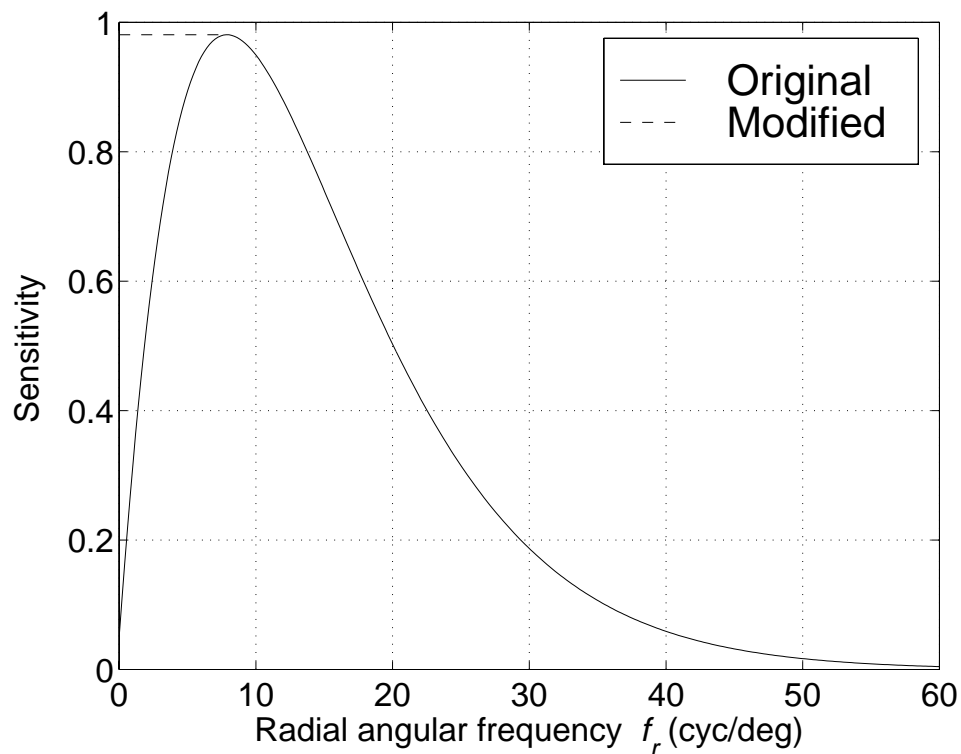
- At Nyquist frequency

$$f_a = \frac{N\pi d}{360l} \text{ cycles/degree}$$

# ***CONTRAST SENSITIVITY (CSF)***

- Minimum contrast to distinguish sine grating from uniform field
- Model [Mannos & Sakrison 1974]
- Modification [Mitsa & Varkur 1993]

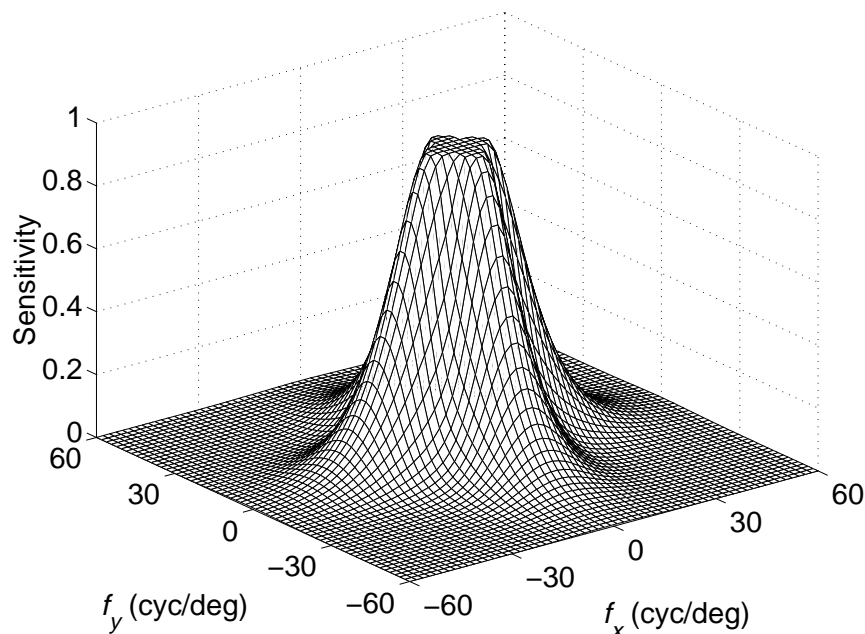
$$\text{CSF} = 2.6 (0.02 + 0.1 f_a) e^{-(0.1 f_a)^{1.1}}$$



- Orientation-independent

# WEIGHTED SNR METRIC

- Include orientation (angular dependence) in CSF  
[Sullivan, Miller & Pios 1993]

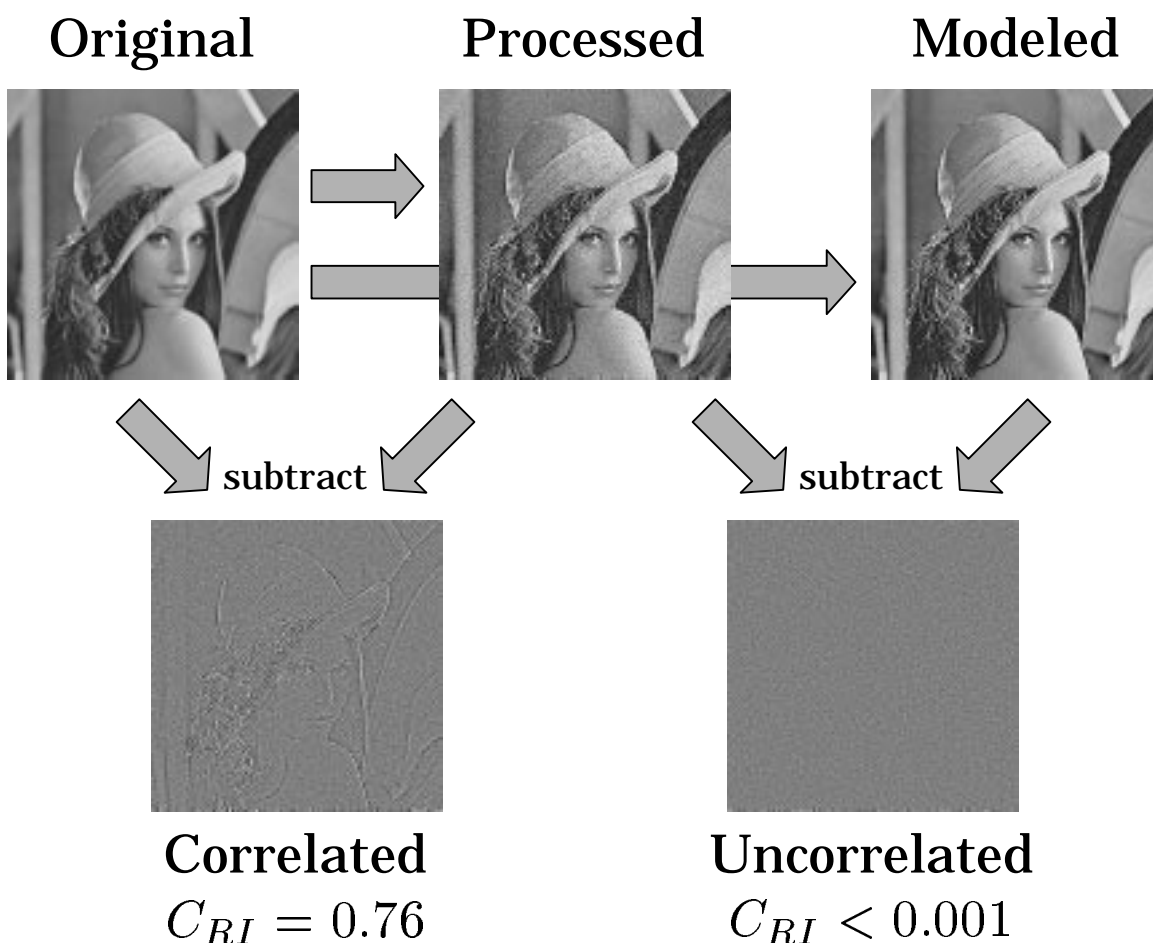


- Compute weighted signal-to-noise ratio between original image  $x$  and processed image  $y$

$$\text{WSNR} = 10 \log_{10} \left( \frac{\sum |\text{CSF} \times X(u, v)|^2}{\sum |\text{CSF} \times (X(u, v) - Y(u, v))|^2} \right)$$

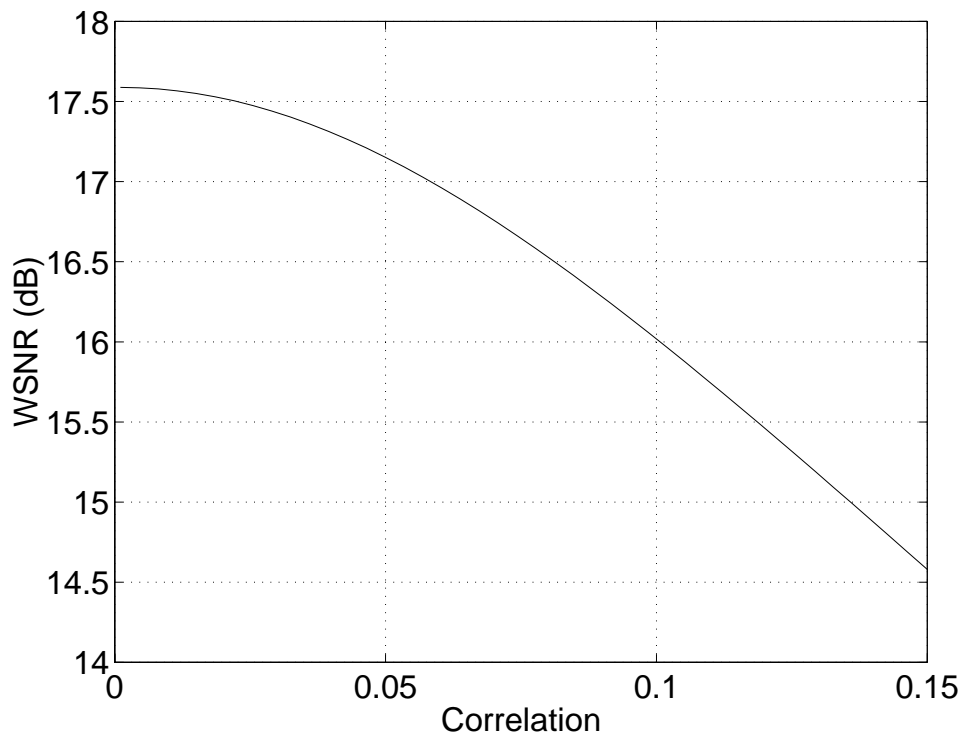
# MODELING DISTORTION

- WSNR is a *noise* metric
  - ▶ Difference (residual) between original and processed images must be noise
  - ▶ Model, compensate for other distortions
  - ▶ Measure correlation of residual and original; for accuracy,  $C_{RI} < 0.020$



## ***WSNR vs. CORRELATION***

- **WSNR increasingly inaccurate as correlation increases**

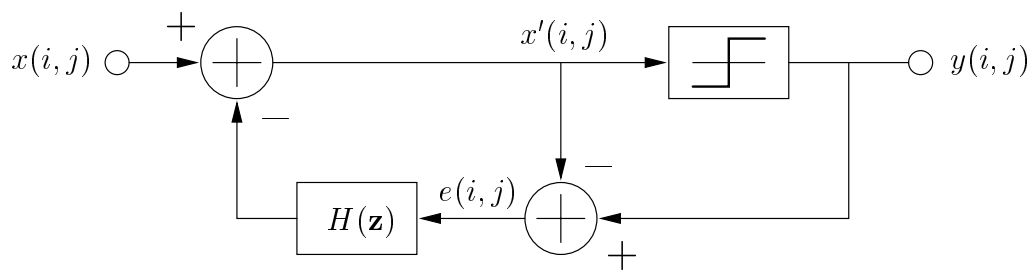


# ***OUTLINE***

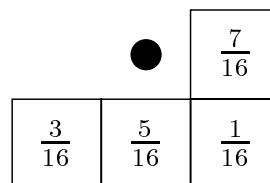
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# ERROR DIFFUSION

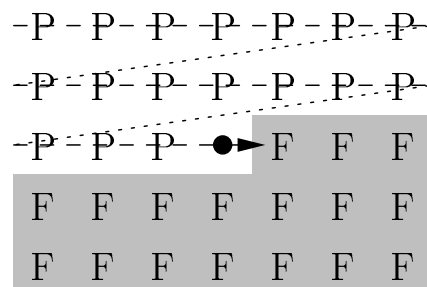
- 2-D delta-sigma modulator
- Noise shaping feedback coder



- Error filter



- Raster scan order



**P** = Past  
**F** = Future

- Serpentine scan also used

## ***ERROR DIFFUSION (contd.)***

- **Quantizer**

$$y(i, j) = \begin{cases} 0, & x'(i, j) < 0.5 \\ 1, & x'(i, j) \geq 0.5 \end{cases}$$

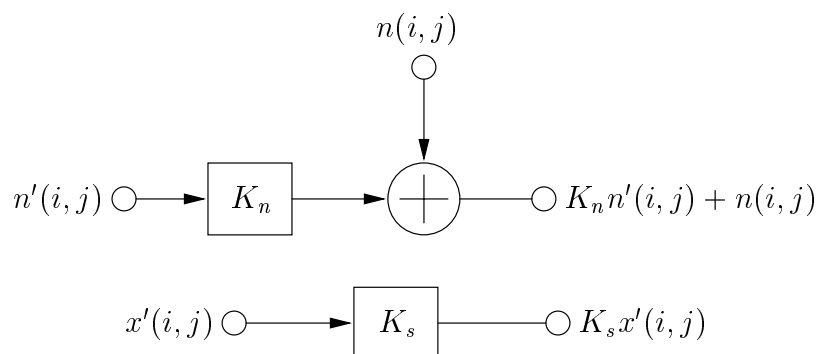
- **Governing equations**

$$\begin{aligned} e(i, j) &= y(i, j) - x'(i, j) \\ x'(i, j) &= x(i, j) - h(i, j) * e(i, j) \end{aligned}$$

- **Non-linearity difficult to analyze**

- **Linearize quantizer**

[Kite, Evans, Bovik & Sculley 1997]



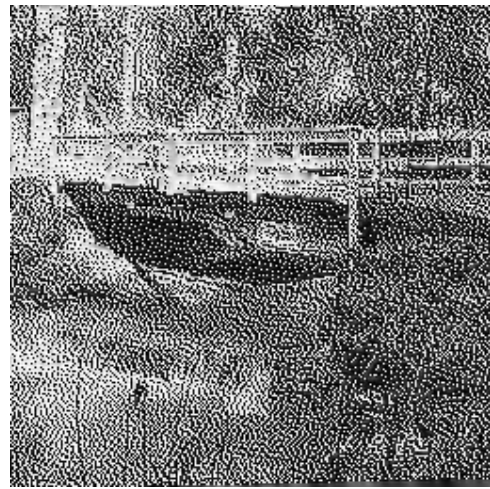
- **Separate signal and noise paths**  
[Ardalan & Paulos 1987]

## ***LINEAR GAIN MODEL***

- Quantization error correlated with input [Knox 1992]



Floyd-Steinberg



Jarvis, Judice & Ninke

- Least squares fit of quantizer input to output defines signal gain

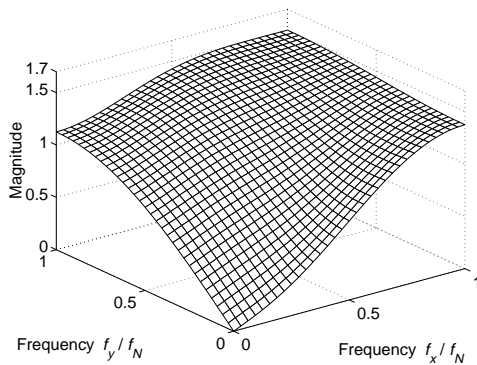
$$K_s = \frac{E[|x'(i, j)|]}{2E[x'(i, j)^2]}$$

- Signal gain:  $K_s \approx \text{constant}$
- Noise gain:  $K_n = 1$

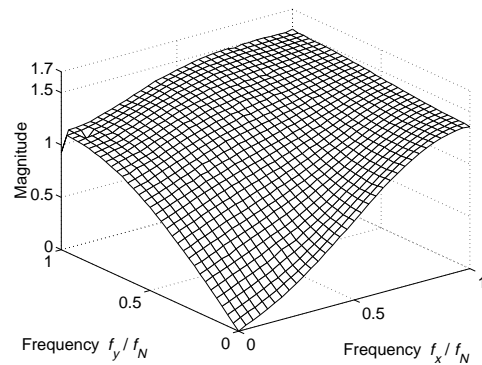
# GAIN MODEL PREDICTIONS

## ■ Noise transfer function (NTF)

$$\text{NTF} = 1 - H(\mathbf{z})$$



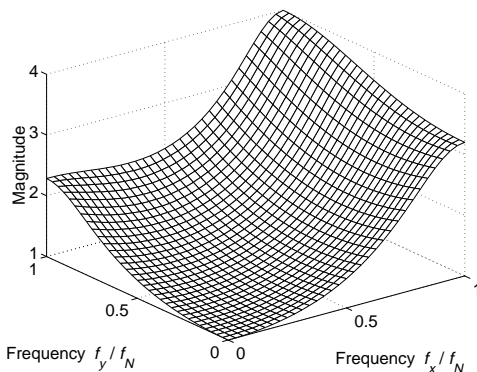
Predicted



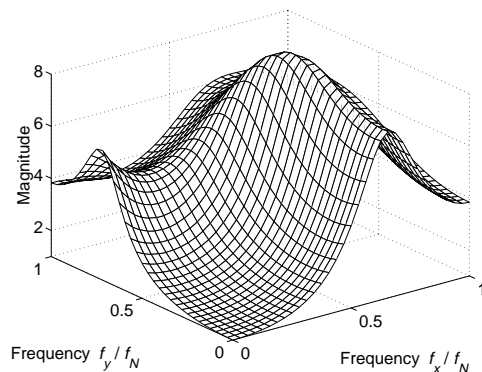
Measured

## ■ Signal transfer function (STF)

$$\text{STF} = \frac{K_s}{1 + (K_s - 1)H(\mathbf{z})}$$



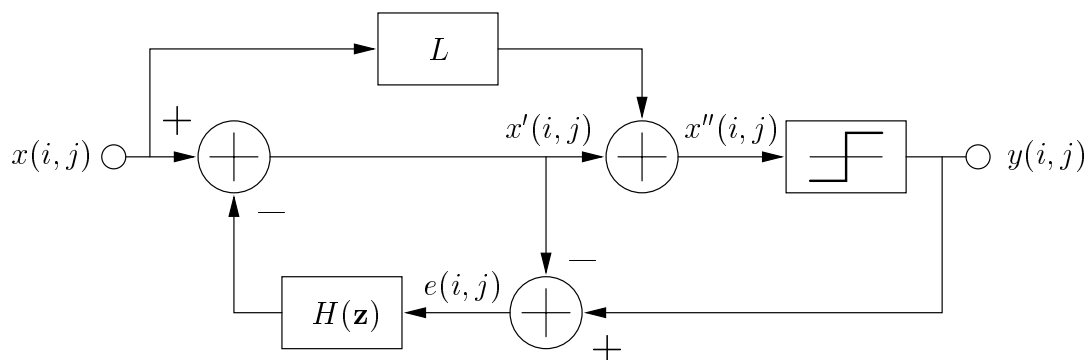
Floyd-Steinberg



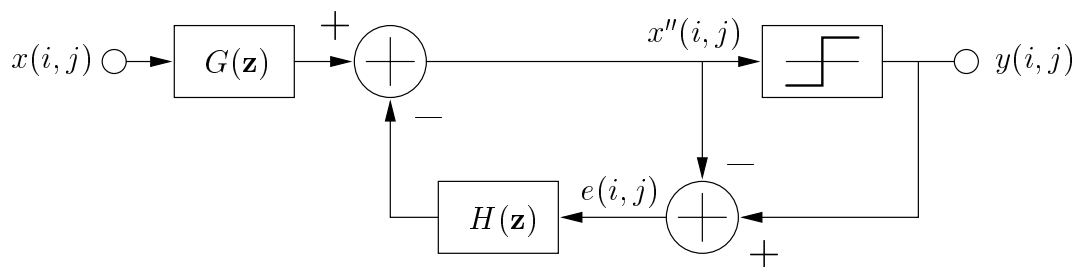
Jarvis *et al.*

# MODIFIED ERROR DIFFUSION

- Efficient method of adjusting sharpness [Eschbach & Knox 1991]



- Equivalent circuit: pre-filter



$$G(\mathbf{z}) = 1 + L(1 - H(\mathbf{z}))$$

# UNSHARPENED HALFTONES

- If  $L = \frac{1 - K_s}{K_s}$  then STF = 1 (flat)



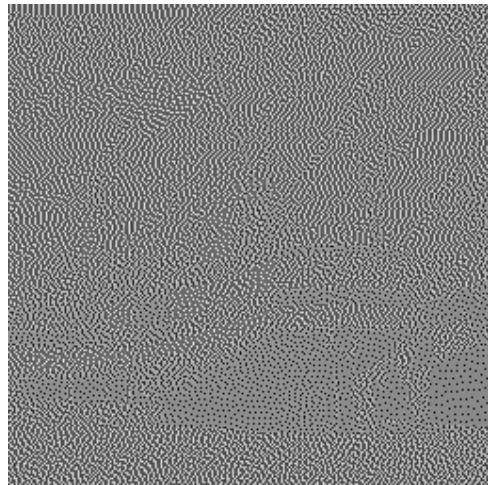
Original image



Jarvis halftone



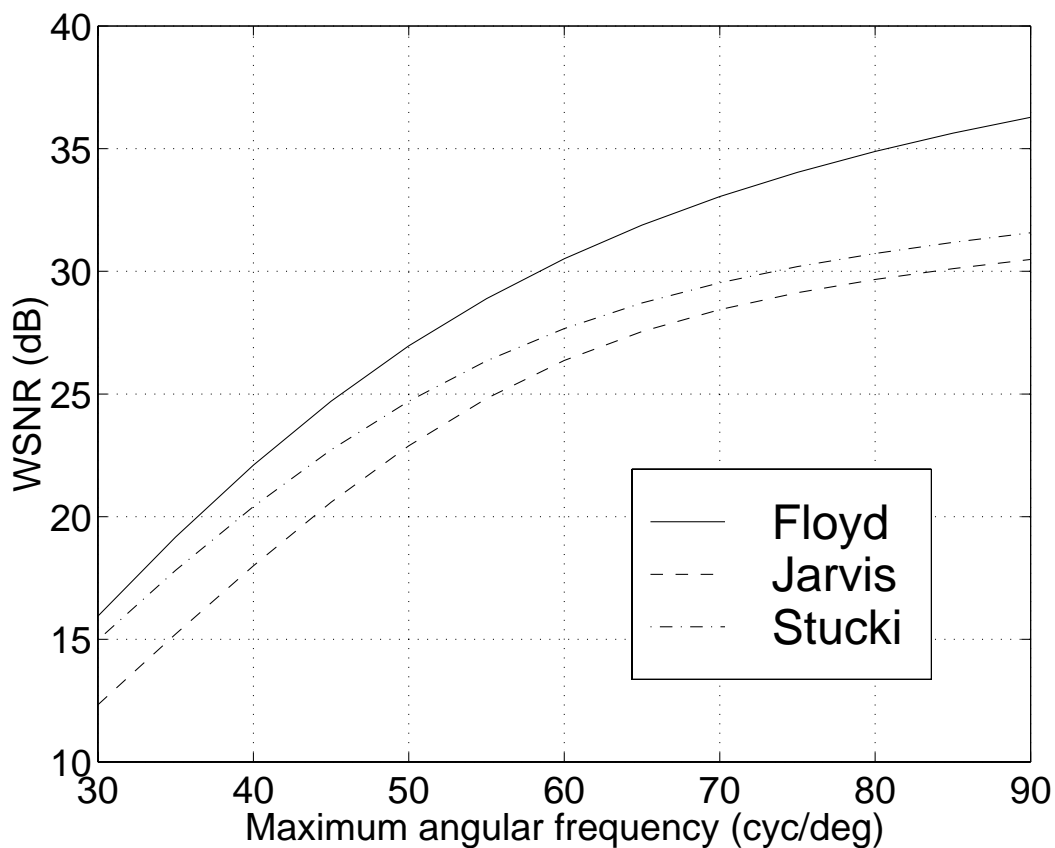
Unsharpened halftone



Residual

# OBJECTIVE NOISE METRIC

- To find WSNR
  - ▶ Compute signal gain  $K_s$ , or use average
  - ▶ Generate unsharpened halftone using modified error diffusion
  - ▶ Compute WSNR of unsharpened halftone relative to original image



## ***OBJECTIVE TONALITY METRIC***

- Limit cycles cause visual ‘worm’ artifacts [Fan & Eschbach 1994]
- Larger filters and serpentine scan result in lower tonality
- Define tonality metric
  - ▶ Measure total distortion of sine grating

$$T = \left[ \frac{1}{Y(e^{j\omega_f})Y^*(e^{j\omega_f})} \sum_{\omega \in \{\omega_d\}} Y(e^{j\omega})Y^*(e^{j\omega}) \right]^{\frac{1}{2}}$$

- ▶ Average  $T$  over grating frequencies
- Agrees with visual results
  - ▶ Correct ordering of error filters
  - ▶ Serpentine scan less tonal

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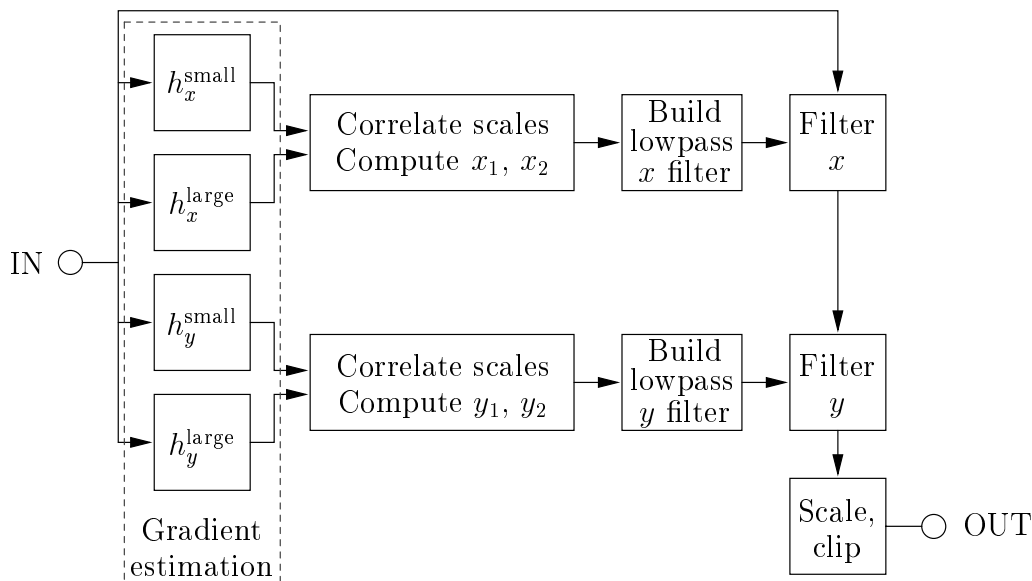
# ***INVERSE HALFTONING***

- Attempt to recover grayscale images from halftones
- Applications
  - ▶ Digital copiers
  - ▶ Scanner software
- Many approaches:
  - ▶ Bayesian estimation  
[Schweizer & Stevenson 1993]
  - ▶ Vector quantization  
[Ting & Riskin 1994]
  - ▶ Projection onto convex sets  
[Hein & Zakhor 1995]
  - ▶ Lowpass smoothing and nonlinear filtering [Wong 1995]
  - ▶ Wavelet denoising  
[Xiong, Orchard & Ramchandran 1997]
- Most are iterative and slow
- Best results from wavelet scheme

## ***PROPOSED METHOD***

- **Apply anisotropic diffusion**  
[Kite, Damera-Venkata, Evans & Bovik 1998]
  - ▶ Estimate image gradients
  - ▶ Compute diffusion coefficient
  - ▶ Smooth within areas, preserve edges
- **Unique environment**
  - ▶ Highpass noise, SNR  $\approx$  3 dB
  - ▶ Tonal
- **Solution**
  - ▶ Specialized gradient estimator
  - ▶ Correlate estimate across scales  
[Mallat & Zhong 1992]
  - ▶ Separable—smooth parallel to edges
- **Local operations**
  - ▶ Low memory requirement
  - ▶ Low computational cost

## ***PROPOSED METHOD (contd.)***

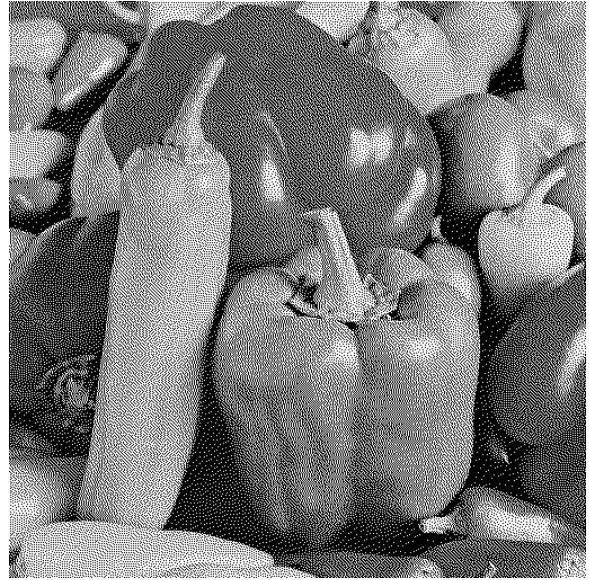


- **Estimate gradients at two scales**
  - ▶  $7 \times 7, 5 \times 5$  FIR filters
  - ▶ Integer additions only
- **Correlate gradients across scales**
  - ▶ 5 dB improvement in gradient SNR
- **Construct parametric smoother**
  - ▶  $7 \times 7$  separable FIR filter
  - ▶ Family optimized for halftones
  - ▶ Quantized integer coefficients

# ***INVERSE HALFTONE RESULTS***



**Original image**



**Halftone**



**Proposed method**



**Wavelet method**

# ***INVERSE HALFTONING MODEL***

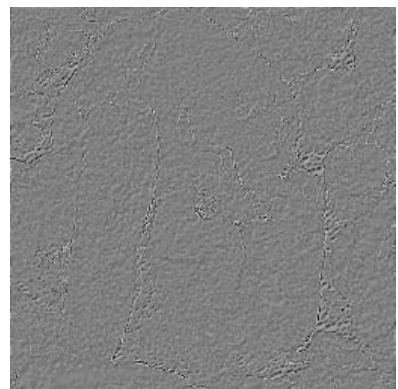
- Forward/inverse halftoning system blurs image and adds noise
- Model inverse halftoning
  - ▶ Compute unsharpened halftone
  - ▶ Inverse halftone; save filter parameters at each pixel
  - ▶ Filter original image using saved filters
- Typical correlation
  - ▶ Inverse halftone:  $C_{RI} = 0.32$
  - ▶ Model inverse halftone:  $C_{RI} = 0.01$



Inverse halftone



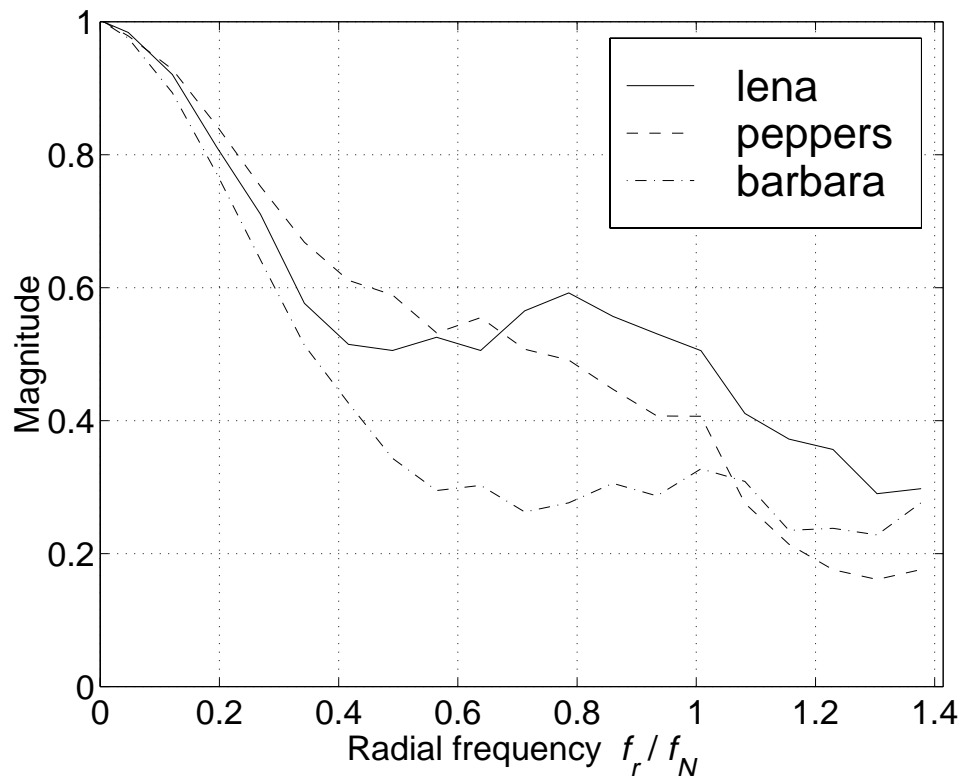
Modeled



Residual ( $\times 4$ )

# INVERSE HALFTONE QUALITY

- Compute WSNR
- Compute effective transfer function
  - ▶ Divide FFT of model inverse halftone by FFT of original image
  - ▶ Radially average over annuli

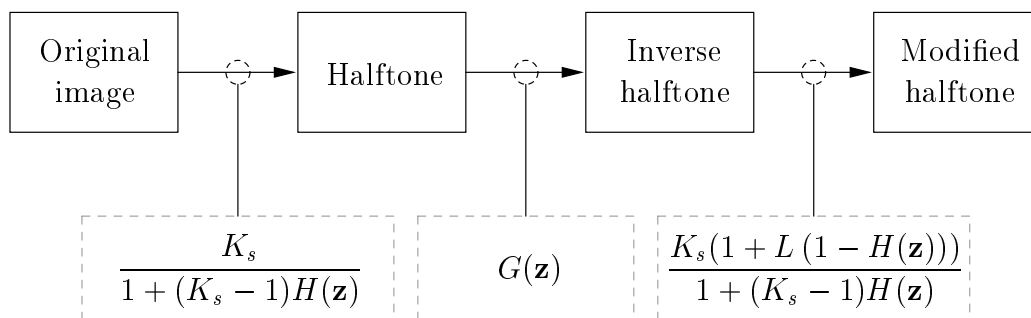


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- **Rehalftoning and interpolation**
  - **Contributions**

# REHALFTONING

- Halftone conversion, manipulation
- Assume input and output are error diffused halftones
- Fixed lowpass inverse halftoning filter, compromise cut-off frequency
  - ▶ Noise leakage masked by halftoning
  - ▶ Blurring correctable by modified error diffusion
  - ▶ Computationally efficient



- Use linear gain model to design  $L$  for flat response
- Use approximation for digital frequency:  $e^{j\omega} \approx 1 + j\omega - \omega^2/2$

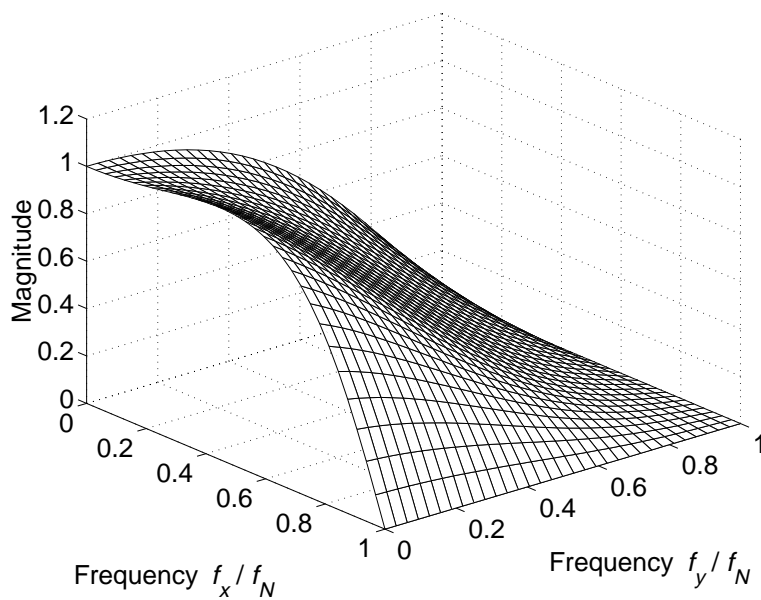
# REHALFTONING RESULTS



Original image



Rehalthtone



Signal transfer function

# ***INTERPOLATION***

- Image resizing
- Different methods (increasing cost)
  - ▶ Nearest neighbor
  - ▶ Bilinear
  - ▶ Bicubic, cubic splines, lowpass filtering
- Nearest neighbor, bilinear methods
  - ▶ Low computational cost
  - ▶ Artifacts masked by quantization noise in halftone
  - ▶ Blurring correctable by modified error diffusion
- Examine  $\times 2$  interpolation; method applies to any scaling factor
- Design  $L$  for flat transfer function using linear gain model
- $L$  constant for given interpolation scheme

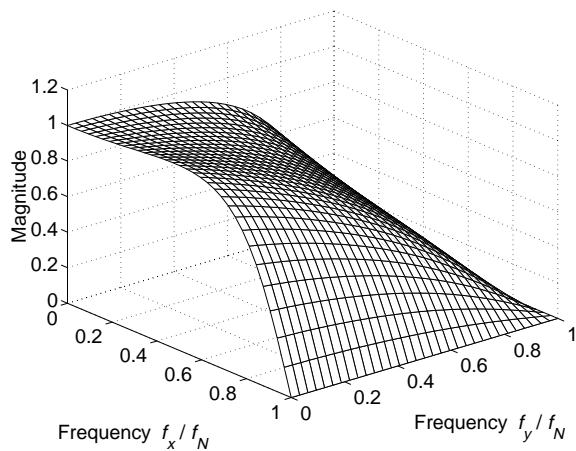
# INTERPOLATION RESULTS



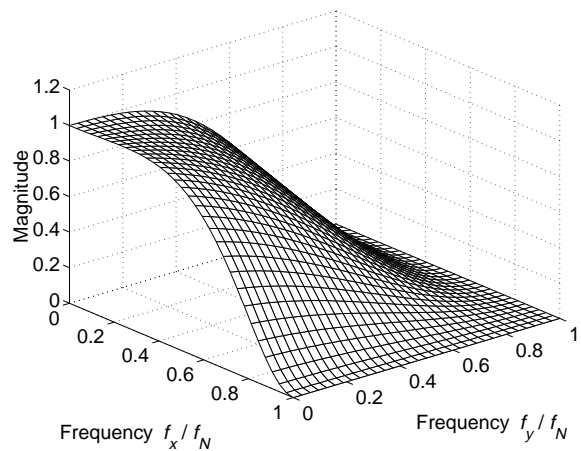
Nearest neighbor  $\times 2$



Bilinear  $\times 2$



Transfer function  
 $L = -0.0105$



Transfer function  
 $L = 0.340$

# ***CONTRIBUTIONS***

- **Visual quality metrics for forward and inverse halftones**
  - ▶ Restriction on correlation for accuracy of WSNR metric
- **Linear gain model of error diffusion**
  - ▶ Accuracy of model established
  - ▶ Tonality metric for artifacts
  - ▶ Link between filter gain and signal gain
- **Inverse halftoning**
  - ▶ New efficient method, suitable for hardware and embedded software
  - ▶ Model for inverse halftoning
  - ▶ Quality metrics for inverse halftones
- **Rehalftoning and interpolation**
  - ▶ Efficient algorithms
  - ▶ Verifies validity of linear gain model