DESIGN AND QUALITY ASSESSMENT OF FORWARD AND INVERSE ERROR DIFFUSION HALFTONING ALGORITHMS

Ph.D. Defense

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OUTLINE

- Introduction to halftoning
- Visual quality metrics for forward and inverse halftoning
  - Human visual system
  - Weighted noise metric (WSNR)
  - Modeling other distortions
- Halftoning by error diffusion
  - Linear gain model
  - Modified error diffusion
  - Noise metric
  - Tonality metric
- Inverse halftoning
  - Algorithm design and results
  - Modeling inverse halftoning
  - Quality metrics
- Rehalftoning and interpolation
- Contributions
INTRODUCTION: HALFTONING

- Was analog, now digital image processing
- Wordlength reduction for images
  - 8-bit to 1-bit for grayscale
  - 24-bit RGB to 8-bit for color displays
  - 24-bit RGB to CMY for color printers

- Applications
  - Printers
  - Digital copiers
  - Liquid crystal displays
  - Video cards

- Halftoning methods
  - Screening
  - Error diffusion
  - Direct binary search
  - Hybrids
FOURIER TRANSFORMS

Original image  Direct binary search
Clustered dot screen  Error diffusion I
Dispersed dot screen  Error diffusion II
PROBLEMS TO BE SOLVED

- Visual quality metrics for forward and inverse halftones
  - Quantify frequency distortion
  - Quantify artifacts
  - Quantify quantization noise

- Modeling error diffusion
  - Develop tractable model
  - Demonstrate accuracy of model
  - Use model to design applications

- Inverse halftoning
  - Develop efficient algorithm
  - Develop model for inverse halftoning
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HUMAN VISUAL SYSTEM (HVS)

- Non-linear, spatially varying
- Assuming linearity, spatial invariance explains [Cornsweet 1970]
  - Mach band effect
  - Apparent brightness vs. intensity

- Weight by spatial frequency to quantify visual impact of noise

White noise
SNR = 10 dB

Blue (highpass) noise
SNR = 10 dB
Sensitivity depends on angular frequency subtended at eye

Compute angular frequency from image size (pixels), printed image size (mm), viewing distance (mm)

At Nyquist frequency

\[ f_a = \frac{N \pi d}{360l} \text{ cycles/degree} \]
CONTRAST SENSITIVITY (CSF)

- Minimum contrast to distinguish sine grating from uniform field
- Model [Mannos & Sakrison 1974]
- Modification [Mitsa & Varkur 1993]

\[ CSF = 2.6 \left( 0.02 + 0.1f_a \right) e^{- (0.1f_a)^{1.1}} \]

- Orientation-independent
**WEIGHTED SNR METRIC**

- Include orientation (angular dependence) in CSF [Sullivan, Miller & Pios 1993]

- Compute weighted signal-to-noise ratio between original image \( x \) and processed image \( y \)

\[
WSNR = 10 \log_{10} \left( \frac{\sum |\text{CSF} \times X(u, v)|^2}{\sum |\text{CSF} \times (X(u, v) - Y(u, v))|^2} \right)
\]
- WSNR is a noise metric
  - Difference (residual) between original and processed images must be noise
  - Model, compensate for other distortions
  - Measure correlation of residual and original; for accuracy, $C_{RI} < 0.020$

Original → Processed → Modeled

- Subtract
- Correlated $C_{RI} = 0.76$
- Uncorrelated $C_{RI} < 0.001$
WSNR vs. CORRELATION

- WSNR increasingly inaccurate as correlation increases
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ERROR DIFFUSION

- 2-D delta-sigma modulator
- Noise shaping feedback coder

\[ x(i, j) \xrightarrow{+} e(i, j) \xrightarrow{H(z)} x'(i, j) \xrightarrow{-} y(i, j) \]

- Error filter

\[
\begin{array}{|c|c|c|}
\hline
3/16 & 5/16 & 1/16 \\
\hline
\end{array}
\]

- Raster scan order

\[ P = \text{Past} \]
\[ F = \text{Future} \]

- Serpentine scan also used
ERROR DIFFUSION (contd.)

- Quantizer
  \[ y(i, j) = \begin{cases} 
  0, & x'(i, j) < 0.5 \\
  1, & x'(i, j) \geq 0.5 
\end{cases} \]

- Governing equations
  \[ e(i, j) = y(i, j) - x'(i, j) \]
  \[ x'(i, j) = x(i, j) - h(i, j) \ast e(i, j) \]

- Non-linearity difficult to analyze
- Linearize quantizer
  [Kite, Evans, Bovik & Sculley 1997]

- Separate signal and noise paths
  [Ardalan & Paulos 1987]
LINEAR GAIN MODEL

- Quantization error correlated with input [Knox 1992]

Floyd-Steinberg Jarvis, Judice & Ninke

- Least squares fit of quantizer input to output defines signal gain

\[ K_s = \frac{E[|x'(i, j)|]}{2E[x'(i, j)^2]} \]

- Signal gain: \( K_s \approx \text{constant} \)
- Noise gain: \( K_n = 1 \)
GAIN MODEL PREDICTIONS

- Noise transfer function (NTF)
  \[ NTF = 1 - H(z) \]

- Signal transfer function (STF)
  \[ STF = \frac{K_s}{1 + (K_s - 1)H(z)} \]
Efficient method of adjusting sharpness [Eschbach & Knox 1991]

Equivalent circuit: pre-filter

\[ G(z) = 1 + L (1 - H(z)) \]
If \( L = \frac{1 - K_s}{K_s} \) then \( STF = 1 \) (flat)

Original image  
Unsharpened halftone

Jarvis halftone  
Residual
Objective Noise Metric

To find WSNR

- Compute signal gain $K_s$, or use average
- Generate unsharpened halftone using modified error diffusion
- Compute WSNR of unsharpened halftone relative to original image
OBJECTIVE TONALITY METRIC

- Limit cycles cause visual ‘worm’ artifacts [Fan & Eschbach 1994]
- Larger filters and serpentine scan result in lower tonality
- Define tonality metric
  - Measure total distortion of sine grating
    \[
    T = \left[ \frac{1}{Y(e^{j\omega_f})Y^*(e^{j\omega_f})} \sum_{\omega \in \{\omega_d\}} Y(e^{j\omega})Y^*(e^{j\omega}) \right]^{\frac{1}{2}}
    \]
  - Average T over grating frequencies
- Agrees with visual results
  - Correct ordering of error filters
  - Serpentine scan less tonal
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INVERSE HALFTONING

- Attempt to recover grayscale images from halftones
- Applications
  - Digital copiers
  - Scanner software
- Many approaches:
  - Bayesian estimation
    [Schweizer & Stevenson 1993]
  - Vector quantization
    [Ting & Riskin 1994]
  - Projection onto convex sets
    [Hein & Zakhor 1995]
  - Lowpass smoothing and nonlinear filtering [Wong 1995]
  - Wavelet denoising
    [Xiong, Orchard & Ramchandran 1997]
- Most are iterative and slow
- Best results from wavelet scheme
PROPOSED METHOD

- Apply anisotropic diffusion [Kite, Damera-Venkata, Evans & Bovik 1998]
  - Estimate image gradients
  - Compute diffusion coefficient
  - Smooth within areas, preserve edges

- Unique environment
  - Highpass noise, SNR \( \approx 3 \) dB
  - Tonal

- Solution
  - Specialized gradient estimator
  - Correlate estimate across scales [Mallat & Zhong 1992]
  - Separable—smooth parallel to edges

- Local operations
  - Low memory requirement
  - Low computational cost
- **Estimate gradients at two scales**
  - $7 \times 7$, $5 \times 5$ FIR filters
  - Integer additions only
- **Correlate gradients across scales**
  - 5 dB improvement in gradient SNR
- **Construct parametric smoother**
  - $7 \times 7$ separable FIR filter
  - Family optimized for halftones
  - Quantized integer coefficients
INVERSE HALFTONE RESULTS

Original image

Proposed method

Halftone

Wavelet method
INVERSE HALFTONING MODEL

- Forward/inverse halftoning system blurs image and adds noise
- Model inverse halftoning
  - Compute unsharpened halftone
  - Inverse halftone; save filter parameters at each pixel
  - Filter original image using saved filters
- Typical correlation
  - Inverse halftone: $C_{RI} = 0.32$
  - Model inverse halftone: $C_{RI} = 0.01$

Inverse halftone  Modeled  Residual (×4)
INVERSE HALFTONE QUALITY

- Compute WSNR
- Compute effective transfer function
  - Divide FFT of model inverse halftone by FFT of original image
  - Radially average over annuli

![Graph showing the magnitude of four different images (lena, peppers, barbara) against radially averaged radial frequency](image-url)
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Halftone conversion, manipulation

Assume input and output are error diffused halftones

Fixed lowpass inverse halftoning filter, compromise cut-off frequency
- Noise leakage masked by halftoning
- Blurring correctable by modified error diffusion
- Computationally efficient

Use linear gain model to design $L$ for flat response

Use approximation for digital frequency: $e^{j\omega} \approx 1 + j\omega - \omega^2 / 2$
REHALFTONING RESULTS

Original image

Rehalftone

Signal transfer function
INTERPOLATION

- Image resizing
- Different methods (increasing cost)
  - Nearest neighbor
  - Bilinear
  - Bicubic, cubic splines, lowpass filtering
- Nearest neighbor, bilinear methods
  - Low computational cost
  - Artifacts masked by quantization noise in halftone
  - Blurring correctable by modified error diffusion
- Examine $\times 2$ interpolation; method applies to any scaling factor
- Design $L$ for flat transfer function using linear gain model
- $L$ constant for given interpolation scheme
INTERPOLATION RESULTS

Nearest neighbor ×2

Bilinear ×2

Transfer function
$L = -0.0105$

Transfer function
$L = 0.340$
CONTRIBUTIONS

- Visual quality metrics for forward and inverse halftones
  - Restriction on correlation for accuracy of WSNR metric
- Linear gain model of error diffusion
  - Accuracy of model established
  - Tonality metric for artifacts
  - Link between filter gain and signal gain
- Inverse halftoning
  - New efficient method, suitable for hardware and embedded software
  - Model for inverse halftoning
  - Quality metrics for inverse halftones
- Rehalftoning and interpolation
  - Efficient algorithms
  - Verifies validity of linear gain model