# Delta-Sigma Analysis Applied to Digital Halftoning

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# The Problem

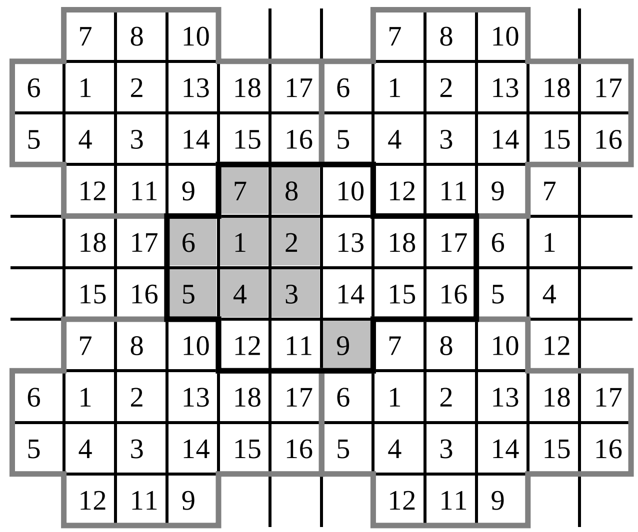
- How do we reproduce grayscale imagery on inherently binary media?
- Some printers, facsimile machines, visual displays are monochrome only
- Some devices are unreliable at placing isolated dots
- Ink spread nonlinearity is a problem

# The Solution: Halftoning

- Create the illusion of grayscale by varying average dot density in local area
- Darker areas have more activated dots (printers) or fewer bright pixels (displays)
- Sacrifice spatial resolution for grayscale resolution, unless sufficient oversampling is used (as it may be for printers)

# The Classical Screen

- Image is compared to a tessellated threshold screen of fixed size
- Pixels greater than the threshold are activated
- Trade-off between spatial and graylevel resolution
- Parallel implementation




19-level clustered-dot screen

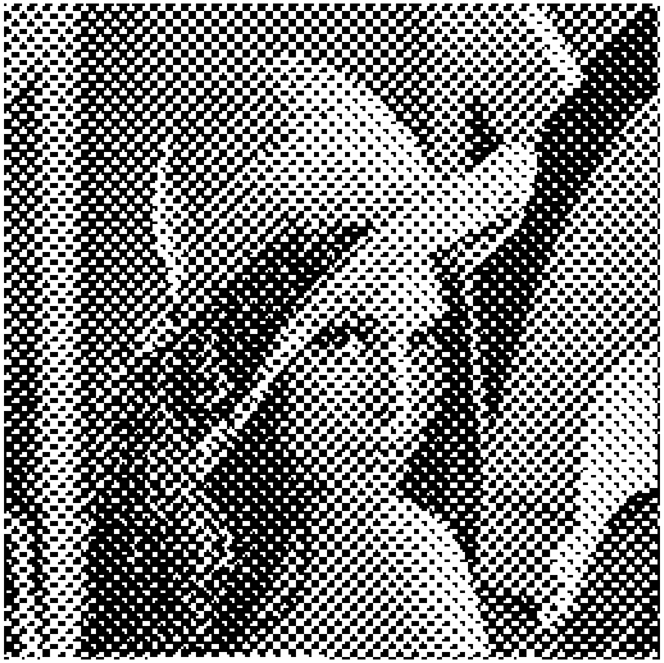
# The Classical Screen II

- Clustered-dot dither generates large clumps
- Visual displays can support isolated dots
- Dispersed-dot screen is more appealing
- Implementation does not change

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 9 | 5 | 12 | 8 | 9 | 5 | 12 | 8 | 9 | 5 | |
| | 3 | 13 | 2 | 16 | 3 | 13 | 2 | 16 | 3 | 13 | |
| 6 | 11 | 7 | 10 | 6 | 11 | 7 | 10 | 6 | 11 | 7 | 10 |
| 14 | 1 | 15 | 4 | 14 | 1 | 15 | 4 | 14 | 1 | 15 | 4 |
| 8 | 9 | 5 | 12 | 8 | 9 | 5 | 12 | 8 | 9 | 5 | 12 |
| 16 | 3 | 13 | 2 | 16 | 3 | 13 | 2 | 16 | 3 | 13 | 2 |
| | 11 | 7 | 10 | 6 | 11 | 7 | 10 | 6 | 11 | 7 | |
| | 1 | 15 | 4 | 14 | 1 | 15 | 4 | 14 | 1 | 15 | |

17-level dispersed-dot screen

# Classical Screen Results

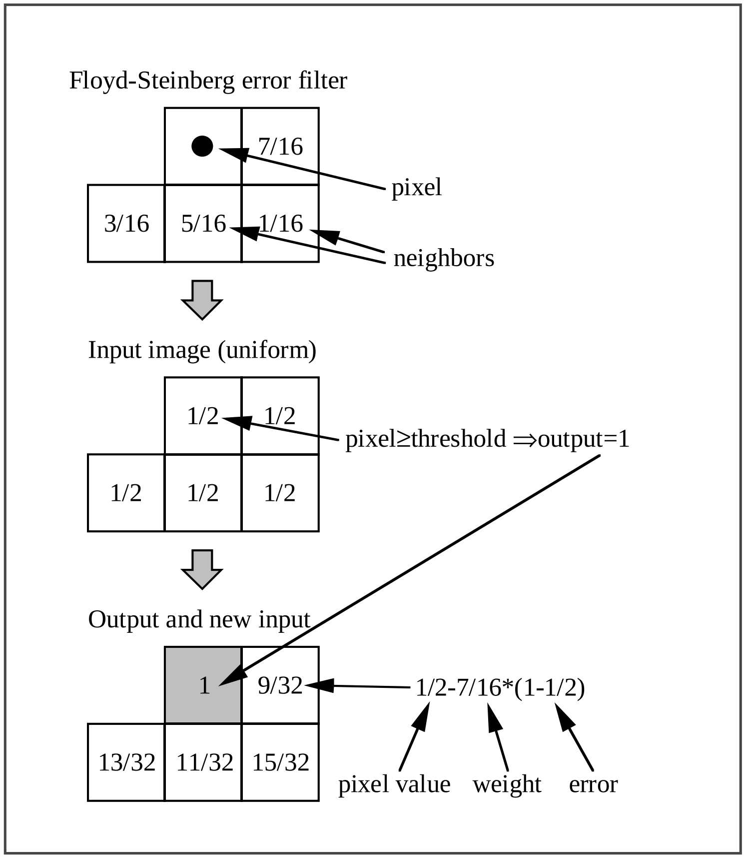


Clustered-dot screen

Dispersed-dot screen

# Error Diffusion

- Scan image (usually in raster form)
- Threshold pixel
- Distribute threshold error amongst neighbors
- Error distribution determines quality and ease of computation



# Error Diffusion Results

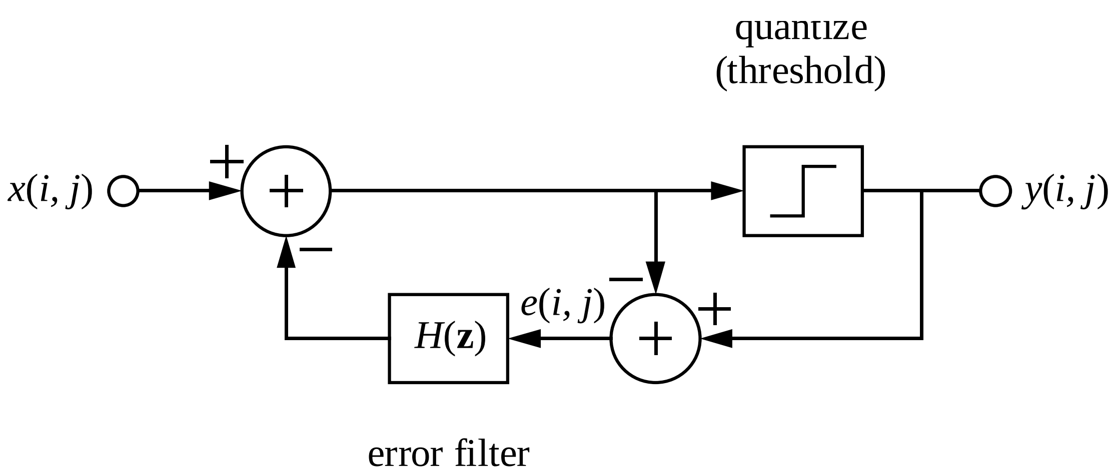
Floyd-Steinberg error filter

Jarvis error filter

# Error Diffusion: Analysis

- Why does it work? Floyd and Steinberg used trial and error to choose filter weights
- Why do the larger filters (Jarvis, Stucki) sharpen the image?
- Can we predict visual quality given the error filter?
- Can we define a metric for halftone quality?

# Noise-Shaping Feedback Coder




- A form of DSM used for wordlength reduction
- Error diffusion is identical, with a two-dimensional *H*(*z*)

# Analysis of the NSFC

- Linear analysis predicts

$$Y(z) = X(z) + [1 - H(z)]N(z)$$

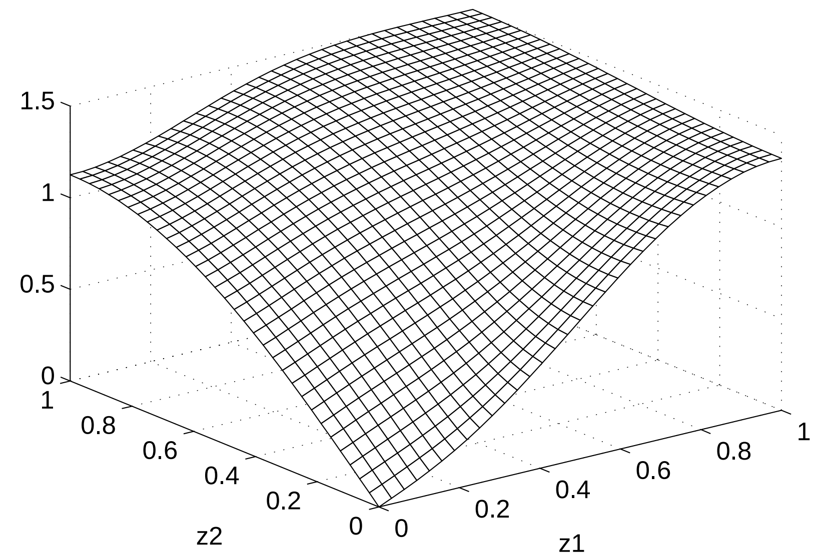
- Signal (image) passes unchanged
- Quantization error (noise) is shaped as $1 - H(z)$
- All common filters have a zero at DC

# Floyd-Steinberg as a NSFC

- Noise transfer function:

$$NTF(z_1, z_2) = 1 - \tfrac{1}{16}(z_1^{-1} z_2^{-1} + 5z_2^{-1} + 3z_1 z_2^{-1} + 7z_1^{-1})$$

- Zero at DC
- Noise is shaped to higher frequencies
- Theory agrees well with measurement




NTF of Floyd-Steinberg scheme

# Comments on NSFC analysis

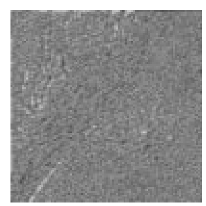
- Linear analysis predicts the correct NTF, but not the form of the noise (distortion)
- As in audio, quantization error is tonal
- Tonality perhaps less of a problem in images than in audio
- More investigation needed into the nature of quantization error for various image models

# Edge Sharpening

- Floyd-Steinberg neutral, but larger filters tend to sharpen the image
- Linear analysis shows that STF = 1
- Sharpening must come from quantization error
- We are investigating!

Jarvis filter output

Quantization error



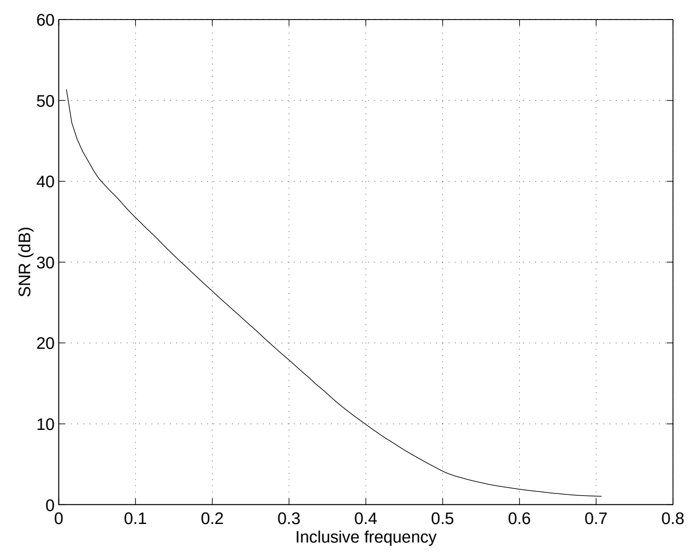
Difference between error and linear fit to output

# Quality of Halftoned Images

- ‘Blue noise’ characteristic is desirable
- Edge sharpening is possibly desirable, but should be applicable separately
- Linear NSFC analysis does not predict edge sharpening; other models may
- Signal-to-noise ratio vs. radial frequency is a strong indicator of visual quality

# SNR Metric

- Noise-shaping produces an image whose SNR decreases with frequency
- Apart from sharpening, halftoning artifacts are essentially noise-like
- A plot of SNR against inclusive frequency is useful




Floyd-Steinberg SNR vs. frequency

# Conclusions

- Error diffusion is still an open problem
- Techniques from delta-sigma analysis are directly applicable and should be used
- More understanding of the nature of quantization error in images is needed
- SNR metric appears more valid here than in other fields (e.g., image compression)