1. Find an FIR filter $H(z)$ which can be written in the form $A(e^{j\omega})e^{j\phi(\omega)}$, where the amplitude response is given by $A(e^{j\omega}) = \sin(2\omega)$.

Using Euler's identity, we have

$$\sin(2\omega) = \frac{e^{j2\omega} - e^{-j2\omega}}{2j} \quad . \tag{1}$$

We are free to choose any phase function, so let's choose one that, when multiplied by (1), will give real coefficients. That is, we choose

$$\begin{aligned} e^{j\phi} &= j \\ \therefore \quad \phi &= \pi/2 \ , \end{aligned} \tag{2}$$

where we understand that we have chosen only one of an infinite series of possible angles that satisfy the criterion. The filter now has the form

$$\begin{aligned} H(e^{j\omega}) &= \tfrac{1}{2}\left[e^{2j\omega} - e^{-2j\omega}\right] \\ \therefore \quad H(z) &= \tfrac{1}{2}\left[z^{2} - z^{-2}\right] \ . \end{aligned} \tag{3}$$

We can make this causal if we desire by multiplying by z^{-2} to give

$$H(z) = \tfrac{1}{2}\left[1 - z^{-4}\right] \ . \tag{4}$$

2. Bilinear transformation.

From the information about the pole, zero, and DC gain, we can immediately write

$$G_a(s) = \frac{1}{3}\left[\frac{s+2}{s+\frac{2}{3}}\right] \ . \tag{5}$$

We apply the bilinear transformation with $\alpha = \frac{2}{T}$:

$$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right], \tag{6}$$

to arrive at

$$G_d(z) = \frac{1}{3}\left[\frac{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+2}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+\frac{2}{3}}\right] \ . \tag{7}$$

Now multiply top and bottom by $(1+z^{-1})$ and collect terms to arrive at

$$G_d(z) = \frac{(T+1) + z^{-1}(T-1)}{(T+3) + z^{-1}(T-3)} \quad . \tag{8}$$

Comparing (8) with the expression given in the assignment, namely

$$G_d(z) = \frac{K}{1-\beta z^{-1}} \quad , \tag{9}$$

we conclude by inspection that

$$\begin{aligned} T &= 1; \\ K &= \tfrac{1}{2}; \\ \beta &= \tfrac{1}{2} \quad . \end{aligned} \tag{10}$$