EE 381K **PROBLEM OF THE WEEK #1** **SPRING 1998**

(a) Show that the response $y(n)$ to input $x(n)$ of a system with unit step response $s(n)$ can be expressed $y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]x(n-k)$**.**

The unit impulse can be expressed in terms of unit steps:

$$\delta(n) = u(n) - u(n-1) \tag{1}$$

and hence the unit pulse response can be expressed in terms of the response to a unit step (since the system is LTI):

$$\begin{aligned} h(n) &= h(n) * \delta(n) \\ &= h(n) * [u(n) - u(n-1)] \\ &= h(n) * u(n) - h(n) * u(n-1) \quad \text{(linearity)} \\ &= s(n) - s(n-1) \quad . \qquad \text{(time invariance)} \end{aligned} \tag{2}$$

Since the response of the system is given by the convolution of the impulse response of the system with the input, and the impulse response is given by (2), we obtain the result

$$\begin{aligned} y(n) &= h(n) * x(n) \\ &= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]x(n-k) \quad . \end{aligned} \tag{3}$$

(b) Show that $y(n)$ can also be expressed $y(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]s(n-k)$**.**

We start with the result from (a):

$$y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]x(n-k)$$

and rearrange to give

$$y(n) = \sum_{k=-\infty}^{\infty} s(k)x(n-k) - \sum_{k=-\infty}^{\infty} s(k-1)x(n-k) \quad . \tag{4}$$

Since $a * b = b * a$, we rearrange (4) to give

$$y(n) = \sum_{k=-\infty}^{\infty} s(n-k)x(k) - \sum_{k=-\infty}^{\infty} s(n-k)x(k-1) \tag{5}$$

and recombine into a single sum to arrive at the result

$$y(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]s(n-k) \quad .$$