COMPUTER HOMEWORK THREE

We begin by writing down the transfer function of the analog filter by inspection:

$$T = \frac{V_{OUT}}{V_{IN}} = \frac{R}{R + j\omega L + 1/j\omega C} = \frac{sCR}{1 + sCR + s^2 LC} \ , \tag{1}$$

where we have made the substitution $s = j\omega$. The function has a maximum at the center frequency; this occurs when the imaginary part of the denominator is zero (since the real part is fixed). This is easier to see with the first $(j\omega)$ expression of (1), rather than the second (s). We therefore have

$$\begin{aligned} \omega_0 L &= \tfrac{1}{\omega_0 C} \\ \therefore \quad \omega_0 &= \tfrac{1}{\sqrt{LC}} \ . \end{aligned} \tag{2}$$

Since the imaginary part drops out at resonance, we see from (1) that $T = 1$. At the 3 dB down points, the magnitude of the transfer function is $1/\sqrt{2}$; in other words, the real and imaginary parts of T have equal size. Let us denote the upper 3 dB down point $\omega_0 + \Delta\omega$; then we have

$$(\omega_0 + \Delta\omega)L - \frac{1}{(\omega_0 + \Delta\omega)C} = R \ . \tag{3}$$

To avoid solving the quadratic, we expand the second term on the left hand side as a Taylor series:

$$\begin{aligned} \frac{1}{(\omega_0 + \Delta\omega)C} &= \frac{1}{C\omega_0}(1 + \tfrac{\Delta\omega}{\omega_0})^{-1} \\ &\approx \frac{1}{C\omega_0}(1 - \tfrac{\Delta\omega}{\omega_0}) \\ &= \frac{1}{C\omega_0} - L\Delta\omega \ , \end{aligned} \tag{4}$$

where the approximation comes from assuming that $\Delta\omega << \omega_0$ (Q is high), and we have substituted for ω_0 in the last line. We now solve (3):

$$\Delta\omega = \tfrac{R}{2L} \ , \tag{5}$$

and therefore the bandwidth is $2\Delta\omega = R/L$. Thus $Q = \omega_0 / \Delta\omega = (1/R)\sqrt{L/C}$. Now we apply the bilinear transformation to (1):

$$s = \alpha\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right] , \tag{6}$$

to obtain

$$T = \frac{\alpha CR\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}{1 + \alpha CR\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + \alpha^2 LC\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2} \ . \tag{7}$$

We now tidy up by multiplying top and bottom by $[1 + z^{-1}]$ and collect terms to obtain

$$T = \frac{\frac{\alpha CR}{\beta}[1 - z^{-2}]}{1 + 2\,\frac{(1-\alpha^2 LC)}{\beta} z^{-1} + \frac{(1-\alpha CR-\alpha^2 LC)}{\beta} z^{-2}} \ , \tag{8}$$

where $\beta = 1 + \alpha CR + \alpha^2 LC$. We choose $\alpha = 2/T_s$ and pre-warp the center frequencies according to $\omega_d = 2 f_s \tan\left(\frac{\omega_a}{2 f_s}\right)$, where ω_d and ω_a are the digital and analog frequencies in radians per second, respectively, and $f_s = 1/T_s$ is the sampling frequency. The code is shown next.

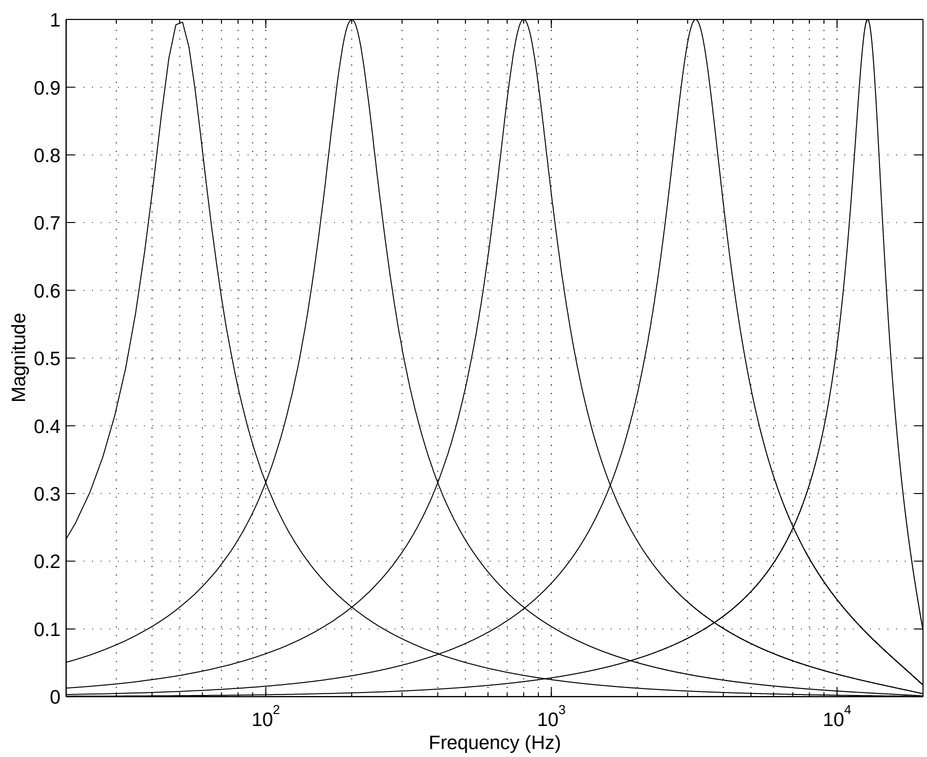
```
% Set up the problem

fs=44100;                                  % sampling frequency
ts=1/fs;                                   % sampling period
al=2/ts;                                   % bilinear alpha...
al2=al*al;                                 % ...and alpha squared
c=1e-6;                                    % choose a capacitor
q=2;                                       % filter Q


% Compute the numerators and denominators of the digital filters

cf=[50 200 800 3200 12800]*2*pi;           % center frequencies
w0=2*fs*tan(cf/(2*fs))';                   % prewarped
l=1./(c*w0.^2);                            % inductors
r=sqrt(l/c)/q;                             % resistors
beta=1+al*c*r+al2*l*c;
b=al*c*r./beta*[1 0 -1];
a=[ones(5,1) 2*(1-al2*l*c)./beta (1-al*c*r+al2*l*c)./beta];
```

The frequency responses of the filters look like this:



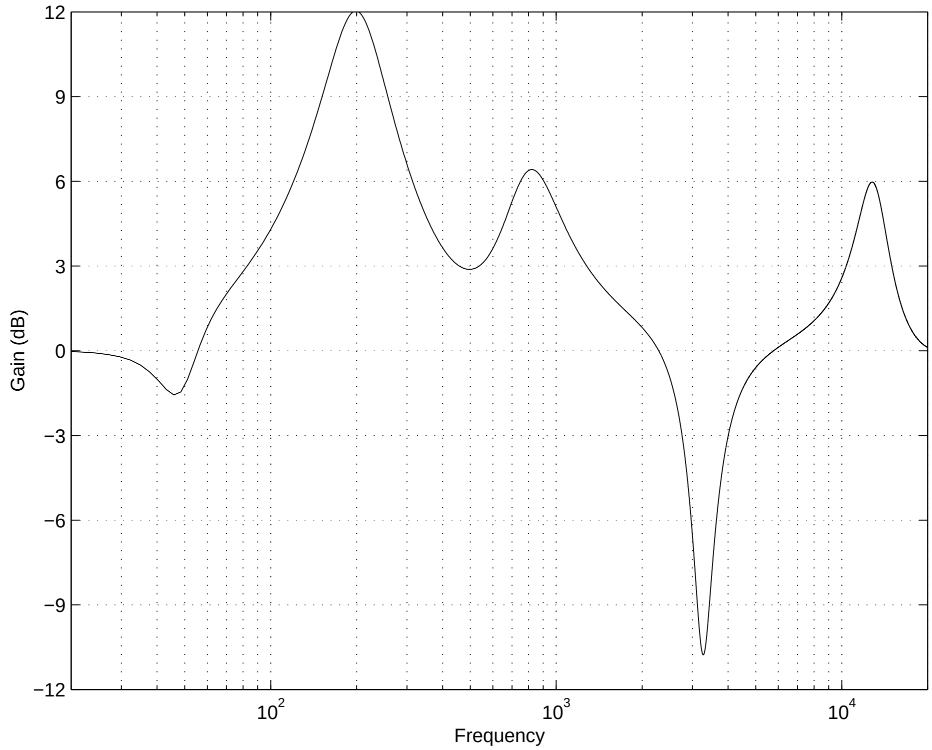
Now we compute the frequency response of the system for the given filter settings:

```
levdB=[-3 12 6 -12 6];                     % gain in each band
K=10.^(levdB/20)-1;                        % compute gains

ti=1:16384;                                % generate chirp
x=sin(ti.*ti/16384*pi/2);

y=x;                                       % straight through output
for t=1:5                                  % add each filter output
  y=y+filter(b(t,:),a(t,:),x)*K(t);
end
h=fft(y)./fft(x);                          % compute transfer function
```

which looks like this:



We see that the equalizer is indeed close to being graphic. Each band peaks at approximately the correct gain, although the interaction between bands (due to low filter Q) means that the gains deviate slightly from the specification. If more bands are required, the Q should be increased to reduce filter interaction.