

1. There are many ways of writing the `stp.m` function. Here is one of them.

```
function [x,y,z]=stp(in)
%STP Return sine, pulse and triangle waves.
% [X, Y, Z] = STP(IN) computes the sine of input vector IN (X),
% a triangle wave with peaks at the peaks of the sine wave (Y).
% a pulse wave whose value is 1 at the sine wave peaks, and 0 else (Z).

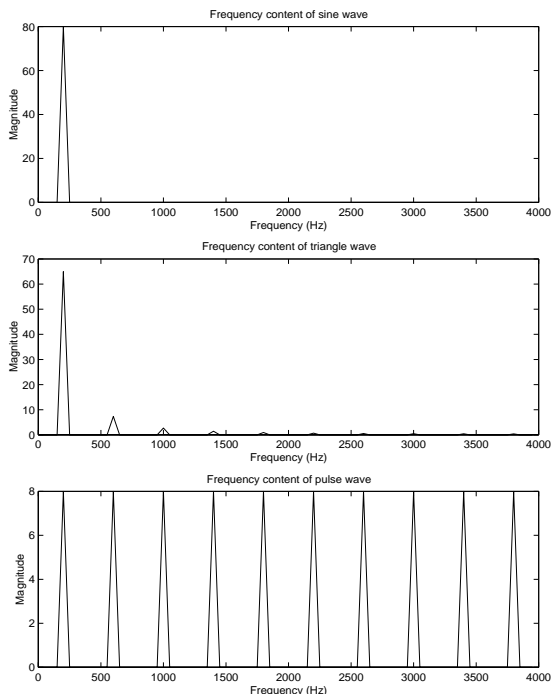
% T Kite February 1997, 1998

l=length(in);
x=sin(in);
y=asin(x)*2/pi;
xpad=[0 abs(x) 0];
z=sign(x) .* (xpad(2:l+1) >= xpad(3:l+2) & xpad(2:l+1) > xpad(1:l));
```

First, we define the form of the function and include a `help` file. The sine vector is computed trivially. The triangle vector is computed by finding the `asin` (arcsine) of the sine vector; this returns values between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, and is therefore multiplied by $\frac{2}{\pi}$ to normalise it. The pulse wave is computed by using a conditional to isolate points which are higher than their neighbors. The `>=` conditional ensures that a pulse is produced for 'flat top' sine waves, where two successive samples have the maximum value.

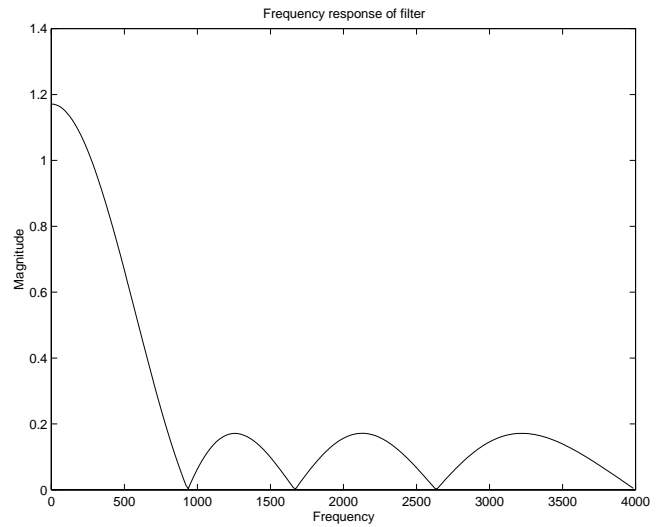
2. The waves are generated and plotted as follows. The plot was shown in the assignment.

```
fs=8000;
ts=1/fs;
t=0:ts:0.02-ts;
[si,tr,pul]=spt(2*pi*t*200);
plot(t*1000,si,'--',t*1000,tr,'-.',t*1000,pul,'-')
axis([min(t*1000) max(t*1000) -1 1])
title 'Output of spt function'
xlabel 'Time (ms)'; ylabel 'Amplitude'
```

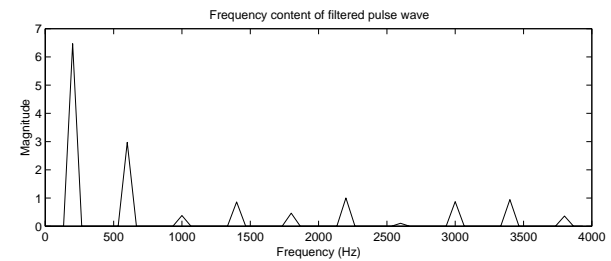
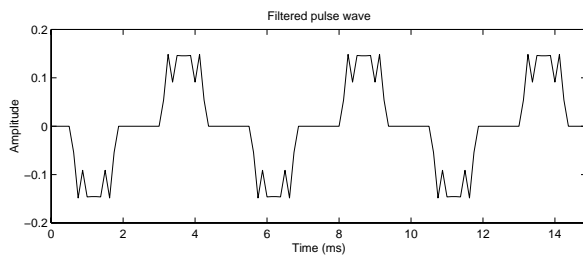
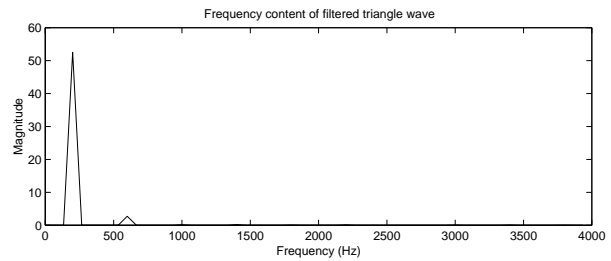
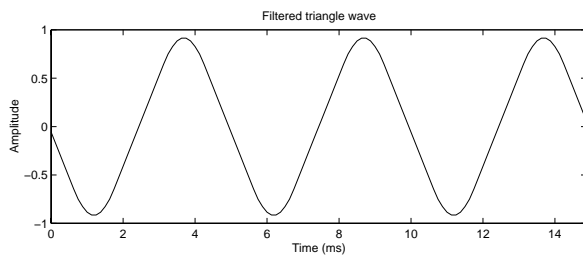
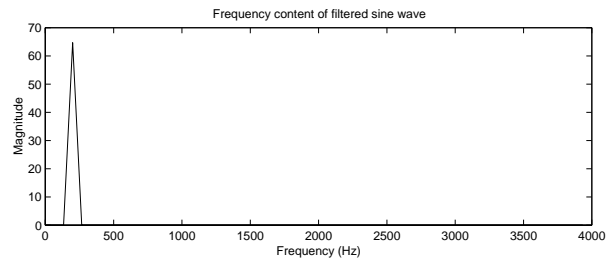
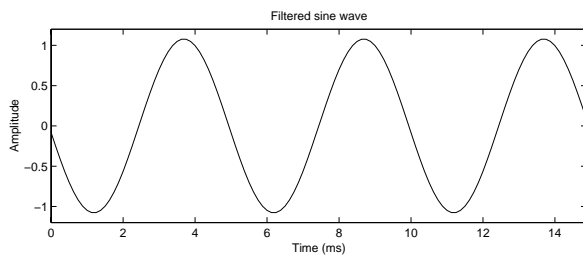


3. The frequency content of the three signals is shown at the left. Note that if 80 points are not used in `freqz`, you will get a different (and incorrect) result. The sine wave spectrum consists of a single delta function at 200 Hz. It only appears to have width because MATLAB joins points on a graph; to avoid that, use `stem` instead of `plot`. The triangle wave consists of a fundamental at 200 Hz and odd harmonics which decay approximately as $\frac{1}{n^2}$. There are no even harmonics because the wave possesses symmetry about the x -axis. (It is nothing to do with it being an odd function, as some asserted.) The pulse wave consists of the 200 Hz fundamental and equally-sized harmonics; this stems from the fact that the Fourier transform of a pulse train is also a pulse train. Because the time domain functions are periodic, their Fourier transforms consist of discrete spectral lines; they are distributions, not functions. This is true whether the system is discrete-time or not.

4. The frequency response of the filter is shown at the right. It is a lowpass filter with a cutoff frequency (-3 dB point) of around 470 Hz. Although the filter is obviously not 'ideal', it is in fact the best filter of that length (10 coefficients), in the sense that the transition band is as narrow as possible for the given passband ripple and minimum stopband attenuation. It's a Parks-McClellan filter, also called an equiripple filter. It is FIR and therefore consists only of zeros. Some of these zeros lie on the unit circle. Its frequency response is definitely NOT a sinc function; a look at its impulse response confirms this, since it is not a top hat function. You will see many more filters like this throughout the course.



5, 6. The time domain signals are on the left; the frequency domain on the right.



The sine wave is unaffected by the filter, apart from a slight gain due to the fact that the filter gain is not exactly unity in the passband. Because the filter is a linear system, it cannot introduce extra frequencies; this would imply distortion. (Another way of saying this is that the sine wave is an eigenfunction of the system.) The triangle wave has its harmonics attenuated, and therefore starts to resemble the sine wave in the time domain, since its fundamental now dominates. Its peak-to-peak height is less than it was, because the power in the fundamental for the triangle wave is lower than that of a pure sine wave of the same peak-to-peak height. However, for a square wave, this is not true. A square wave with a peak height of unity turns into a sine wave of peak height $\frac{4}{\pi}$ when its harmonics are stripped! It's therefore incorrect to say that the peak-to-peak height is decreased by filtering—sometimes it's increased. The pulse wave has also had its harmonics attenuated, giving it a somewhat square appearance. Repeated applications of the filter will eventually result in a reasonably pure sine wave at the output.