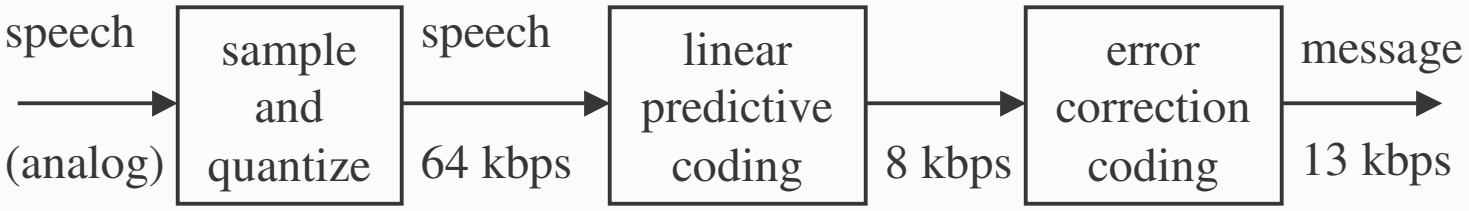
Spread Spectrum Communications

- **Prof. Brian L. Evans**
- **Dept. of Electrical and Computer Engineering**
- **The University of Texas at Austin**

Lecture 21

Sprint PCS

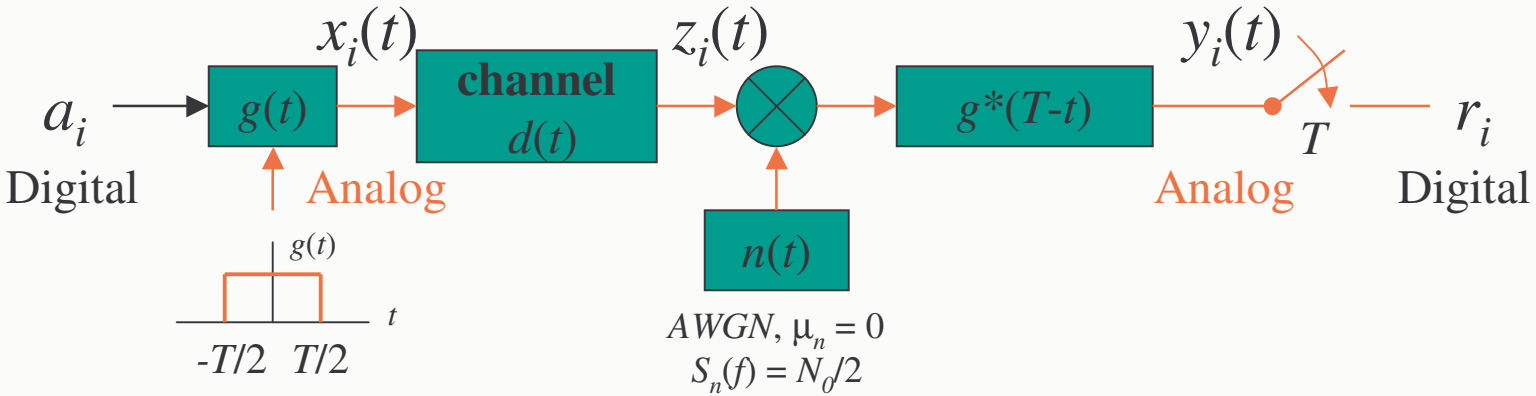
- **Speech compression and coding in transmitter**



- **Transmit message signal using spread system**
  - For every message bit, generate $L = 64$ bits of a pseudo noise sequence with user's code as initial value
  - Send and receive the L bits bit-by-bit using 2-PAM on a radio frequency carrier of 1.9 GHz
- **Speech decompression and decoding in receiver**

Matched Filtering for 2-PAM

- **Transmit equally probable bits, $a_i \in \{-1, 1\}$**



- **Send single pulse, ignore noise $n(t)$, and assume that channel $d(t)$ has been equalized**

$$y_i(t) = a_i \; g(t) * g^*(T-t)$$

where $g^*(T-t)$ is the matched filter

$$r_i(t) = y_i(t)\big|_{t=T} = a_i \int g(\tau)\, g^*(\tau)\, d\tau = a_i \; E_b$$

where E_b is the energy per bit. Hence,

$$r_i(t) \in \{-E_b, E_b\}$$

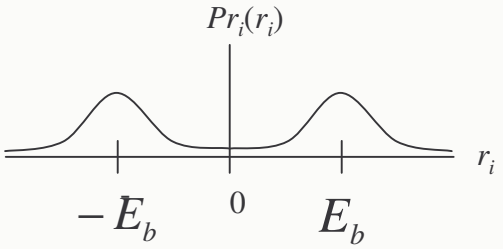
Probability of Error for 2-PAM

- **General case: one bit in isolation down channel**

$$y_i(t) = a_i g(t) * g^*(T-t) + n(t) * g^*(T-t)$$

$$r_i(t) = y_i(t)\big|_{t=T} = a_i \int_{-\infty}^{\infty} g(\tau) g^*(\tau) d\tau + n(t) * g^*(T-t)\big|_{t=T}$$

- **Since $a_i \in \{-1, 1\}$, r_i clusters around $+E_b$ and $-E_b$**



$$E_b = \int_{-\infty}^{\infty} g(\tau) g^*(\tau) d\tau = \int_{-\infty}^{\infty} |g(\tau)|^2 d\tau$$

$$\sigma_r^2 = \frac{N_0}{2} \int_{-\infty}^{\infty} |G(f)|^2 df = \frac{N_0}{2} \int_{-\infty}^{\infty} |g(\tau)|^2 d\tau = \frac{N_0}{2} E_b$$

$$\text{SNR}_{ri} = \frac{E_b^2}{\sigma_r^2} = \frac{2E_b}{N_0}$$

  - Determine which bit was sent: threshold at 0
  - Bit errors due to noise (when tails of Gaussians overlap)
  - For chain of bits, assume each bit is independent

Probability of Error for 2-PAM

- **Probability that tail of r_i centered at $+E_b$ is positive and tail of r_i centered at $-E_b$ is negative**

$$P_e = P(e|1)P(1) + P(e|-1)P(-1)$$

$$= \left[1 - Q\left(-\frac{E_b}{\sqrt{\frac{N_0}{2}E_b}}\right)\right]P(1) + \left[Q\left(\frac{E_b}{\sqrt{\frac{N_0}{2}E_b}}\right)\right]P(-1)$$

$$= \left[Q\left(\frac{E_b}{\sqrt{\frac{N_0}{2}E_b}}\right)\right]P(1) + \left[Q\left(\frac{E_b}{\sqrt{\frac{N_0}{2}E_b}}\right)\right]P(-1)$$

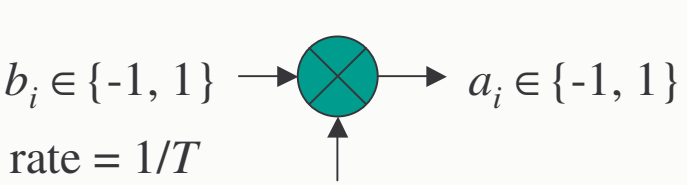
Because $P(1) = P(-1) = 1/2$,

$$P_e = Q\left(\frac{E_b}{\sqrt{\frac{N_0}{2}E_b}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

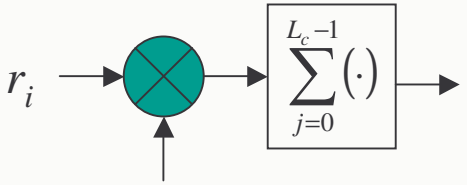
where $Q(t) = \frac{1}{\sqrt{2\pi}}\int_t^\infty e^{-\frac{x^2}{2}}\,dx$

Spread Spectrum Communications

- **Enhance modulator/demodulator to spread spectrum to make it look more like noise and convert it from narrowband to a wider band**



Pre-processing (digital)



Post-processing (digital)

$T/T_c = L_c$ = number of chips

c_{ij} is pseudo-noise sequence generated by Galois Field (GF) binary polynomials

c_{ij} are known in advance and must be synchronized

Spread Spectrum Communications

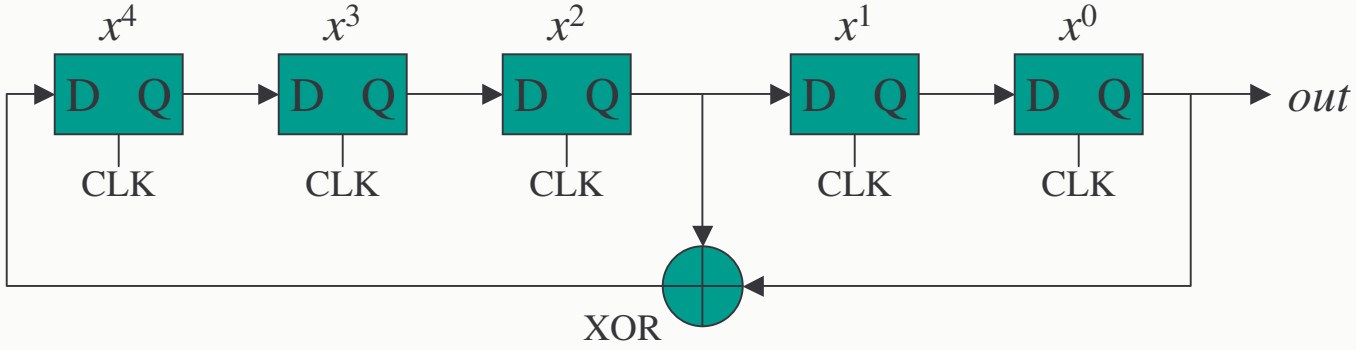
- ***g*(*t*) scaled in time by L_c : system has same P_e**
- ***GF*(*N*) generates sequences of *N*-1 bits**

  Almost uncorrelated noise (pseudo-noise): $\sum_{j=1}^{31} c_j c_{j+n} = \begin{cases} 31 & \text{for } n = 0 \\ -1 & \text{for } n \neq 0 \end{cases}$

  Polynomials and polynomial variable take binary values of 0 and 1

  Fast hardware implementations using *D* flip-flops

- ***GF*(32); $32 = 2^5$; $p(x) = x^5 + x^2 + 1$. Note $x^0 = 1$.**



CDMA QualComm Standard

- **800 & 1900 MHz bands**
- **Each user**
  - Has unique spreading code
  - Receives from 2 closest base stations (handoff is robust)
- **Reverse link (from users to base station)**
  - Walsh codes for *M*-ary mod
  - Power adjust in user transmission: base receiver sees all users at equal power
- **Forward link (base station to user)**
  - Transmitter uses Walsh codes for each user
  - User signals orthogonal: requires each user to be synchronized to xmitter, but not to each other
  - Transmission power increases as number of users increase

Other Applications of PN Sequences

- **Training for wireline transceivers (voiceband, ADSL, etc.)**
- **From search of "pseudo noise sequence" in an on-line database**

  Echo cancellers

  Pulse compression sonar

  Analysis of tape recorders
- **IEEE online database**

  http://ieeexplore.ieee.org/Xplore/DynWel.jsp